1 Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

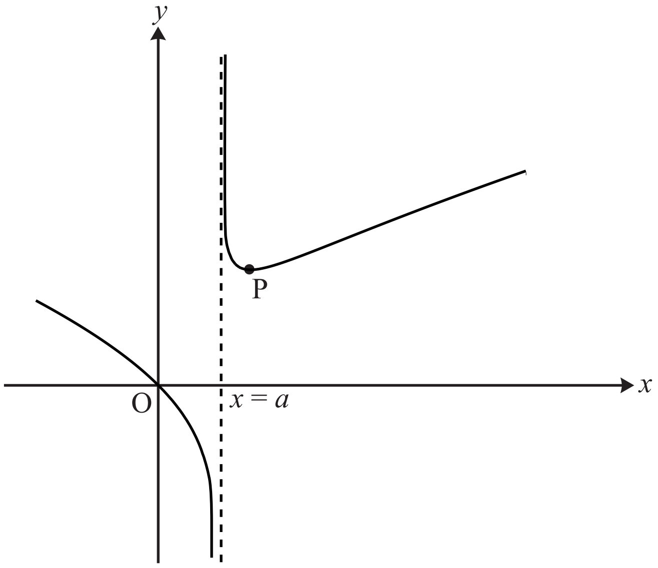

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. **[9]**

(iii) Show that the substitution $u = 2x - 1$ transforms $\displaystyle\int \frac{x}{\sqrt[3]{2x-1}}\,dx$ to $\displaystyle\frac{1}{4}\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right) du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$. **[8]**

2 Fig. 8 shows the curve $y = \dfrac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$.

The curve meets these lines at P and Q respectively. R is the point (11, 11).

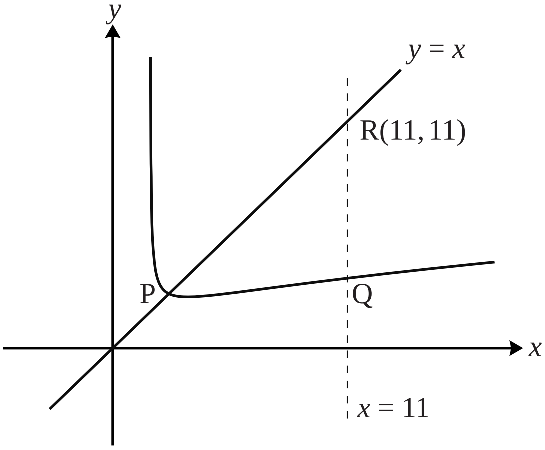

Fig. 8

(i) Verify that the x-coordinate of P is 3. **[2]**

(ii) Show that, for the curve, $\dfrac{dy}{dx} = \dfrac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. **[7]**

(iii) Using the substitution $u = x - 2$, show that $\displaystyle\int_3^{11} \frac{x}{\sqrt{x-2}}\,dx = 25\tfrac{1}{3}$.

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. **[9]**

3 Fig. 9 shows the curve $y = \mathrm{f}(x)$, which has a y-intercept at P(0, 3), a minimum point at Q(1, 2), and an asymptote $x = -1$.

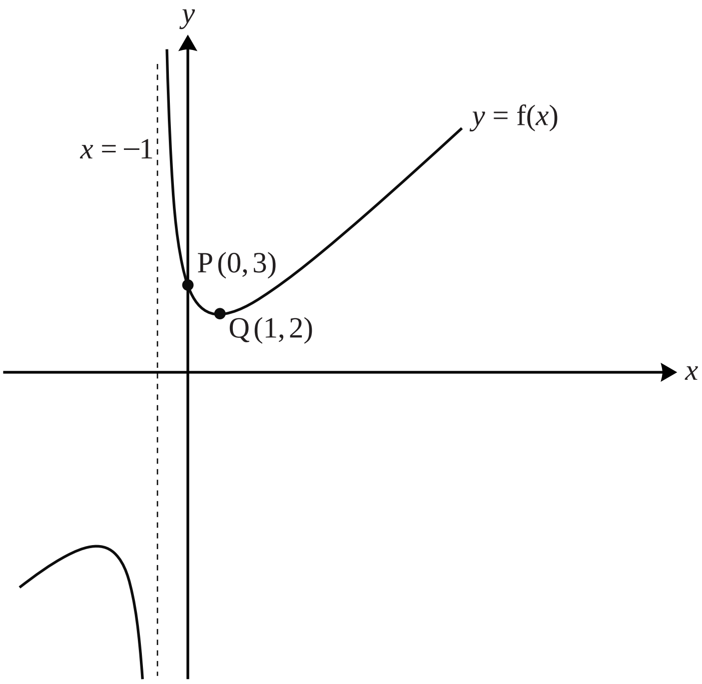

Fig. 9

(i) Find the coordinates of the images of the points P and Q when the curve $y = \mathrm{f}(x)$ is transformed to

(*A*) $y = 2\mathrm{f}(x)$,

(*B*) $y = \mathrm{f}(x + 1) + 2$. **[4]**

You are now given that $\mathrm{f}(x) = \dfrac{x^2 + 3}{x + 1}$, $x \neq -1$.

(ii) Find $\mathrm{f}'(x)$, and hence find the coordinates of the other turning point on the curve $y = \mathrm{f}(x)$. **[6]**

(iii) Show that $\mathrm{f}(x - 1) = x - 2 + \dfrac{4}{x}$. **[3]**

(iv) Find $\displaystyle\int_a^b \left(x - 2 + \frac{4}{x}\right) \mathrm{d}x$ in terms of a and b.

Hence, by choosing suitable values for a and b, find the exact area enclosed by the curve $y = \mathrm{f}(x)$, the x-axis, the y-axis and the line $x = 1$. **[5]**

4 **(i)** Differentiate $\frac{\ln x}{x^2}$, simplifying your answer. **[4]**

(ii) Using integration by parts, show that $\int \frac{\ln x}{x^2}\,\mathrm{d}x = -\frac{1}{x}(1 + \ln x) + c$. **[4]**