1(i) $y=(1+3x^2)^{1/2}$ $\Rightarrow$ $dy/dx=\frac{1}{2}(1+3x^2)^{-1/2}.6x$ $= 3x(1+3x^2)^{-1/2}$	M1 B1 A1 [3]	chain rule $\frac{1}{2}u^{-1/2}$ o.e., but must be '3'	 can isw here
(ii) $y=x(1+3x^2)^{1/2}$ $\Rightarrow$ $dy/dx=x.\frac{3x}{\sqrt{1+3x^2}}+1.(1+3x^2)^{1/2}$ $=\frac{3x^2+1+3x^2}{\sqrt{1+3x^2}}$ $=\frac{1+6x^2}{\sqrt{1+3x^2}}$ *	M1 A1ft M1 E1 [4]	product rule ft their dy/dx from (i) common denominator or factoring $(1+3x^2)^{-1/2}$ www	 must show this step for M1 E1

2 $y^3 = xy - x^2$ $\Rightarrow$ $3y^2$ dy/dx = x dy/dx + $y - 2x$ $\Rightarrow$ $3y^2$ dy/dx – x dy/dx = $y - 2x$ $\Rightarrow$ $(3y^2 - x)$ dy/dx = $y - 2x$ $\Rightarrow$ dy/dx = $(y - 2x)/(3y^2 - x)$ * TP when dy/dx = 0 $\Rightarrow$ $y - 2x = 0$ $\Rightarrow$ $y = 2x$ $\Rightarrow$ $(2x)^3 = x.2x - x^2$ $\Rightarrow$ $8x^3 = x^2$ $\Rightarrow$ $x = 1/8$ *(or 0)	B1 B1 M1 E1 M1 M1 E1 [7]	$3y^2$dy/dx x dy/dx + $y - 2x$ collecting terms in dy/dx only or $x = 1/8$ and dy/dx = 0 $\Rightarrow$ $y = ¼$ or $(1/4)^3 = (1/8)(1/4) - (1/8)^2$ or verifying e.g. 1/64 = 1/64	 must show 'x dy/dx + y' on one side or $x = 1/8 \Rightarrow y^3 = (1/8)y - 1/64$ M1 verifying that $y = ¼$ is a solution (must show evidence*) M1 $\Rightarrow$ dy/dx = (¼ – 2(1/8))/(…) = 0 E1 *just stating that $y = ¼$ is M1 M0 E0

3(i) When $x = 1$ $y = 1^2 - (\ln 1)/8 = 1$ Gradient of PR = $(1 + 7/8)/1 = 1\frac{7}{8}$	B1 M1 A1 [3]	 1.9 or better
(ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ When $x = 1$, $dy/dx = 2 - 1/8 = 1\frac{7}{8}$ Same as gradient of PR, so PR touches curve	B1 B1dep E1 [3]	cao 1.9 or better dep 1st B1 dep gradients exact
(iii) Turning points when $dy/dx = 0$ $\Rightarrow \quad 2x - \frac{1}{8x} = 0$ $\Rightarrow \quad 2x = \frac{1}{8x}$ $\Rightarrow \quad x^2 = 1/16$ $\Rightarrow \quad x = ¼\ (x > 0)$ When $x = ¼$, $y = \frac{1}{16} - \frac{1}{8}\ln\frac{1}{4} = \frac{1}{16} + \frac{1}{8}\ln 4$ So TP is $(\frac{1}{4}, \frac{1}{16} + \frac{1}{8}\ln 4)$	M1 M1 A1 M1 A1cao [5]	setting their derivative to zero multiplying through by x allow verification substituting for x in y o.e. but must be exact, not $1/4^2$. Mark final answer.
(iv) $\frac{d}{dx}(x\ln x - x) = x.\frac{1}{x} + 1.\ln x - 1 = \ln x$	M1 A1	product rule $\ln x$
Area = $\int_1^2 (x^2 - \frac{1}{8}\ln x)dx$ $= \left[\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)\right]_1^2$ $= (\frac{8}{3} - \frac{1}{4}\ln 2 + \frac{1}{4}) - (\frac{1}{3} - \frac{1}{8}\ln 1 + \frac{1}{8})$ $= \frac{7}{3} + \frac{1}{8} - \frac{1}{4}\ln 2$ $= \frac{59}{24} - \frac{1}{4}\ln 2$ *	M1 M1 A1 M1 E1 [7]	correct integral and limits (soi) – condone no dx $\int \ln x dx = x\ln x - x$ used (or derived using integration by parts) $\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)$ – bracket required substituting correct limits must show at least one step

4(i) $e^{2y} = 1 + \sin x$ $\Rightarrow$ $2e^{2y}dy/dx = \cos x$ $\Rightarrow$ $dy/dx = \dfrac{\cos x}{2e^{2y}}$	M1 B1 A1 [3]	Their $2e^{2y} \times dy/dx$ $2e^{2y}$ o.e. cao
(ii) $2y = \ln(1 + \sin x)$ $\Rightarrow$ $y = ½ \ln(1 + \sin x)$ $\Rightarrow$ $dy/dx = \dfrac{1}{2}\dfrac{\cos x}{1+\sin x}$ $= \dfrac{\cos x}{2e^{2y}}$ as before	 B1 M1 B1 E1 [4]	 chain rule (can be within 'correct' quotient rule with $dv/dx = 0$) $1/u$ or $1/(1 + \sin x)$ soi www

5 (i) $e^{2y} = x^2 + y$ $\Rightarrow$ $2e^{2y}\dfrac{dy}{dx} = 2x + \dfrac{dy}{dx}$ $\Rightarrow$ $(2e^{2y} - 1)\dfrac{dy}{dx} = 2x$ $\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{2x}{2e^{2y} - 1}$ *	M1 A1 M1 E1 [4]	Implicit differentiation – allow one slip (but with dy/dx both sides) collecting terms
(ii) Gradient is infinite when $2e^{2y} – 1 = 0$ $\Rightarrow$ $e^{2y} = ½$ $\Rightarrow$ $2y = \ln ½$ $\Rightarrow$ $y = ½ \ln ½ = –0.347$ (3 s f.) $x^2 = e^{2y} – y = ½ – (–0.347)$ $= 0.8465$ $\Rightarrow$ $x = 0.920$	M1 A1 M1 A1 [4]	 must be to 3 s f. substituting their y and solving for x cao – must be to 3 s f., but penalise accuracy once only.