1 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x^2 - 1}{x^2 + 1}$ for the domain $0 \leqslant x \leqslant 2$.

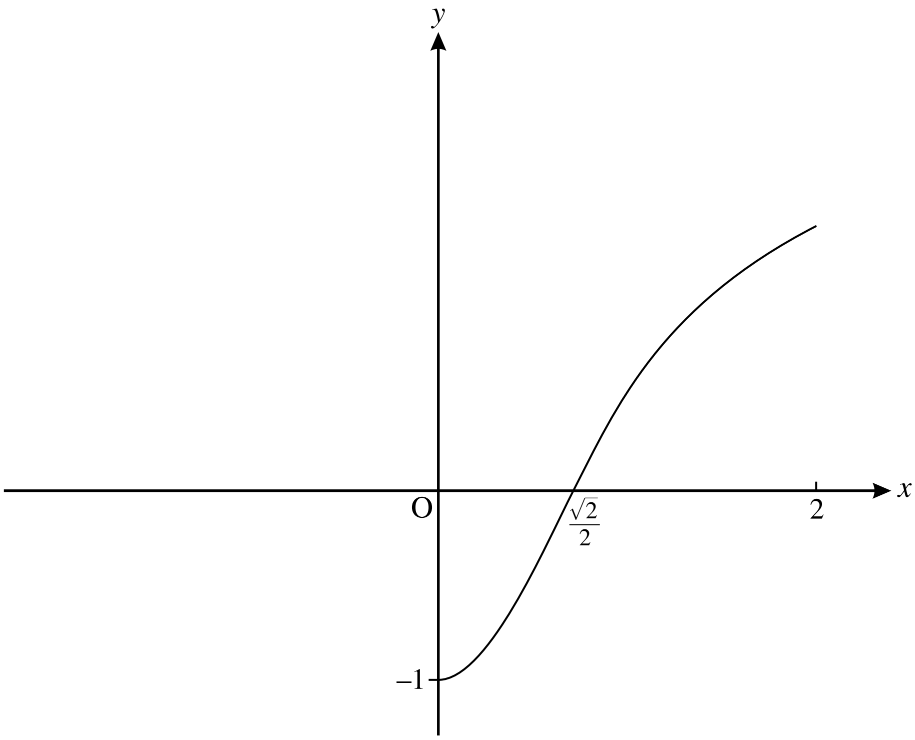

Fig. 9

(i) Show that $\mathrm{f}'(x) = \dfrac{6x}{(x^2 + 1)^2}$, and hence that $\mathrm{f}(x)$ is an increasing function for $x > 0$. **[5]**

(ii) Find the range of $\mathrm{f}(x)$. **[2]**

(iii) **Given** that $\mathrm{f}''(x) = \dfrac{6 - 18x^2}{(x^2 + 1)^3}$, find the maximum value of $\mathrm{f}'(x)$. **[4]**

The function $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

(iv) Write down the domain and range of $\mathrm{g}(x)$. Add a sketch of the curve $y = \mathrm{g}(x)$ to a copy of Fig. 9. **[4]**

(v) Show that $\mathrm{g}(x) = \sqrt{\dfrac{x + 1}{2 - x}}$. **[4]**

2 The variables x and y satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$. **[4]**

Both x and y are functions of t.

(ii) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 1$, $y = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 6$. **[3]**

3 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 1 + \sin 2x$ for $-\frac{1}{4}\pi \leqslant x \leqslant \frac{1}{4}\pi$.

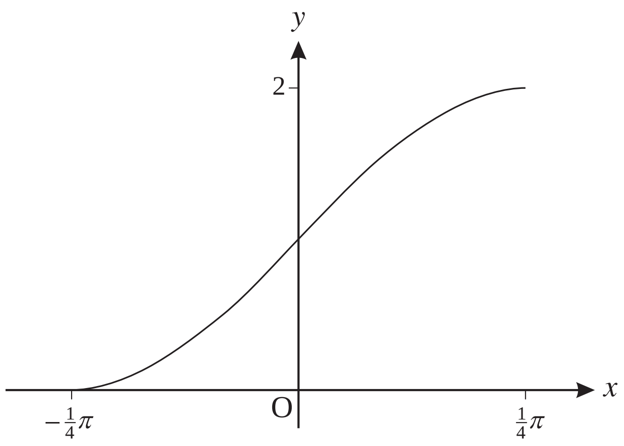

Fig. 8

(i) State a sequence of two transformations that would map part of the curve $y = \sin x$ onto the curve $y = \mathrm{f}(x)$. [4]

(ii) Find the area of the region enclosed by the curve $y = \mathrm{f}(x)$, the x-axis and the line $x = \frac{1}{4}\pi$. [4]

(iii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 1)$. Hence write down the gradient of the curve $y = \mathrm{f}^{-1}(x)$ at the point $(1, 0)$. [4]

(iv) State the domain of $\mathrm{f}^{-1}(x)$. Add a sketch of $y = \mathrm{f}^{-1}(x)$ to a copy of Fig. 8. [3]

(v) Find an expression for $\mathrm{f}^{-1}(x)$. [2]