**1** Given that $x^2 + xy + y^2 = 12$, find $\frac{dy}{dx}$ in terms of $x$ and $y$. [5]

**2** The function $f(x)$ is defined by $f(x) = \sqrt{4 - x^2}$ for $-2 \leqslant x \leqslant 2$.

**(i)** Show that the curve $y = \sqrt{4 - x^2}$ is a semicircle of radius 2, and explain why it is not the whole of this circle. [3]

Fig. 9 shows a point P $(a, b)$ on the semicircle. The tangent at P is shown.

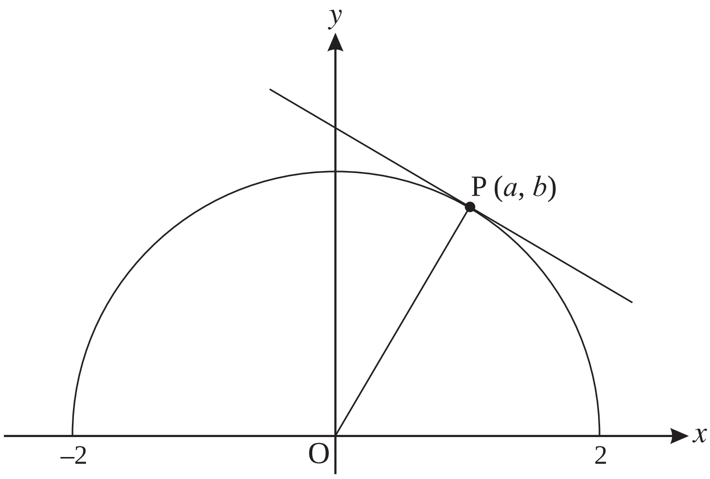


**Fig. 9**

**(ii)** ($A$) Use the gradient of OP to find the gradient of the tangent at P in terms of $a$ and $b$.

($B$) Differentiate $\sqrt{4 - x^2}$ and deduce the value of $f'(a)$.

($C$) Show that your answers to parts ($A$) and ($B$) are equivalent. [6]

The function $g(x)$ is defined by $g(x) = 3f(x - 2)$, for $0 \leqslant x \leqslant 4$.

**(iii)** Describe a sequence of two transformations that would map the curve $y = f(x)$ onto the curve $y = g(x)$.

Hence sketch the curve $y = g(x)$. [6]

**(iv)** Show that if $y = g(x)$ then $9x^2 + y^2 = 36x$. [3]

**3** Fig. 6 shows the triangle OAP, where O is the origin and A is the point $(0, 3)$. The point $P(x, 0)$ moves on the positive $x$-axis. The point $Q(0, y)$ moves between O and A in such a way that $AQ + AP = 6$.

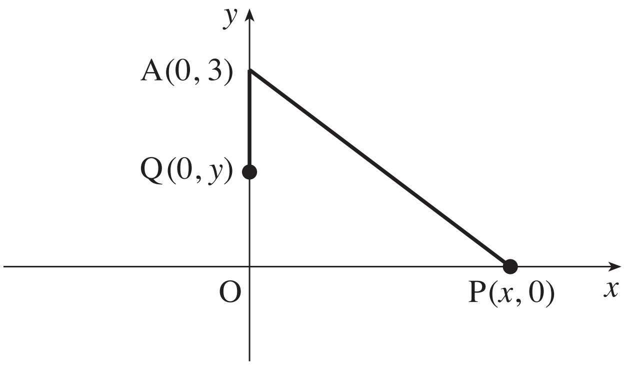


**Fig. 6**

**(i)** Write down the length AQ in terms of $y$. Hence find AP in terms of $y$, and show that

$$(y+3)^2 = x^2 + 9.$$ [3]

**(ii)** Use this result to show that $\frac{dy}{dx} = \frac{x}{y+3}$. [2]

**(iii)** When $x = 4$ and $y = 2$, $\frac{dx}{dt} = 2$. Calculate $\frac{dy}{dt}$ at this time. [3]

**4** A curve has equation $2y^2 + y = 9x^2 + 1$.

**(i)** Find $\frac{dy}{dx}$ in terms of $x$ and $y$. Hence find the gradient of the curve at the point $A(1, 2)$. [4]

**(ii)** Find the coordinates of the points on the curve at which $\frac{dy}{dx} = 0$. [4]

**5** Given that $y = (1+6x)^{\frac{1}{3}}$, show that $\frac{dy}{dx} = \frac{2}{y^2}$. [4]

**6** A curve is defined implicitly by the equation

$$y^3 = 2xy + x^2.$$

**(i)** Show that $\frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x}$. [4]

**(ii)** Hence write down $\frac{dx}{dy}$ in terms of $x$ and $y$. [1]