1 Fig. 7 shows the curve $y =$ $x - 1$. It has a minimum at the point P. The line l is an asymptote to the curve.

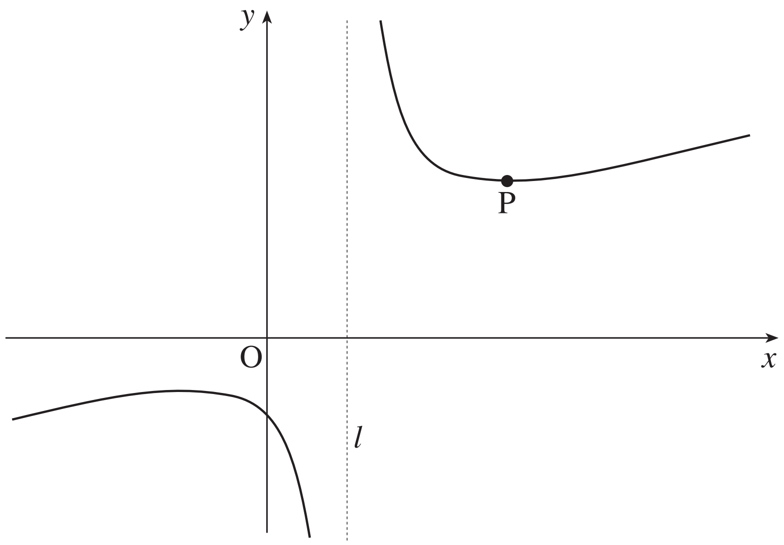


Fig. 7

(i) Write down the equation of the asymptote l. [1]

(ii) Find the coordinates of P. [6]

(iii) Using the substitution $u = x - 1$, show that the area of the region enclosed by the x-axis, the curve and the lines $x = 2$ and $x = 3$ is given by

$$\int_1^2 \left(u + 2 + \frac{4}{u}\right) du.$$

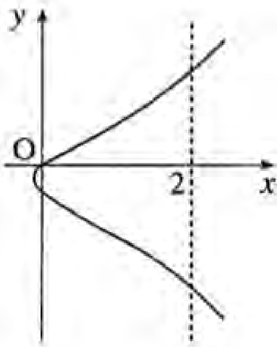
Evaluate this area exactly. [7]

(iv) Another curve is defined by the equation $e^y = \frac{x^2 + 3}{x - 1}$. Find $\frac{dy}{dx}$ in terms of x and y by differentiating implicitly. Hence find the gradient of this curve at the point where $x = 2$. [4]

2 Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line $x = 2$.



Not to scale

Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\frac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at each of these two points. [8]

3 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{x}{\sqrt{2+x^2}}$.

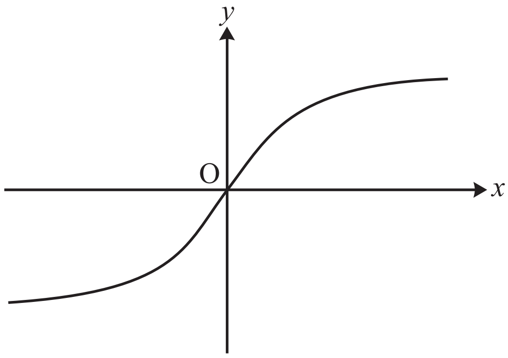


Fig. 8

(i) Show algebraically that $\mathrm{f}(x)$ is an odd function. Interpret this result geometrically. **[3]**

(ii) Show that $\mathrm{f}'(x) = \frac{2}{(2+x^2)^{\frac{3}{2}}}$. Hence find the exact gradient of the curve at the origin. **[5]**

(iii) Find the exact area of the region bounded by the curve, the x-axis and the line $x = 1$. **[4]**

(iv) (A) Show that if $y = \frac{x}{\sqrt{2+x^2}}$, then $\frac{1}{y^2} = \frac{2}{x^2} + 1$. **[2]**

(B) Differentiate $\frac{1}{y^2} = \frac{2}{x^2} + 1$ implicitly to show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^3}{x^3}$. Explain why this expression cannot be used to find the gradient of the curve at the origin. **[4]**