1 Fig. 4 shows the curve $y = f(x)$, where

$$f(x) = a + \cos bx,\ 0 \leqslant x \leqslant 2\pi,$$

and a and b are positive constants. The curve has stationary points at (0, 3) and $(2\pi, 1)$.

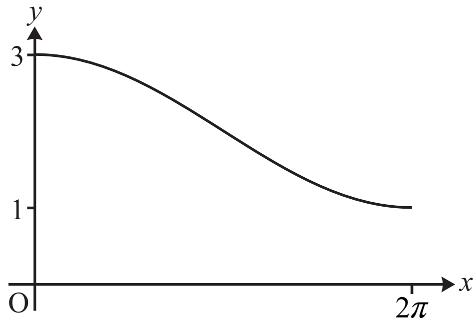

Fig. 4

(i) Find a and b. **[2]**

(ii) Find $f^{-1}(x)$, and state its domain and range. **[5]**

2 Fig. 9 shows the line $y = x$ and the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{1}{2}(\mathrm{e}^x - 1)$. The line and the curve intersect at the origin and at the point $\mathrm{P}(a, a)$.

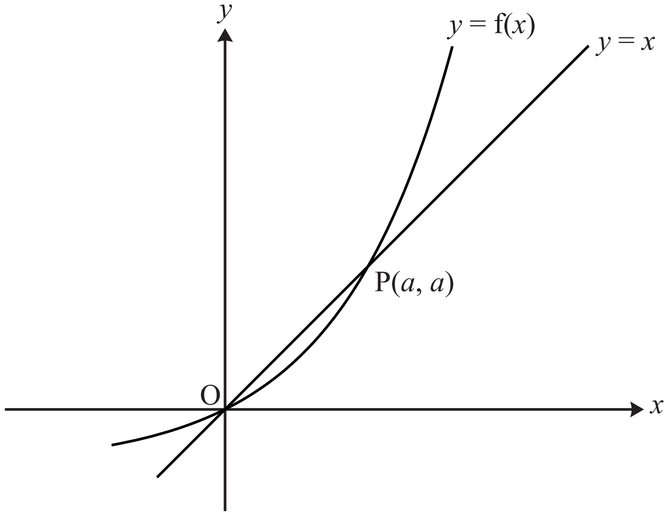

Fig. 9

(i) Show that $\mathrm{e}^a = 1 + 2a$. **[1]**

(ii) Show that the area of the region enclosed by the curve, the x-axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a, the area enclosed by the curve and the line $y = x$. **[6]**

(iii) Show that the inverse function of $\mathrm{f}(x)$ is $\mathrm{g}(x)$, where $\mathrm{g}(x) = \ln(1 + 2x)$. Add a sketch of $y = \mathrm{g}(x)$ to the copy of Fig. 9. **[5]**

(iv) Find the derivatives of $\mathrm{f}(x)$ and $\mathrm{g}(x)$. Hence verify that $\mathrm{g}'(a) = \dfrac{1}{\mathrm{f}'(a)}$.

Give a geometrical interpretation of this result. **[7]**

3 The function $f(x)$ is defined by $f(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

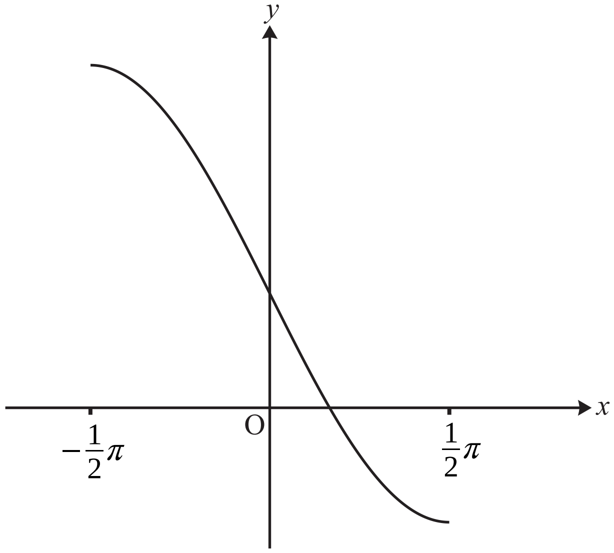

Fig. 3

(i) Write down the range of the function $f(x)$. **[2]**

(ii) Find the inverse function $f^{-1}(x)$. **[3]**

(iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. **[3]**

4 Fig. 6 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = 2\arcsin x$, $-1 \nless \;\; \leqslant 1$.

Fig. 6 also shows the curve $y = \mathrm{g}(x)$, where $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

P is the point on the curve $y = \mathrm{f}(x)$ with x-coordinate $\frac{1}{2}$.

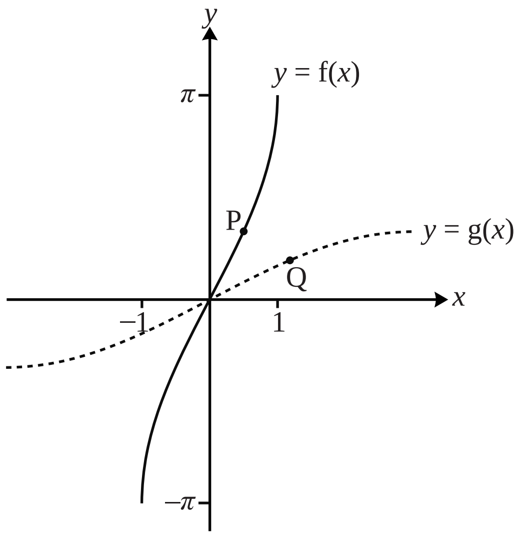

Fig. 6

(i) Find the y-coordinate of P, giving your answer in terms of π. **[2]**

The point Q is the reflection of P in $y = x$.

(ii) Find $\mathrm{g}(x)$ and its derivative $\mathrm{g}'(x)$. Hence determine the exact gradient of the curve $y = \mathrm{g}(x)$ at the point Q.

Write down the exact gradient of $y = \mathrm{f}(x)$ at the point P. **[6]**

5 Fig. 9 shows the curve $y = \mathrm{f}(x)$. The endpoints of the curve are P $(-\pi, 1)$ and Q $(\pi, 3)$, and $\mathrm{f}(x) = a + \sin bx$, where a and b are constants.

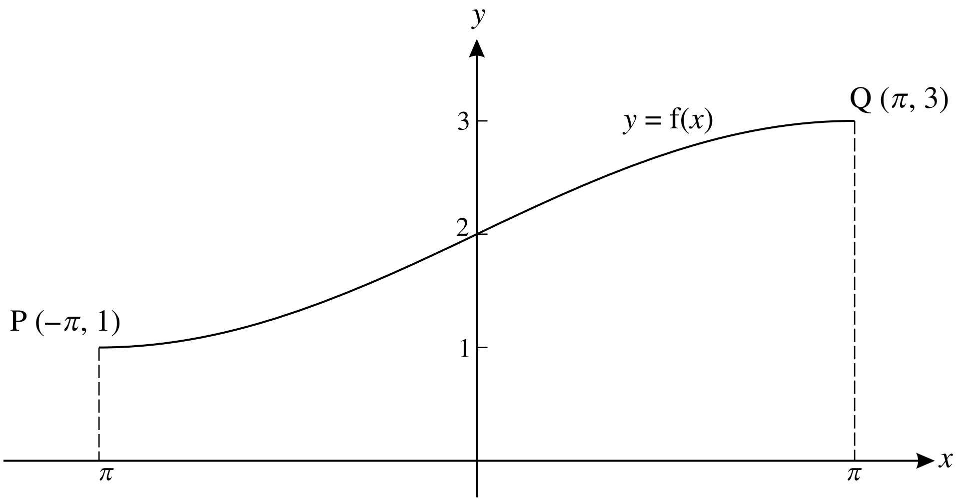

Fig. 9

(i) Using Fig. 9, show that $a = 2$ and $b = \frac{1}{2}$. **[3]**

(ii) Find the gradient of the curve $y = \mathrm{f}(x)$ at the point $(0, 2)$.

Show that there is no point on the curve at which the gradient is greater than this. **[5]**

(iii) Find $\mathrm{f}^{-1}(x)$, and state its domain and range.

Write down the gradient of $y = \mathrm{f}^{-1}(x)$ at the point $(2, 0)$. **[6]**

(iv) Find the area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = \pi$. **[4]**