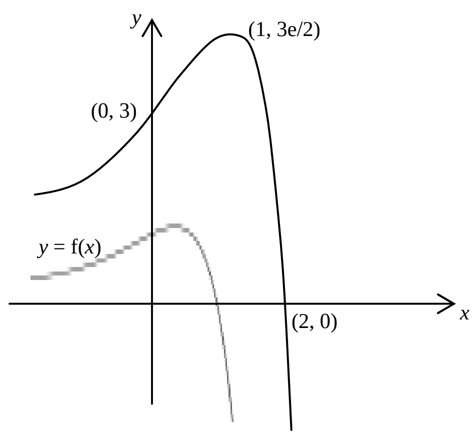
Question			Answer	Marks	Gui
1	**(i)**		(1, 0) and (0, 1)	B1B1	$x = 0, y = 1$; $y = 0, x = 1$
				[2]	
	(ii)		$f'(x) = 2(1 - x)e^{2x} - e^{2x}$	B1	$d/dx(e^{2x}) = 2e^{2x}$
				M1	product rule consistent with their derivatives
			$= e^{2x}(1 - 2x)$	A1	correct expression, so $(1 - x)e^{2x} - e^{2x}$ is B0M1A0
			$f'(x) = 0$ when $x = ½$	M1dep	setting their derivative to 0 dep 1st M1
				A1cao	$x = ½$
			$y = ½ e$	B1	allow $½ e^1$ isw
				[6]	
	(iii)		$A = \int_0^1 (1-x)e^{2x}dx$	B1	correct integral and limits; condone no dx (limits may be seen later)
			$u = (1 - x), u' = -1, v' = e^{2x}, v = ½ e^{2x}$	M1	u, u', v', v, all correct; or if split up $u = x, u' = 1, v' = e^{2x}, v = ½ e^{2x}$
			$\Rightarrow \quad A = \left[\frac{1}{2}(1-x)e^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}.(-1)dx$	A1	condone incorrect limits; or, from above, $...\left[\frac{1}{2}xe^{2x}\right]_0^1 - \int_0^1 \frac{1}{2}e^{2x}dx$
			$= \left[\frac{1}{2}(1-x)e^{2x} + \frac{1}{4}e^{2x}\right]_0^1$	A1	o.e. if integral split up; condone incorrect limits
			$= ¼ e^2 - ½ - ¼$		
			$= ¼ (e^2 - 3)$ *	A1cao	**NB AG**
				[5]	

Question			Answer	Marks	Gui
1	**(iv)**		$g(x) = 3f(\tfrac{1}{2}x) = 3(1 - \tfrac{1}{2}x)e^x$	B1	o.e; mark final answer
				B1	through (2,0) and (0,3) – condone errors in writing coordinates (e.g. (0,2)).
				B1dep	reasonable shape, dep previous B1
				B1	TP at (1, 3e/2) or (1, 4.1) (or better). (Must be evidence that $x = 1$, $y = 4.1$ is indeed the TP − appearing in a table of values is not enough on its own.)
				[4]	
	(v)		$6 \times \tfrac{1}{4}(e^2 - 3)$ [$= 3(e^2 - 3)/2$]	B1 **[1]**	o.e. mark final answer

Question			Answer	Marks	Gui
2	**(i)**		$a = ½$	B1 **[1]**	allow $x = ½$
	(ii)		$y^3 = \dfrac{x^3}{2x-1}$		
			$\Rightarrow \quad 3y^2\dfrac{dy}{dx} = \dfrac{(2x-1)3x^2 - x^3.2}{(2x-1)^2}$	B1 M1 A1	$3y^2 dy/dx$ Quotient (or product) rule consistent with their derivatives; $(v\,du + u\,dv)/v^2$ M0 correct RHS expression – condone missing bracket
			$= \dfrac{6x^3 - 3x^2 - 2x^3}{(2x-1)^2} = \dfrac{4x^3 - 3x^2}{(2x-1)^2}$	A1	
			$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ *	A1	**NB AG** penalise omission of bracket in QR at this stage
			$dy/dx = 0$ when $4x^3 - 3x^2 = 0$	M1	
			$\Rightarrow x^2(4x - 3) = 0$, $x = 0$ or ¾	A1	if in addition $2x - 1 = 0$ giving $x = ½$, A0
			$y^3 = (¾)^3/½ = 27/32$,	M1	must use $x = ¾$; if (0, 0) given as an additional TP, then A0
			$y = 0.945$ (3sf)	A1 **[9]**	can infer M1 from answer in range 0.94 to 0.95 inclusive

Question			Answer	Marks	Gui
2	(iii)		$u = 2x - 1 \Rightarrow du = 2dx$		
			$\int \frac{x}{\sqrt[3]{2x-1}}dx = \int \frac{\frac{1}{2}(u+1)}{u^{1/3}}\frac{1}{2}du$	M1	$\frac{\frac{1}{2}(u+1)}{u^{1/3}}$ if missing brackets, withhold A1
				M1	× ½ du condone missing du here, but withhold A1
			$= \frac{1}{4}\int \frac{u+1}{u^{1/3}}du = \frac{1}{4}\int (u^{2/3} + u^{-1/3})du$ *	A1	**NB AG**
			area $= \int_1^{4.5} \frac{x}{\sqrt[3]{2x-1}}dx$	M1	correct integral and limits – may be inferred from a change of limits andP their attempt to integrate (their) ¼ $(u^{2/3} + u^{-1/3})$
			when $x = 1$, $u = 1$, when $x = 4.5$, $u = 8$	A1	$u = 1, 8$ (or substituting back to x's and using 1 and 4.5)
			$= \frac{1}{4}\int_1^8 (u^{2/3} + u^{-1/3})du$		
			$= \frac{1}{4}\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]_1^8$	B1	$\left[\frac{3}{5}u^{5/3} + \frac{3}{2}u^{2/3}\right]$ o.e. e.g. [$u^{5/3}/(5/3) + u^{2/3}/(2/3)$]
			$= \frac{1}{4}\left[\frac{96}{5} + 6 - \frac{3}{5} - \frac{3}{2}\right]$	A1	o.e. correct expression (may be inferred from a correct final answer)
			$= 5\frac{31}{40} = 5.775$ or $\frac{231}{40}$	A1	cao, must be exact; mark final answer
				[8]	

3	**(i)**		When $x = 1$, f(1) =ln(2/2)= ln 1 = 0 so P is (1, 0) f(2) = ln(4/3)	B1 B1 **[2]**	or $\ln(2x/1+x) = 0 \Rightarrow 2x/(1+x) = 1$ $\Rightarrow 2x = 1+x \Rightarrow x = 1$	 if approximated, can isw after ln(4/3)
	(ii)		$y = \ln(2x) - \ln(1 + x)$ $\Rightarrow \quad \frac{dy}{dx} = \frac{2}{2x} - \frac{1}{1+x}$ **OR** $\frac{d}{dx}\left(\frac{2x}{1+x}\right) = \frac{(1+x)2 - 2x.1}{(1+x)^2} = \frac{2}{(1+x)^2}$ $\frac{dy}{dx} = \frac{2}{(1+x)^2}.\frac{1}{2x/(1+x)} \quad = \frac{1}{x(1+x)}$ At P, $dy/dx = 1 - ½ = ½$	M1 M1 A1cao B1 M1 A1 A1cao **[4]**	 one term correct mark final ans correct quotient or product rule chain rule attempted o.e., but mark final ans	condone lack of brackets $2/2x$ or $-1/(1+x)$ need not be simplified need not be simplified

3	**(iii)**		$x = \ln[2y/(1+y)]$ or $\Rightarrow e^x = 2y/(1+y)$ $\Rightarrow e^x(1+y) = 2y$ $\Rightarrow e^x = 2y - e^x y = y(2 - e^x)$ $\Rightarrow y = e^x/(2 - e^x)$ [$= g(x)$] **OR** $gf(x)=g(2x/(1+x)) = e^{\ln[2x/(1+x)]}/\{2-e^{\ln[2x/(1+x)]}\}$ $= \dfrac{2x/(1+x)}{2-2x/(1+x)}$ $= \dfrac{2x}{2+2x-2x} = \dfrac{2x}{2} = x$ gradient at R = 1/ ½ = 2	 B1 B1 B1 B1 M1 A1 M1A1 B1 ft **[5]**	($x \leftrightarrow y$ here or at end to complete) completion forming gf or fg 1/their ans in (ii) unless ±1 or 0	$x = e^y/(2 - e^y)$ $x(2 - e^y) = e^y$ B1 $2x = e^y + xe^y = e^y(1 + x)$ B1 $2x/(1+x) = e^y$ B1 $\ln[2x/(1+x)] = y$ [$= f(x)$] B1 $fg(x) = \ln\{2e^x/(2-e^x)/[1+ e^x/(2-e^x)]\}$ M1 $= \ln[2e^x/(2 - e^x + e^x)]$ A1 $= \ln(e^x) = x$ M1A1 2 must follow ½ for 9(ii) unless $g'(x)$ used (*see additional notes*)
	(iv)		let $u = 2 - e^x \Rightarrow du/dx = -e^x$ $x = 0$, $u = 1$, $x = \ln(4/3)$, $u = 2 - 4/3 = 2/3$ $\Rightarrow \int_0^{\ln(4/3)} g(x)\,dx = \int_1^{2/3} -\frac{1}{u}\,du$ $= \left[-\ln(u)\right]_1^{2/3} = -\ln(2/3) + \ln 1 = \ln(3/2)$* Shaded region = rectangle – integral = 2ln(4/3) – ln(3/2) = ln(16/9 × 2/3) = ln(32/27)*	 B1 M1 A1 A1cao M1 B1 A1cao **[7]**	$2-e^0 = 1$, and $2 - e^{\ln(4/3)} = 2/3$ seen $\int -1/u\,du$ condone $\int 1/u\,du$ [–ln(u)] (could be [ln$u$] if limits swapped) NB AG rectangle area = 2ln(4/3) NB AG must show at least one step from 2ln(4/3) – ln(3/2)	here or later (i.e. after substituting 0 and ln(4/3) into $\ln(2 - e^x)$) or by inspection [$k \ln(2 - e^x)$] $k = -1$ Allow full marks here for correctly evaluating $\int_1^2 \ln(\frac{2x}{1+x})\,dx$ (*see additional notes)*