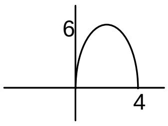
1 $x^2 + xy + y^2 = 12$ $\Rightarrow \quad 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$ $\Rightarrow \quad (x + 2y)\frac{dy}{dx} = -2x - y$ $\Rightarrow \quad \frac{dy}{dx} = -\frac{2x + y}{(x + 2y)}$	M1 B1 A1 M1 A1 [5]	Implicit differentiation $x\frac{dy}{dx} + y$ correct equation collecting terms in d*y*/d*x* and factorising oe cao

2(i) y = $\sqrt{4 - x2}$ $\Rightarrow \quad y^2 = 4 - x^2$ $\Rightarrow \quad x^2 + y^2 = 4$ which is equation of a circle centre O radius 2 Square root does not give negative values, so this is only a semi-circle.	M1 A1 B1 [3]	squaring $x^2 + y^2 = 4$ + comment (correct) oe, e.g. f is a function and therefore single valued
(ii) (*A*) Grad of OP = b/a $\Rightarrow$ grad of tangent = $-\frac{a}{b}$ (*B*) $f'(x) = \frac{1}{2}(4 - x^2)^{-1/2}.(-2x)$ $= -\frac{x}{\sqrt{4 - x^2}}$ $\Rightarrow \quad f'(a) = -\frac{a}{\sqrt{4 - a^2}}$ (*C*) $b = \sqrt{(4 - a^2)}$ so $f'(a) = -\frac{a}{b}$ as before	M1 A1 M1 A1 B1 E1 [6]	 chain rule or implicit differentiation oe substituting *a* into their f'(*x*)
(iii) Translation through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ followed by stretch scale factor 3 in *y*-direction 	M1 A1 M1 A1 M1 A1 [6]	Translation in *x*-direction through $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ or 2 to right ('shift', 'move' M1 A0) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ alone is SC1 stretch in *y*-direction (condone *y* 'axis') (scale) factor 3 elliptical (or circular) shape through (0, 0) and (4, 0) and (2, 6) (soi) –1 if whole ellipse shown
(iv) $y = 3f(x - 2)$ $= 3\sqrt{(4 - (x - 2)^2)}$ $= 3\sqrt{(4 - x^2 + 4x - 4)}$ $= 3\sqrt{(4x - x^2)}$ $\Rightarrow \quad y^2 = 9(4x - x^2)$ $\Rightarrow \quad 9x^2 + y^2 = 36x$ *	 M1 A1 E1 [3]	 or substituting $3\sqrt{(4 - (x - 2)^2)}$ oe for *y* in $9x^2 + y^2$ $4x - x^2$ www

3(i) QA = 3 – y, PA = 6 – (3 – y) = 3 + y By Pythagoras, $PA^2 = OP^2 + OA^2$ ⇒ $(3 + y)^2 = x^2 + 3^2 = x^2 + 9$. *	B1 B1 E1 [3]	 must show some working to indicate Pythagoras (e.g. $x^2 + 3^2$)
(ii) Differentiating implicitly: $2(y+3)\frac{dy}{dx} = 2x$ ⇒ $\frac{dy}{dx} = \frac{x}{y+3}$ *	 M1 E1	 Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $9 + 6y + y^2 = x^2 + 9$ ⇒ $6y + y^2 = x^2$ ⇒ $(6+2y)\frac{dy}{dx} = 2x$ ⇒ $\frac{dy}{dx} = \frac{x}{y+3}$	 M1 E1	 Allow errors in RHS derivative (but not LHS) - notation should be correct brackets must be used
or $y = \sqrt{(x^2 + 9)} - 3 \Rightarrow dy/dx = ½(x^2+9)^{-1/2}.2x$ $= \frac{x}{\sqrt{x^2+9}} = \frac{x}{y+3}$	M1 E1	(cao)
(iii) $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$ $= \frac{4}{2+3} \times 2$ $= \frac{8}{5}$	 M1 A1 A1 [3]	 chain rule (soi)

4(i) Differentiating implicitly: $(4y \pm 1)\frac{dy}{dx} = 18x$ $\Rightarrow \frac{dy}{dx} = \frac{18x}{4y+1}$ When x = 1, y = 2, $\frac{dy}{dx} = \frac{18}{9} = 2$	M1 A1 M1 A1cao [4]	$(4y+1)\frac{dy}{dx} = ...$ allow $4y+1\frac{dy}{dx} = ...$ condone omitted bracket if intention implied by following line. $4y\frac{dy}{dx}+1$ M1 A0 substituting x = 1, y = 2 into their derivative (provided it contains x's and y's). Allow unsupported answers.
(ii) $\frac{dy}{dx}$ = 0 when x = 0 $\Rightarrow$ $2y^2 + y = 1$ $\Rightarrow$ $2y^2 + y - 1 = 0$ $\Rightarrow$ $(2y - 1)(y + 1) = 0$ $\Rightarrow$ y = ½ or y = –1 So coords are (0, ½) and (0, –1)	B1 M1 A1 A1 [4]	x = 0 from their numerator = 0 (must have a denominator) Obtaining correct quadratic and attempt to factorise or use quadratic formula $y = \frac{-1 \pm \sqrt{1-4\times -2}}{4}$ cao allow unsupported answers provided quadratic is shown

5 $y = (1 + 6x)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1+6x)^{-2/3}.6$ $= 2(1 + 6x)^{-2/3}$ $= 2[(1 + 6x)^{1/3}]^{-2}$ $= \frac{2}{y^2}$ *	M1 B1 A1 E1	Chain rule $\frac{1}{3}(1+6x)^{-2/3}$ or $\frac{1}{3}u^{-2/3}$ any correct expression for the derivative www
or $y^3 = 1 + 6x$ $\Rightarrow$ $x = (y^3 - 1)/6$ $\Rightarrow$ $dx/dy = 3y^2/6 = y^2/2$ $\Rightarrow$ $dy/dx = 1/(dx/dy) = 2/y^2$ *	M1 A1 B1 E1	Finding x in terms of y $y^2/2$ o.e.
or $y^3 = 1 + 6x$ $\Rightarrow$ $3y^2\, dy/dx = 6$ $\Rightarrow$ $dy/dx = 6/3y^2 = 2/y^2$ *	M1 A1 A1 E1 [4]	together with attempt to differentiate implicitly $3y^2\, dy/dx$ = 6

6(i) $y^3 = 2xy + x^2$ $\Rightarrow 3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y + 2x$ $\Rightarrow (3y^2 - 2x)\frac{dy}{dx} = 2y + 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x}$ *	B1 B1 M1 E1	$3y^2\frac{dy}{dx} =$ $2x\frac{dy}{dx} + 2y + 2x$ collecting dy/dx terms on one side www
(ii) $\frac{dx}{dy} = \frac{3y^2 - 2x}{2(x+y)}$	B1cao [5]	