Question			Answer	Marks	Gui
1	**(i)**		$a = ½$	B1	allow $x = ½$
				[1]	
	(ii)		$y^3 = \dfrac{x^3}{2x-1}$		
			$\Rightarrow \quad 3y^2\dfrac{dy}{dx} = \dfrac{(2x-1)3x^2 - x^3.2}{(2x-1)^2}$	B1	$3y^2dy/dx$
				M1	Quotient (or product) rule consistent with their derivatives; $(v\,du + u\,dv)/v^2$ M0
				A1	correct RHS expression – condone missing bracket
			$= \dfrac{6x^3 - 3x^2 - 2x^3}{(2x-1)^2} = \dfrac{4x^3 - 3x^2}{(2x-1)^2}$	A1	
			$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ *	A1	**NB AG** penalise omission of bracket in QR at this stage
			$dy/dx = 0$ when $4x^3 - 3x^2 = 0$	M1	
			$\Rightarrow x^2(4x - 3) = 0$, $x = 0$ or ¾	A1	if in addition $2x - 1 = 0$ giving $x = ½$, A0
			$y^3 = (¾)^3/½ = 27/32$,	M1	must use $x = ¾$; if (0, 0) given as an additional TP, then A0
			$y = 0.945$ (3sf)	A1	can infer M1 from answer in range 0.94 to 0.95 inclusive
				[9]	

Question			Answer	Marks	Gui
1	**(iii)**		$u = 2x - 1 \Rightarrow du = 2dx$		
			$\int \frac{x}{\sqrt[3]{2x-1}}dx = \int \frac{\frac{1}{2}(u+1)}{u^{1/3}}\frac{1}{2}du$	M1 M1	$\frac{\frac{1}{2}(u+1)}{u^{1/3}}$ if missing brackets, withhold A1 × ½ d*u* condone missing d*u* here, but withhold A1
			$= \frac{1}{4}\int \frac{u+1}{u^{1/3}}du = \frac{1}{4}\int (u^{2/3} + u^{-1/3})du$ *	A1	**NB AG**
			area $= \int_1^{4.5} \frac{x}{\sqrt[3]{2x-1}}dx$	M1	correct integral and limits – may be inferred from a change of limits andP their attempt to integrate (their) ¼ $(u^{2/3} + u^{-1/3})$
			when $x = 1$, $u = 1$, when $x = 4.5$, $u = 8$	A1	$u = 1, 8$ (or substituting back to x's and using 1 and 4.5)
			$= \frac{1}{4}\int_1^8 (u^{2/3} + u^{-1/3})du$		
			$= \frac{1}{4}\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]_1^8$	B1	$\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]$ o.e. e.g. [$u^{5/3}/(5/3) + u^{2/3}/(2/3)$]
			$= \frac{1}{4}\left[\frac{96}{5} + 6 - \frac{3}{5} - \frac{3}{2}\right]$	A1	o.e. correct expression (may be inferred from a correct final answer)
			$= 5\frac{31}{40} = 5.775$ or $\frac{231}{40}$	A1	cao, must be exact; mark final answer
				[8]	

2	**(i)**		When $x = 3$, $y = 3/\sqrt{(3-2)} = 3$ So P is (3, 3) which lies on $y = x$	M1 A1 **[2]**	substituting $x = 3$ (both x's) $y = 3$ and completion ('3 = 3' is enough)	or $x = x/\sqrt{(x-2)}$ M1 $\Rightarrow x = 3$ A1(by solving or verifying)
	(ii)		$\frac{dy}{dx} = \frac{\sqrt{x-2}.1 - x.\frac{1}{2}.(x-2)^{-1/2}}{x-2}$ $= \frac{x-2-\frac{1}{2}x}{(x-2)^{3/2}} = \frac{\frac{1}{2}x-2}{(x-2)^{3/2}}$ $= \frac{x-4}{2(x-2)^{3/2}}$ * When $x = 3$, $dy/dx = -\frac{1}{2} \times 1^{3/2}$ $= -\frac{1}{2}$ This gradient would be -1 if curve were symmetrical about $y = x$	M1 A1 M1 A1 M1 A1 A1cao **[7]**	Quotient or product rule PR: $-\frac{1}{2}x(x-2)^{-3/2} + (x-2)^{-1/2}$ correct expression $\times$ top and bottom by $\sqrt{(x-2)}$ o.e. e.g. taking out factor of $(x-2)^{-3/2}$ NB **AG** substituting $x = 3$ or an equivalent valid argument	If correct formula stated, allow one error; otherwise QR must be on correct u and v, with numerator consistent with their derivatives and denominator correct initially allow ft on correct equivalent algebra from their incorrect expression

2	**(iii)**		$u = x - 2 \Rightarrow du/dx = 1 \Rightarrow du = dx$	B1	or $dx/du = 1$	No credit for integrating initial integral by parts. Condone $du = 1$.Condone missing du's in subsequent working.
			When $x = 3$, $u = 1$ when $x = 11$, $u = 9$ $\Rightarrow \int_3^{11} \frac{x}{\sqrt{x-2}} dx = \int_1^9 \frac{u+2}{u^{1/2}} du$	B1	$\int \frac{u+2}{u^{1/2}}(du)$	
			$= \int_1^9 (u^{1/2} + 2u^{-1/2}) du$	M1	splitting their fraction (correctly) and $u/u^{1/2} = u^{1/2}$ (or $\sqrt{u}$)	or integration by parts: $2u^{1/2}(u+2) - \int 2u^{1/2}du$ (must be fully correct – condone missing bracket
			$= \left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]_1^9$	A1	$\left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]$ (o.e)	by parts: $[2u^{1/2}(u+2) - 4u^{3/2}/3]$
			$= (18 + 12) - (2/3 + 4)$	M1	substituting correct limits	$F(9) - F(1)$ (u) or $F(11) - F(3)$ (x)
			$= 25\frac{1}{3}$ *	A1cao	NB AG	dep substitution and integration attempted
			Area under $y = x$ is ½ (3 + 11) × 8 = 56	B1	o.e. (e.g. 60.5 – 4.5)	
			Area = (area under $y = x$) – (area under curve)	M1	soi from working	must be trapezium area: $60.5 - 25\frac{1}{3}$ is M0
			so required area $= 56 - 25\frac{1}{3} = 30\frac{2}{3}$	A1cao **[9]**	30.7 or better	

Question			Answer	Marks	Guidance	
3	(i)		(A) (0, 6) and (1, 4 (B) −1, 5) and (0, 4)	B1B1 B1B1 **[4]**	Condone P and Q incorrectly labelled (or unlabelled)	
	(ii)		$\text{f}'(x)=\frac{(x+1).2x-(x^2+3).1}{(x+1)^2}$ $\text{f}'(x) = 0 \Rightarrow 2x\,(x + 1) - (x^2 + 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$ $\Rightarrow (x - 1)(x + 3) = 0$ $\Rightarrow x = 1$ or $x = -3$ When $x = -3$, $y = 12/(-2) = -6$ so other TP is $(-3, -6)$	M1 A1 M1 A1dep B1B1cao **[6]**	Quotient or product rule consistent with their derivatives, condone missing brackets correct expression their derivative = 0 obtaining correct quadratic equation (soi) dep 1st M1 but withhold if denominator also set to zero must be from correct work (but see note re quadratic)	PR: $(x^2+3)(-1)(x+1)^{-2} + 2x(x+1)^{-1}$ If formula stated correctly, allow one substitution error. condone missing brackets if subsequent working implies they are intended Some candidates get $x^2 + 2x + 3$, then realise this should be $x^2 + 2x - 3$, and correct back, but not for every occurrence. Treat this sympathetically. Must be supported, but −3 could be verified by substitution into correct derivative
	(iii)		$\text{f}(x - 1) = \frac{(x-1)^2+3}{x-1+1}$ $=\frac{x^2-2x+1+3}{x-1+1}$ $=\frac{x^2-2x+4}{x} = x-2+\frac{4}{x}$ *	M1 A1 A1 **[3]**	substituting $x - 1$ for both x's in f NB **AG**	allow 1 slip for M1
	(iv)		$\int_a^b (x-2+\frac{4}{x})\,\text{d}x=\left[\frac{1}{2}x^2-2x+4\ln x\right]_a^b$ $=(\frac{1}{2}b^2-2b+4\ln b)-(\frac{1}{2}a^2-2a+4\ln a)$ Area is $\int_0^1 \text{f}(x)\,\text{d}x$ So taking $a = 1$ and $b = 2$ area = (2 – 4 + 4ln 2) – (½ – 2 + 4ln 1) = 4 ln 2 – ½	B1 M1 A1 M1 A1 cao **[5]**	$\left[\frac{1}{2}x^2-2x+4\ln x\right]$ F(b) – F(a) condone missing brackets oe (mark final answer) must be simplified with ln 1 = 0	F must show evidence of integration of at least one term or $\text{f}(x) = x + 1 - 2 + 4/(x+1)$ $A = \int_0^1 \text{f}(x)\,\text{d}x=\left[\frac{1}{2}x^2-x+4\ln(1+x)\right]_0^1$ M1 = ½ − 1 + 4ln 2 = 4ln2 – ½ A1

4(i)	$\frac{dy}{dx}=\frac{x^2.\frac{1}{x}-\ln x.2x}{x^4}$ $=\frac{x-2x\ln x}{x^4}$ $=\frac{1-2\ln x}{x^3}$	M1 B1 A1 A1 [4]	quotient rule with $u = \ln x$ and $v = x^2$ $d/dx (\ln x) = 1/x$ soi correct expression (o.e.) o.e. cao, mark final answer, but must have divided top and bottom by x	Consistent with their derivatives. $udv \pm vdu$ in the quotient rule is M0 Condone $\ln x.2x = \ln 2x^2$ for this A1 (provided $\ln x.2x$ is shown) e. $\frac{1}{x^3}-\frac{2\ln x}{x^3}$, $x^{-3}-2x^{-3}\ln x$
or	$\frac{dy}{dx}=-2x^{-3}\ln x+x^{-2}(\frac{1}{x})$ $=-2x^{-3}\ln x+x^{-3}$	M1 B1 A1 A1 [4]	product rule with $u = x^{-2}$ and $v = \ln x$ $d/dx (\ln x) = 1/x$ soi correct expression o.e. cao, mark final answer, must simplify the $x^{-2}.(1/x)$ term.	or vice-versa
(ii)	$\int\frac{\ln x}{x^2}dx$ let $u = \ln x$, $du/dx = 1/x$ $dv/dx = 1/x^2$, $v = -x^{-1}$ $=-\frac{1}{x}\ln x+\int\frac{1}{x}.\frac{1}{x}dx$ $=-\frac{1}{x}\ln x+\int\frac{1}{x^2}dx$ $=-\frac{1}{x}\ln x-\frac{1}{x}+c$ $=-\frac{1}{x}(\ln x+1)+c$ *	M1 A1 A1 A1 [4]	Integration by parts with $u = \ln x$, $du/dx = 1/x$, $dv/dx = 1/x^2$, $v = -x^{-1}$ must be correct, condone + c condone missing c NB **AG** must have c shown in final answer	Must be correct at this stage . Need to see $1/x^2$