1(i) $\mathrm{f}(-x)=\dfrac{1}{\mathrm{e}^{-x}+\mathrm{e}^{-(-x)}+2}$ $= \mathrm{f}(x)$, [$\Rightarrow$ f is even *] Symmetrical about O*y*	M1 A1 B1 [3]	substituting $-x$ for x in f(x) condone 'reflection in *y*-axis'	Can imply that $\mathrm{e}^{-(-x)} = \mathrm{e}^x$ from $\mathrm{f}(-x)=\dfrac{1}{\mathrm{e}^{-x}+\mathrm{e}^{x}+2}$ Must mention axis
(ii) $\mathrm{f}'(x)=-(\mathrm{e}^x+\mathrm{e}^{-x}+2)^{-2}(\mathrm{e}^x-\mathrm{e}^{-x})$ or $= \dfrac{(\mathrm{e}^x+\mathrm{e}^{-x}+2).0-(\mathrm{e}^x-\mathrm{e}^{-x})}{(\mathrm{e}^x+\mathrm{e}^{-x}+2)^2}$ $=\dfrac{(\mathrm{e}^{-x}-\mathrm{e}^x)}{(\mathrm{e}^x+\mathrm{e}^{-x}+2)^2}$	B1 M1 A1 [3]	d/d*x* (e^x) = e^x and d/d*x*(e^{-x}) = $-\mathrm{e}^{-x}$ soi chain or quotient rule condone missing bracket on top if correct thereafter o.e. mark final answer	. If differentiating $\dfrac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}$ withhold A1 (unless result in (iii) proved here) e.g. $\dfrac{1}{(\mathrm{e}^x+\mathrm{e}^{-x}+2)^2}\times(\mathrm{e}^{-x}-\mathrm{e}^x)$
(iii) $\mathrm{f}(x)=\dfrac{\mathrm{e}^x}{\mathrm{e}^{2x}+1+2\mathrm{e}^x}$ $=\dfrac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}$ *	M1 A1 [2]	× top and bottom by e^x (correctly) condone e^{x^2} for M1 but not A1 **NB AG**	or $\dfrac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}=\dfrac{\mathrm{e}^x}{\mathrm{e}^{2x}+2\mathrm{e}^x+1}$ M1, $=\dfrac{1}{\mathrm{e}^x+\mathrm{e}^{-x}+2}$ A1 condone no $\mathrm{e}^{2x} = (\mathrm{e}^x)^2$,for both M1 and A1
(iv) $A=\int_0^1\dfrac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}\,\mathrm{d}x$ let $u = \mathrm{e}^x + 1$, du = e^x dx when x = 0, u = 2; when x = 1, u = e + 1 $\Rightarrow$ $A=\int_2^{1+e}\dfrac{1}{u^2}\,\mathrm{d}u$ $=\left[-\dfrac{1}{u}\right]_2^{1+e}$ $=-\dfrac{1}{1+\mathrm{e}}+\dfrac{1}{2}=\dfrac{1}{2}-\dfrac{1}{1+\mathrm{e}}$	B1 M1 A1 M1 A1cao [5]	correct integral and limits $\int\dfrac{1}{u^2}(\mathrm{d}u)$ $\left[-\dfrac{1}{u}\right]$ substituting correct limits (dep 1st M1 and integration) o.e. mark final answer. Must be exact Don't allow e^1.	condone no dx, must use f(x) = $\dfrac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}$. Limits may be implied by subsequent work. If 0.231.. unsupported, allow 1st B1 only or by inspection $\left[\dfrac{k}{e^x+1}\right]$ M1 $\left[-\dfrac{1}{e^x+1}\right]$ A1 upper–lower; 2 and 1+e (or 3.7..)for u, or 0 and 1 for x if substituted back (correctly) e.g. $\dfrac{\mathrm{e}-1}{2(1+\mathrm{e})}$. Can isw 0.231, which may be used as evidence of M1. Can isw numerical ans (e.g. 0.231) but not algebraic errors
(v) Curves intersect when $\mathrm{f}(x)=\dfrac{1}{4}\mathrm{e}^x$ $\Rightarrow$ $(\mathrm{e}^x + 1)^2 = 4$ $\Rightarrow$ e^x = 1 or -3 so as $\mathrm{e}^x > 0$, only one solution $\mathrm{e}^x = 1$ $\Rightarrow x = 0$ when $x = 0$, $y = ¼$	M1 M1 A1 B1 B1 [5]	soi or equivalent quadratic – must be correct getting e^x = 1 and discounting other sol^n x = 0 www (for this value) y = ¼ www (for the x value)	$\dfrac{\mathrm{e}^x}{(\mathrm{e}^x+1)^2}$ *or* $\dfrac{1}{e^x+e^{-x}+2}=\dfrac{1}{4}\mathrm{e}^x$ With e^{2x} or $(\mathrm{e}^x)^2$, condone e^{x^2}, e^0 www e.g. $\mathrm{e}^x = -1$ [or $\mathrm{e}^x + 1 = -2$] not possible www unless verified Do not allow unsupported. A sketch is not sufficient

2 $\int_0^{\pi/6} \cos 3x \, dx = \left[\frac{1}{3}\sin 3x\right]_0^{\pi/6}$ $= \frac{1}{3}\sin\frac{\pi}{2} - 0$ $= 1/3$	M1 B1 A1cao [3]	$k \sin 3x$, $k > 0$, $k \neq 3$ $k = (\pm)1/3$ 0.33 or better	or M1 for $u = 3x \Rightarrow \int \frac{1}{3}\cos u \, du$ condone 90° in limit or M1 for $\left[\frac{1}{3}\sin u\right]$ so: $\sin 3x$: M1B0, $-\sin 3x$: M0B0, $\pm 3\sin 3x$: M0B0, $-1/3 \sin 3x$: M0B1

3(i) $y = x\cos 2x$ $\Rightarrow \frac{dy}{dx} = -2x\sin 2x + \cos 2x$	M1 B1 A1 [3]	product rule $d/dx(\cos 2x) = -2\sin 2x$ oe cao
(ii) $\int x\cos 2x dx = \int x\frac{d}{dx}(\frac{1}{2}\sin 2x)dx$ $= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x dx$ $= \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$	M1 A1 A1ft A1 [4]	parts with $u = x$, $v = ½ \sin 2x$ $+\frac{1}{4}\cos 2x$ cao – must have $+ c$

4(i) Asymptotes when (√)$(2x - x^2) = 0$ $\Rightarrow$ $x(2 - x) = 0$ $\Rightarrow$ $x = 0$ or 2 so $a = 2$ Domain is $0 < x < 2$	M1 A1 B1ft [3]	 or by verification $x > 0$ and $x < 2$, not $\leq$
(ii) $y = (2x - x^2)^{-1/2}$ let $u = 2x - x^2$, $y = u^{-1/2}$ $\Rightarrow$ $dy/du = -\frac{1}{2}u^{-3/2}$, $du/dx = 2 - 2x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -\frac{1}{2}(2x-x^2)^{-3/2}.(2-2x)$ $= \frac{x-1}{(2x-x^2)^{3/2}}$ *	 M1 B1 A1 E1	 chain rule (or within correct quotient rule) $-\frac{1}{2}u^{-3/2}$ or $-\frac{1}{2}(2x-x^2)^{-3/2}$ or $\frac{1}{2}(2x-x^2)^{-1/2}$ in quotient rule $\times (2 - 2x)$ www – penalise missing brackets here
$dy/dx = 0$ when $x - 1 = 0$ $\Rightarrow$ $x = 1$, $y = 1/\sqrt{(2-1)} = 1$ Range is $y \geq 1$	M1 A1 B1 B1ft [8]	extraneous solutions M0
(iii) (*A*) $g(-x) = \frac{1}{\sqrt{1-(-x)^2}} = \frac{1}{\sqrt{1-x^2}} = g(x)$	M1 E1	Expression for g(–x) – must have g(–x) = g(x) seen
(*B*) $g(x-1) = \frac{1}{\sqrt{1-(x-1)^2}}$ $= \frac{1}{\sqrt{1-x^2+2x-1}} = \frac{1}{\sqrt{2x-x^2}} = f(x)$	M1 E1	 must expand bracket
(*C*) f x) is g(x) translated 1 unit to the right. But g(x) is symmetrical about Oy So f(x) is symmetrical about $x = 1$.	M1 M1 A1	 dep both M1s
or $f(1 - x) = g(-x)$, $f(1 + x) = g(x)$ $\Rightarrow$ $f(1 + x) = f(1 - x)$ $\Rightarrow$ f(x) is symmetrical about $x = 1$.	M1 E1 A1 [7]	or $f(1-x) = \frac{1}{\sqrt{2-2x-(1-x)^2}} = \frac{1}{\sqrt{2-2x-1+2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$ $f(1+x) = \frac{1}{\sqrt{2+2x-(1+x)^2}} = \frac{1}{\sqrt{2+2x-1-2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$