1 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (1 - x)\mathrm{e}^{2x}$, with its turning point P.

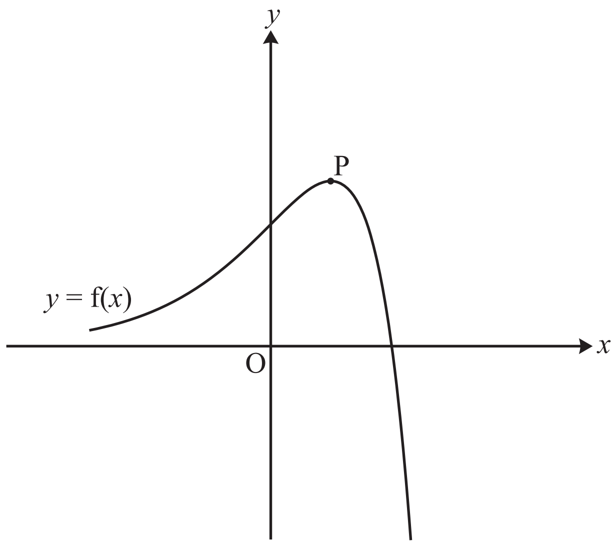

Fig. 8

(i) Write down the coordinates of the intercepts of $y = \mathrm{f}(x)$ with the x- and y-axes. **[2]**

(ii) Find the exact coordinates of the turning point P. **[6]**

(iii) Show that the exact area of the region enclosed by the curve and the x- and y-axes is $\frac{1}{4}(\mathrm{e}^2 - 3)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 3\mathrm{f}\left(\frac{1}{2}x\right)$.

(iv) Express $\mathrm{g}(x)$ in terms of x.

Sketch the curve $y = \mathrm{g}(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x- and y-axes and of its turning point. **[4]**

(v) Write down the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$ and the x- and y-axes. **[1]**

2 Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

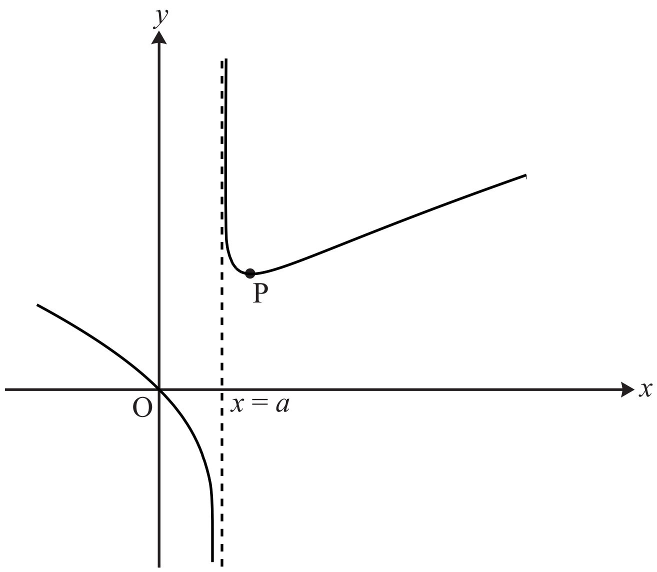

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. **[9]**

(iii) Show that the substitution $u = 2x - 1$ transforms $\displaystyle\int \frac{x}{\sqrt[3]{2x-1}}\,dx$ to $\displaystyle\frac{1}{4}\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right)du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$. **[8]**

3 Fig. 9 shows the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. The function $y = \mathrm{f}(x)$ is given by

$$\mathrm{f}(x) = \ln\left(\frac{2x}{1+x}\right), \quad x > 0.$$

The curve $y = \mathrm{f}(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.

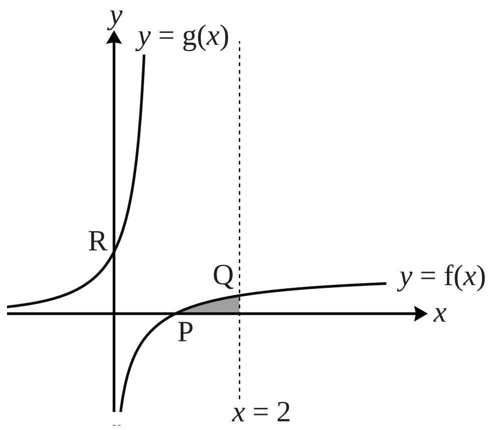

Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. **[2]**

(ii) Find the gradient of the curve at P. [Hint: use $\ln \frac{a}{b} = \ln a - \ln b$.] **[4]**

The function g(x) is given by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x}{2 - \mathrm{e}^x}, \quad x < \ln 2.$$

The curve $y = \mathrm{g}(x)$ crosses the y-axis at the point R.

(iii) Show that $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

Write down the gradient of $y = \mathrm{g}(x)$ at R. **[5]**

(iv) Show, using the substitution $u = 2 - \mathrm{e}^x$ or otherwise, that $\int_0^{\ln\frac{4}{3}} \mathrm{g}(x)\,\mathrm{d}x = \ln \frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln \frac{32}{27}$.
[Hint: consider its reflection in $y = x$.] **[7]**