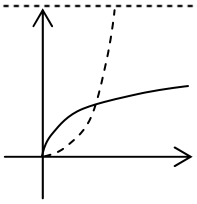
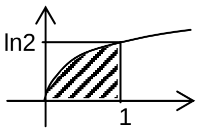
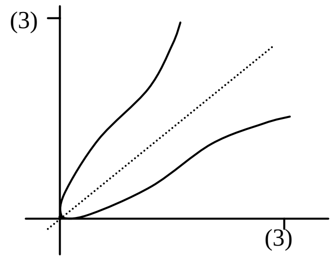
1 (i) $f'(x) = 2(e^x - 1)e^x$ When $x = 0$, $f'(0) = 0$ When $x = \ln 2$, $f'(\ln 2) = 2(2 - 1)2$ $= 4$	M1 A1 B1dep M1 A1cao [5]	or $f(x) = e^{2x} - 2e^x + 1$ M1 (or $(e^x)^2 - 2e^x + 1$ plus correct deriv of $(e^x)^2$) $\Rightarrow f'(x) = 2e^{2x} - 2e^x$ A1 derivative must be correct, www $e^{\ln 2} = 2$ soi
(ii) $y = (e^x - 1)^2 \quad x \leftrightarrow y$ $x = (e^y - 1)^2$ $\Rightarrow \sqrt{x} = e^y - 1$ $\Rightarrow 1 + \sqrt{x} = e^y$ $\Rightarrow y = \ln(1 + \sqrt{x})$	M1 M1 E1	reasonable attempt to invert formula taking lns similar scheme of inverting $y = \ln(1 + \sqrt{x})$
or $gf(x) = g((e^x - 1)^2)$ $= \ln(1 + e^x - 1)$ $= x$	M1 M1 E1	constructing gf or fg $\ln(e^x) = x$ or $e^{\ln(1+\sqrt{x})} = 1 + \sqrt{x}$
 Gradient at (1, ln 2) = ¼	B1 B1ft [5]	reflection in $y = x$ (must have infinite gradient at origin)
(iii) $\int (e^x - 1)^2 dx = \int (e^{2x} - 2e^x + 1) dx$ $= \frac{1}{2}e^{2x} - 2e^x + x + c$ * $\int_0^{\ln 2} (e^x - 1)^2 dx = \left[\frac{1}{2}e^{2x} - 2e^x + x\right]_0^{\ln 2}$ $= \frac{1}{2}e^{2\ln 2} - 2e^{\ln 2} + \ln 2 - (\frac{1}{2} - 2)$ $= 2 - 4 + \ln 2 - \frac{1}{2} + 2$ $= \ln 2 - \frac{1}{2}$	M1 E1 M1 M1 A1 [5]	expanding brackets (condone e^{x^2}) substituting limits $e^{\ln 2} = 2$ used must be exact
(iv)  Area = $1 \times \ln 2 - (\ln 2 - \frac{1}{2})$ $= \frac{1}{2}$	M1 B1 A1cao [3]	subtracting area in (iii) from rectangle rectangle area = $1 \times \ln 2$ must be supported

2 (i) $-\pi/2 < \arctan x < \pi/2$ $\Rightarrow$ $-\pi/4 < f(x) < \pi/4$ $\Rightarrow$ range is $-\pi/4$ to $\pi/4$	M1 A1cao [2]	$\pi/4$ or $-\pi/4$ or 45 seen not $\leq$
(ii) $y = ½ \arctan x$ $x \leftrightarrow y$ $x = ½ \arctan y$ $\Rightarrow$ $2x = \arctan y$ $\Rightarrow$ $\tan 2x = y$ $\Rightarrow$ $y = \tan 2x$ *either* $\frac{dy}{dx} = 2\sec^2 2x$	M1 A1cao M1 A1cao	tan(arctan y or x) = y or x derivative of tan is $\sec^2$ used
or $y = \frac{\sin 2x}{\cos 2x} \Rightarrow \frac{dy}{dx} = \frac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x}$ $= \frac{2}{\cos^2 2x}$	M1 A1cao	quotient rule (need not be simplified but mark final answer)
When $x = 0$, $dy/dx = 2$	B1 [5]	www
(iii) So gradient of $y = ½ \arctan x$ is ½ .	B1ft [1]	ft their '2', but not 1 or 0 or ∞

3 (i) $f(-x) = \ln[1 + (-x)^2]$ $= \ln[1 + x^2] = f(x)$ Symmetrical about Oy	M1 E1 B1 [3]	If verifies that $f(-x) = f(x)$ using a particular point, allow SCB1 For $f(-x) = \ln(1 + x^2) = f(x)$ allow M1E0 For $f(-x) = \ln(1 +- x^2) = f(x)$ allow M1E0 or 'reflects in Oy', etc
(ii) $y = \ln(1 + x^2)$ let $u = 1 + x^2$ $dy/du = 1/u$, $du/dx = 2x$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ $= \frac{1}{u}.2x = \frac{2x}{1+x^2}$ When $x = 2$, $dy/dx = 4/5$.	M1 B1 A1 A1cao [4]	Chain rule $1/u$ soi
(iii) The function is not one to one for this domain	B1 [1]	Or many to one
(iv)  Domain for g(x) $= 0 \le x \le \ln 10$ $y = \ln(1 + x^2)$ $x \leftrightarrow y$ $x = \ln(1 + y^2)$ $\Rightarrow$ $e^x = 1 + y^2$ $\Rightarrow$ $e^x - 1 = y^2$ $\Rightarrow$ $y = \sqrt{(e^x - 1)}$ so $g(x) = \sqrt{(e^x - 1)}$* *or* $g\,f(x) = g[\ln(1 + x^2)]$ $= \sqrt{e^{\ln(1+x^2)} - 1}$ $= (1 + x^2) - 1$ $= x$	M1 A1 B1 M1 M1 E1 M1 M1 E1 [6]	g(x) is f(x) reflected in $y = x$ Reasonable shape and domain, i.e. no –ve x values, inflection shown, does not cross $y = x$ line Condone y instead of x Attempt to invert function Taking exponentials $g(x) = \sqrt{(e^x - 1)}$* www forming g f(x) or f g(x) $e^{\ln(1+x^2)} = 1 + x^2$ or $\ln(1 + e^x - 1) = x$ www
(v) $g'(x) = ½\,(e^x - 1)^{-1/2}.\,e^x$ $\Rightarrow g'(\ln 5) = ½\,(e^{\ln 5} - 1)^{-1/2}.\,e^{\ln 5}$ $= ½\,(5 - 1)^{-1/2}.\,5$ $= 5/4$ Reciprocal of gradient at P as tangents are reflections in $y = x$.	B1 B1 M1 E1cao B1 [5]	½ $u^{-1/2}$ soi $\times\, e^x$ substituting ln 5 into g′ - must be some evidence of substitution Must have idea of reciprocal. Not 'inverse'.