Question			Answer	Marks	Guidance	
1	(i)		$a = 2$, $b = ½$	B1B1 **[2]**		
	(ii)		$y = 2 + \cos ½ x \quad x \leftrightarrow y$ $x = 2 + \cos ½ y$ $\Rightarrow \quad x - 2 = \cos ½ y$ $\Rightarrow \quad \arccos(x - 2) = ½ y$ $\Rightarrow \quad y = f^{-1}(x) = 2\arccos(x - 2)$ Domain $1 \le x \le 3$ Range $0 \le y \le 2\pi$	 M1 M1 A1 M1 A1 **[5]**	(may be seen later) subtracting [their] a from both sides (first) arccos(x –[their] a) = [their] $b \times y$ cao or $2\cos^{-1}(x - 2)$ domain 1 to 3, range 0 to 2π correctly specified: must be $\le$, x for domain, y or f^{-1} or $f^{-1}(x)$ for range	 need not substitute for a, b or with $x \leftrightarrow y$, need not subst for a, b may be implied by flow diagram if not stated, assume first is domain allow [1, 3], [0, 2π] not 360° (not f)

2	(i)		At P(a, a) g(a) = a so $½(e^a - 1) = a$ $\Rightarrow \quad e^a = 1 + 2a$ *	 B1 **[1]**	 **NB AG**	
	(ii)		$A = \int_0^a \frac{1}{2}(e^x - 1)\,dx$ $= \frac{1}{2}\left[e^x - x\right]_0^a$ $= ½\,(e^a - a - e^0)$ $= ½\,(1 + 2a - a - 1) = ½\,a$ * area of triangle = $½\,a^2$ area between curve and line = $½\,a^2 - ½\,a$	M1 B1 A1 A1 B1 B1cao **[6]**	correct integral and limits integral of $e^x - 1$ is $e^x - x$ **NB AG** mark final answer	limits can be implied from subsequent work

Question			er	Marks	Guidance	
2	(iii)		$y = ½(e^x – 1)$ swap x and y $x = ½ (e^y − 1)$			
			$\Rightarrow$ $2x = e^y − 1$	M1	Attempt to invert – one valid step	merely swapping x and y is not 'one step'
			$\Rightarrow$ $2x + 1 = e^y$	A1		
			$\Rightarrow$ $\ln(2x + 1) = y$ *	A1	$y = \ln(2x + 1)$ or $g(x) = \ln(2x + 1)$ **AG**	apply a similar scheme if they start with g(x) and invert to get f(x).
			$\Rightarrow$ $g(x) = \ln(2x + 1)$			or g f(x) = $g((e^x – 1)/2)$ M1
			Sketch: recognisable attempt to reflect in $y = x$	M1	through O and (a, a)	$= \ln(1 + e^x − 1) = \ln(e^x)$ A1 $= x$ A1
			Good shape	A1	no obvious inflexion or TP, extends to third quadrant, without gradient becoming too negative	similar scheme for fg See appendix for examples
				[5]		
	(iv)		$f'(x) = ½ e^x$	B1		
			$g'(x) = 2/(2x + 1)$	M1	$1/(2x + 1)$ (or $1/u$ with $u = 2x + 1$) …	
				A1	… × 2 to get $2/(2x + 1)$	
			$g'(a) = 2/(2a + 1)$, $f'(a) = ½ e^a$	B1	either $g'(a)$ or $f'(a)$ correct soi	
			so $g'(a) = 2/e^a$ or $f'(a) = ½ (2a+1)$	M1	substituting $e^a = 1 + 2a$	either way round
			$= 1/(½ e^a)$ $= (a + 1)/2$	A1	establishing $f'(a) = 1/ g'(a)$	
			[$= 1/f'(a)$] [= 1 ′(a)]			
			tangents are reflections in $y = x$	B1	must mention tangents	
				[7]		

3	**(i)**		Range is $-1 \le y \le 3$	M1 A1 **[2]**	$-1, 3$ $-1 \le y \le 3$ or $-1 \le f(x) \le 3$ or $[-1, 3]$ (not -1 to 3, $-1 \le x \le 3$, $-1 < y < 3$ etc)
3	**(ii)**		$y = 1 - 2\sin x \quad x \leftrightarrow y$ $x = 1 - 2\sin y \Rightarrow x - 1 = -2 \sin y$ $\Rightarrow \quad \sin y = (1 - x)/2$ $\Rightarrow \quad y = \arcsin [(1 - x)/2]$	 M1 A1 A1 **[3]**	[can interchange x and y at any stage] attempt to re-arrange o.e. e.g. $\sin y = (x - 1)/(-2)$ (or $\sin x = (y - 1)/(-2)$) or $f^{-1}(x) = \arcsin [(1 - x)/2]$, not x or $f^{-1}(y) = \arcsin[1 - y)/2]$ (viz must have swapped x and y for final 'A' mark). $\arcsin [(x - 1)/ -2]$ is A0
3	**(iii)**		$f'(x) = -2\cos x$ $\Rightarrow \quad f'(0) = -2$ $\Rightarrow \quad$ gradient of $y = f^{-1}(x)$ at $(1, 0) = -½$	M1 A1 A1 **[3]**	condone $2\cos x$ cao not $1/-2$

Question			Answer	Marks	Guidance	
4	(i)		$y = 2 \text{ arc sin } \frac{1}{2} = 2 \times \pi/6$ $= \pi/3$	M1 A1 **[2]**	$y = 2 \arcsin \frac{1}{2}$ must be in terms of π − can isw approximate answers	1.047… implies M1
	(ii)		$y = 2 \arcsin x$ $\quad x \leftrightarrow y$ $\Rightarrow$ $x = 2 \arcsin y$ $\Rightarrow$ $x/2 = \arcsin y$ $\Rightarrow$ $y = \sin(x/2)$ [so $g(x) = \sin(x/2)$] $\Rightarrow$ $dy/dx = \frac{1}{2}\cos(\frac{1}{2}x)$ At Q, $x = \pi/3$ $\Rightarrow$ $dy/dx = \frac{1}{2}\cos \pi/6 = \frac{1}{2}\sqrt{3}/2 = \sqrt{3}/4$ $\Rightarrow$ gradient at P = $4/\sqrt{3}$	 M1 A1 A1cao M1 A1 B1 ft **[6]**	 or $y/2 = \arcsin x$ but must interchange x and y at some stage substituting their $\pi/3$ into their derivative must be exact, with their $\cos(\pi/6)$ evaluated o.e. e.g. $4\sqrt{3}/3$ but must be exact ft their $\sqrt{3}/4$ unless 1	or $f'(x) = 2/\sqrt{(1-x^2)}$ $f'(\frac{1}{2}) = 2/\sqrt{\frac{3}{4}} = 4/\sqrt{3}$ cao

5(i) When $x = 0$, $f(x) = a = 2$* When $x = \pi$, $f(\pi) = 2 + \sin b\pi = 3$ $\Rightarrow$ $\sin b\pi = 1$ $\Rightarrow$ $b\pi = ½ \pi$, so $b = ½$ * *or* $1 = a + \sin(-\pi b)$ $(= a - \sin \pi b)$ $3 = a + \sin(\pi b)$ $\Rightarrow$ $2 = 2 \sin \pi b$, $\sin \pi b = 1$, $\pi b = \pi/2$, $b = ½$ $\Rightarrow$ $3 = a + 1$ or $1 = a - 1 \Rightarrow a = 2$ (oe for b)	B1 M1 A1 [3]	NB **AG** 'a is the y-intercept' not enough but allow verification (2+sin 0 = 2) or when $x = -\pi$, $f(-\pi) = 2 + \sin(-b\pi) = 1$ $\Rightarrow \sin(-b\pi) = -1$ condone using degrees $\Rightarrow -b\pi = -½ \pi$, $b = ½$ **NB AG** M1 for both points substituted A1 solving for b or a A1 substituting to get a (or b)	or equiv transformation arguments : e.g. 'curve is shifted up 2 so $a = 2$'. e.g. period of sine curve is 4π, or stretched by sf. 2 in x-direction (not squeezed or squashed by ½) $\Rightarrow b = ½$ If verified allow M1A0 If $y = 2 + \sin ½ x$ verified at two points, SC2 A sequence of sketches starting from $y = \sin x$ showing clearly the translation and the stretch (in either order) can earn full marks
(ii) $f'(x) = ½ \cos ½ x$ $\Rightarrow$ $f'(0) = ½$ Maximum value of $\cos ½ x$ is 1 $\Rightarrow$ max value of gradient is ½	M1 A1 A1 M1 A1 [5]	$\pm k \cos ½ x$ cao www or $f''(x) = -¼ \sin ½ x$ $f''(x) = 0 \Rightarrow x = 0$, so max val of $f'(x)$ is ½	
(iii) $y = 2 + \sin ½ x$ $x \leftrightarrow y$ $x = 2 + \sin ½ y$ $\Rightarrow$ $x - 2 = \sin ½ y$ $\Rightarrow$ $\arcsin(x - 2) = ½ y$ $\Rightarrow$ $y = f^{-1}(x) = 2\arcsin(x - 2)$ Domain $1 \le x \le 3$ Range $-\pi \le y \le \pi$ Gradient at (2, 0) is 2	M1 A1 A1 B1 B1 B1ft [6]	Attempt to invert formula or $\arcsin(y - 2) = ½ x$ must be $y = \ldots$ or $f^{-1}(x) = \ldots$ or [1, 3] or $[-\pi, \pi]$ or $-\pi \le f^{-1}(x) \le \pi$ ft their answer in (ii) (except ±1) 1/their ½	viz solve for x in terms of y or vice-versa – one step enough condone use of a and b in inverse function, e.g. $[\arcsin(x - a)]/b$ or $\sin^{-1}(y - 2)$ condone no bracket for 1st A1 only or $2\sin^{-1}(x - 2)$, condone $f'(x)$, must have bracket in final ans but not $1 \le y \le 3$ but not $-\pi \le x \le \pi$. Penalise <'s (or '1 to 3','$-\pi$ to π') once only or by differentiating $\arcsin(x - 2)$ or implicitly
(iv) $A = \int_0^{\pi} (2 + \sin \frac{1}{2}x)\,dx$ $= \left[2x - 2\cos\frac{1}{2}x\right]_0^{\pi}$ $= 2\pi - (-2)$ $= 2\pi + 2$ (= 8.2831…)	M1 M1 A1 A1cao [4]	correct integral and limits $\left[2x - k\cos\frac{1}{2}x\right]$ where k is positive $k = 2$ answers rounding to 8.3	soi from subsequent work, condone no dx but not 180 Unsupported correct answers score 1st M1 only.