**1** Find $\int \sqrt[3]{2x-1}\,dx$. **[4]**

**2** Fig. 8 shows the line $y = 1$ and the curve $y = f(x)$, where $f(x) = \frac{(x-2)^2}{x}$. The curve touches the $x$-axis at P(2, 0) and has another turning point at the point Q.

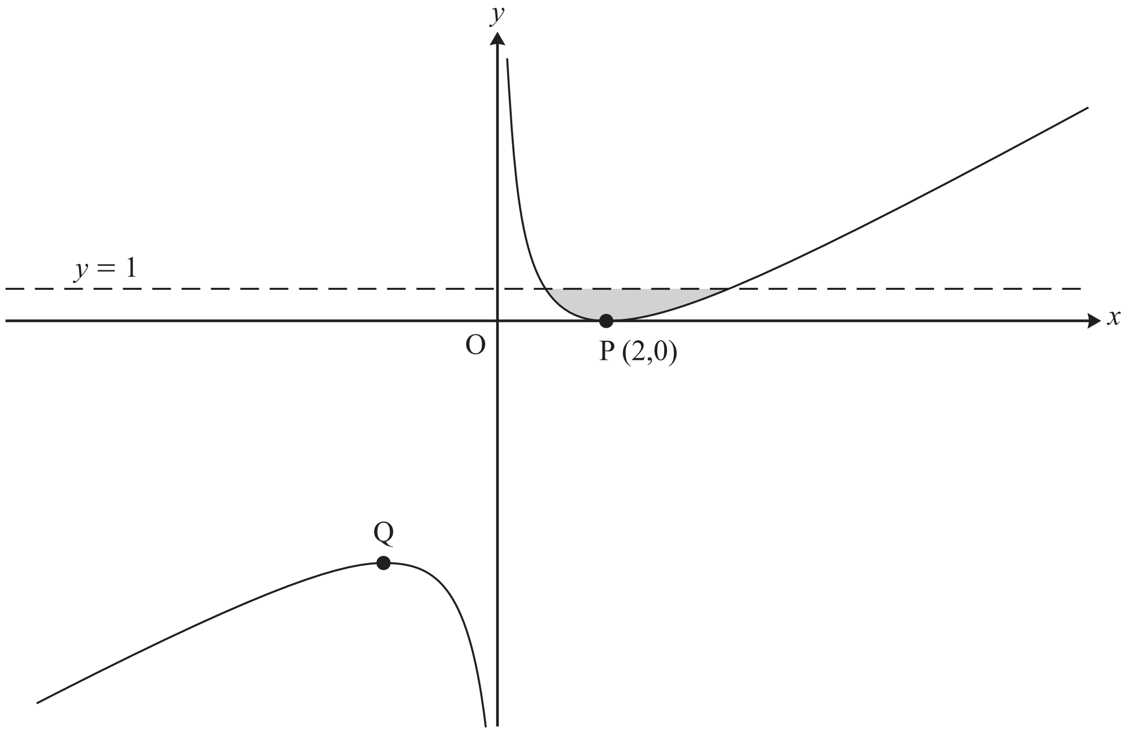


**Fig. 8**

**(i)** Show that $f'(x) = 1 - \frac{4}{x^2}$, and find $f''(x)$.

Hence find the coordinates of Q and, using $f''(x)$, verify that it is a maximum point. **[7]**

**(ii)** Verify that the line $y = 1$ meets the curve $y = f(x)$ at the points with $x$-coordinates 1 and 4. Hence find the exact area of the shaded region enclosed by the line and the curve. **[6]**

The curve $y = f(x)$ is now transformed by a translation with vector $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$. The resulting curve has equation $y = g(x)$.

**(iii)** Show that $g(x) = \frac{x^2 - 3x}{x+1}$. **[3]**

**(iv)** Without further calculation, write down the value of $\int_0^3 g(x)\,dx$, justifying your answer. **[2]**

**3** Evaluate $\int_0^{\frac{1}{6}\pi} (1 - \sin 3x)\,dx$, giving your answer in exact form. **[3]**

**4** Fig. 9 shows the curve $y = xe^{-2x}$ together with the straight line $y = mx$, where $m$ is a constant, with $0 < m < 1$. The curve and the line meet at O and P. The dashed line is the tangent at P.

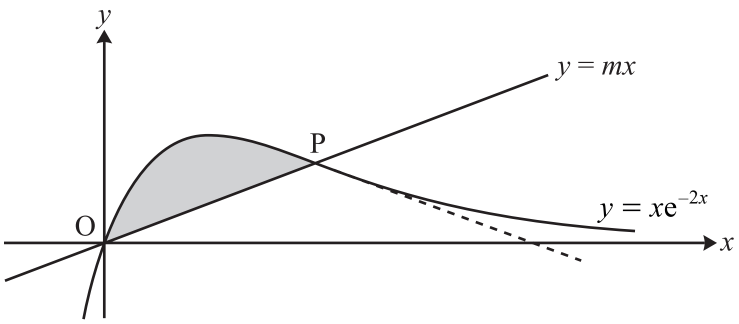


**Fig. 9**

**(i)** Show that the $x$-coordinate of P is $-\frac{1}{2}\ln m$. **[3]**

**(ii)** Find, in terms of $m$, the gradient of the tangent to the curve at P. **[4]**

You are given that OP and this tangent are equally inclined to the $x$-axis.

**(iii)** Show that $m = e^{-2}$, and find the exact coordinates of P. **[4]**

**(iv)** Find the exact area of the shaded region between the line OP and the curve. **[7]**

**5** Using a suitable substitution or otherwise, show that $\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x}\,dx = \frac{1}{2}\ln 2$. **[5]**