1 **(i)** Differentiate $\sqrt{1+3x^2}$. **[3]**

(ii) Hence show that the derivative of $x\sqrt{1+3x^2}$ is $\dfrac{1+6x^2}{\sqrt{1+3x^2}}$. **[4]**

2 Given that $y^3 = xy - x^2$, show that $\dfrac{dy}{dx} = \dfrac{y-2x}{3y^2-x}$.

Hence show that the curve $y^3 = xy - x^2$ has a stationary point when $x = \frac{1}{8}$. **[7]**

3 Fig. 8 shows the curve $y = x^2 - \frac{1}{8}\ln x$. P is the point on this curve with x-coordinate 1, and R is the point $\left(0, -\frac{7}{8}\right)$.

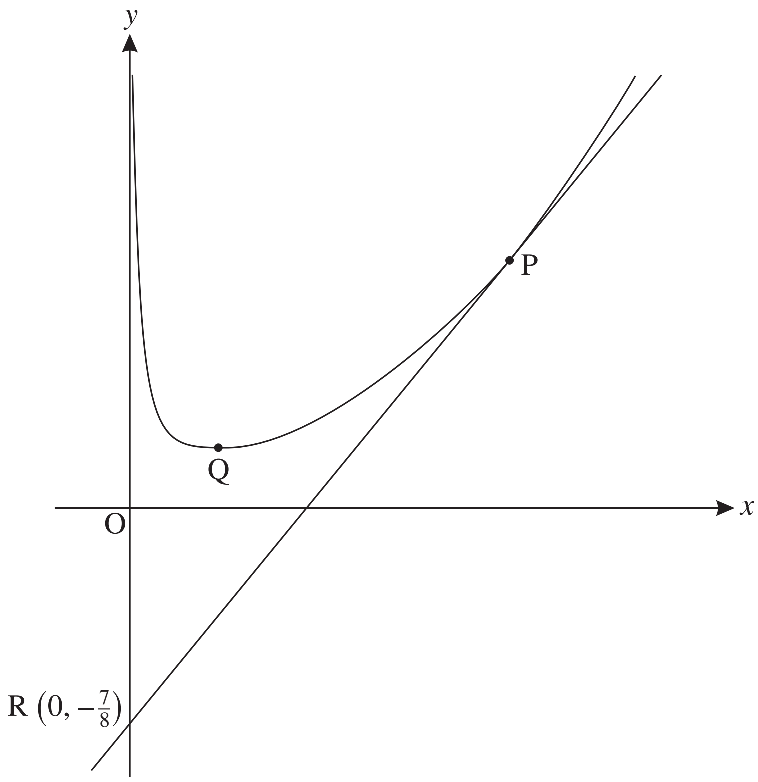

Fig. 8

(i) Find the gradient of PR. **[3]**

(ii) Find $\frac{dy}{dx}$. Hence show that PR is a tangent to the curve. **[3]**

(iii) Find the exact coordinates of the turning point Q. **[5]**

(iv) Differentiate $x\ln x - x$.

Hence, or otherwise, show that the area of the region enclosed by the curve $y = x^2 - \frac{1}{8}\ln x$, the x-axis and the lines $x = 1$ and $x = 2$ is $\frac{59}{24} - \frac{1}{4}\ln 2$. **[7]**

4 The equation of a curve is given by $e^{2y} = 1 + \sin x$.

(i) By differentiating implicitly, find $\dfrac{dy}{dx}$ in terms of x and y. **[3]**

(ii) Find an expression for y in terms of x, and differentiate it to verify the result in part **(i)**. **[4]**

5 Fig. 6 shows the curve $e^{2y} = x^2 + y$.

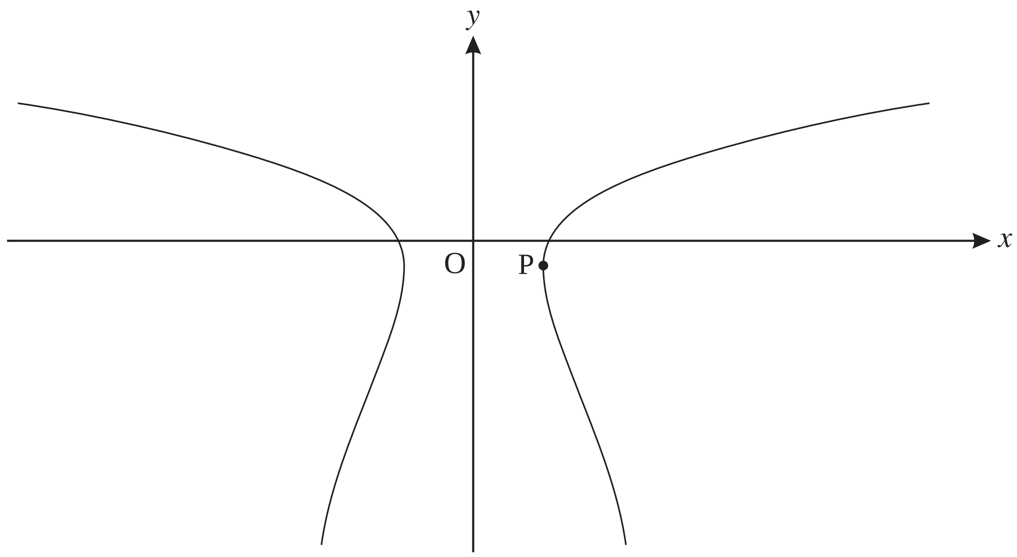

Fig. 6

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x}{2e^{2y} - 1}$. [4]

(ii) Hence find to 3 significant figures the coordinates of the point P, shown in Fig. 6, where the curve has infinite gradient. [4]