

1		<p>let $u = 2x - 1$, $du = 2 dx$</p> $\int \sqrt[3]{2x-1} dx = \int \frac{1}{2} u^{\frac{1}{3}} du$ $= \frac{3}{8} u^{\frac{4}{3}} + c$ $= \frac{3}{8} (2x-1)^{\frac{4}{3}} + c$	<p>M1 M1 M1 A1cao</p>	<p>substituting $u = 2x - 1$ in integral $\times \frac{1}{2}$ o.e.</p> <p>integral of $u^{1/3} = u^{4/3}/(4/3)$ (oe) soi</p> <p>o.e., but must have $+ c$ and single fraction mark final answer</p>	<p>i.e. $u^{1/3}$ or $\sqrt[3]{u}$ seen in integral condone no du, or dx instead of du not $x^{1/3}$</p> <p>so $\frac{3}{4} (2x-1)^{\frac{4}{3}} + c$ is M1M0M1A0</p>
		<p><i>or</i></p> $\int \sqrt[3]{2x-1} dx = \frac{1}{2} \times (2x-1)^{4/3} \div 4/3$ $= \frac{3}{8} (2x-1)^{\frac{4}{3}} + c$	<p>M1 M1 M1 A1cao [4]</p>	<p>$(2x-1)^{4/3}$ seen $\div 4/3$ (oe) soi $\times \frac{1}{2}$</p> <p>o.e., but must have $+ c$ and single fraction mark final ans</p>	<p>e.g. correct power of $(2x-1)$ e.g. $\frac{3}{4} (2x-1)^{4/3}$ seen</p> <p>so $\frac{3}{8} (2x-1)^{\frac{4}{3}}$ is M1M1M1A0</p>

2	(i)	$f(x) = \frac{x \cdot 2(x-2) - (x-2)^2}{x^2}$ $= \frac{2x^2 - 4x - x^2 + 4x - 4}{x^2}$ $= \frac{(x^2 - 4)}{x^2} = 1 - 4/x^2 *$	M1	quotient (or product) rule, condone sign errors only	e.g. $\frac{\pm x \cdot 2(x-2) \pm (x-2)^2}{x^2}$
			A1	correct exp, condone missing brackets here	PR: $(x-2)^2 \cdot (-x^{-2}) + (1/x) \cdot 2(x-2)$
			A1	simplified correctly NB AG	with correct use of brackets
		or $f(x) = (x^2 - 4x + 4)/x$ $= x - 4 + 4/x$ $\Rightarrow f'(x) = 1 - 4/x^2 *$	M1	expanding bracket and dividing each term by x correctly	must be 3 terms: $(x^2 - 4)/x$ is M0
			A1	not from wrong working NB AG	e.g. $x - 4 + 2/x$ is M1A0
		$f''(x) = 8/x^3$ $f'(x) = 0$ when $x^2 = 4$, $x = \pm 2$ so at Q, $x = -2$, $y = -8$. $f''(-2) [= -1] < 0$ so maximum	B1	o.e. e.g. $8x^{-3}$ or $8x/x^4$	
			M1	$x = \pm 2$ found from $1 - 4/x^2 = 0$	allow for $x = -2$ unsupported
			A1	$(-2, -8)$	
			B1dep	dep first B1. Can omit -1 , but if shown must be correct. Must state < 0 or negative.	must use 2 nd derivative test
			[7]		

Question		Answer	Marks	Guidance	
2	(ii)	$f(1) = (-1)^2/1 = 1$ $f(4) = (2)^2/4 = 1$	B1	verifying $f(1) = 1$ and $f(4) = 1$	or $(x-2)^2 = x \Rightarrow x^2 - 5x + 4 = 0$ $\Rightarrow (x-1)(x-4) = 0, x = 1, 4$
		$\int_1^4 \frac{(x-2)^2}{x} dx = \int_1^4 (x-4+4/x) dx$ $= [x^2/2 - 4x + 4 \ln x]_1^4$ $= (8 - 16 + 4 \ln 4) - (1/2 - 4 + 4 \ln 1)$ $= 4 \ln 4 - 4 1/2$ Area enclosed = rectangle – curve $= 3 \times 1 - (4 \ln 4 - 4 1/2) = 7 1/2 - 4 \ln 4$	M1 A1 A1cao M1 A1cao	expanding bracket and dividing each term by x 3 terms: $x - 4/x$ is M0 $x^2/2 - 4x + 4 \ln x$ soi o.e. but must combine numerical terms and evaluate $\ln 1$ – mark final ans	if $u = x - 2$ $\int \frac{u^2}{u+2} du = \int (u-2 + \frac{4}{u+2}) du$ $u^2/2 - 2u + 4 \ln(u+2)$
		or Area = $\int_1^4 [1 - \frac{(x-2)^2}{x}] dx$ $= \int_1^4 (5-x-4/x) dx$ $= [5x - x^2/2 - 4 \ln x]_1^4$ $= 20 - 8 - 4 \ln 4 - (5 - 1/2 - 4 \ln 1)$ $= 7 1/2 - 4 \ln 4$	M1 M1 A1 A1 A1cao [6]	no need to have limits expanding bracket and dividing each term by x $5 - x - 4/x$ $5x - x^2/2 - 4 \ln x$ o.e. but must combine numerical terms and evaluate $\ln 1$ – mark final ans	must be 3 terms in $(x-2)^2$ expansion
	(iii)	$[g(x) =] f(x+1) - 1$ $= \frac{(x+1-2)^2}{x+1} - 1$ $= \frac{x^2 - 2x + 1 - x - 1}{x+1} = \frac{x^2 - 3x}{x+1} *$	M1 A1 A1 [3]	soi [may not be stated] correctly simplified – not from wrong working NB AG	

Question			Answer	Marks	Guidance
2	(iv)		Area is the same as that found in part (ii) $4\ln 4 - 7\frac{1}{2}$	M1 A1cao [2]	award M1 for \pm ans to 8(ii) (unless zero) need not justify the change of sign

Question			Answer	Marks	Guidance
3			$\int_0^{\pi/6} (1 - \sin 3x) dx = \left[x + \frac{1}{3} \cos 3x \right]_0^{\pi/6}$ $= \pi/6 - 1/3$	M1 A1 A1cao [3]	$\pm 1/3 \cos 3x$ seen or $\int \frac{1}{3}(1 - \sin u)[du]$ $\left[x + \frac{1}{3} \cos 3x \right]$ or $\left[\frac{1}{3}(u + \cos u) \right]$ o.e., must be exact i.e. condone sign error condone '+ c' isw after correct answer seen

4	(i)	$xe^{-2x} = mx$ $\Rightarrow e^{-2x} = m$ $\Rightarrow -2x = \ln m$ $\Rightarrow x = -\frac{1}{2} \ln m \quad *$ <p>or</p> <p>If $x = -\frac{1}{2} \ln m$, $y = -\frac{1}{2} \ln m \times e^{\ln m}$ $= -\frac{1}{2} \ln m \times m$</p> <p>so P lies on $y = mx$</p>	M1 M1 A1 M1 A1 A1 [3]	may be implied from 2 nd line dividing by x , or subtracting $\ln x$ NB AG substituting correctly	o.e. e.g. $[\ln x] - 2x = \ln m + [\ln x]$ or factorising: $x(e^{-2x} - m) = 0$
	(ii)	let $u = x$, $u' = 1$, $v = e^{-2x}$, $v' = -2e^{-2x}$ $dy/dx = e^{-2x} - 2xe^{-2x}$ $= e^{-2(-\frac{1}{2}\ln m)} - 2(-\frac{1}{2}\ln m)e^{-2(-\frac{1}{2}\ln m)}$ $= e^{\ln m} + e^{\ln m} \ln m \quad [= m + m \ln m]$	M1* A1 M1dep A1cao [4]	product rule consistent with their derivs o.e. correct expression subst $x = -\frac{1}{2} \ln m$ into their deriv dep M1* condone $e^{\ln m}$ not simplified	but not $-2(-\frac{1}{2} \ln m)$, but mark final ans

Question		Answer	Marks	Guidance
4	(iii)	$m + m \ln m = -m$ $\Rightarrow \ln m = -2$ $\Rightarrow m = e^{-2}$ * or $y + \frac{1}{2} m \ln m = m(1 + \ln m)(x + \frac{1}{2} \ln m) \quad x = -\ln m,$ $y=0 \Rightarrow \frac{1}{2} m \ln m = m(1 + \ln m)(-\frac{1}{2} \ln m)$ $\Rightarrow 1 + \ln m = -1, \ln m = -2, m = e^{-2}$ At P, $x = 1$ $\Rightarrow y = e^{-2}$	M1 A1 B2 B1 B1 [4]	their gradient from (ii) = $-m$ NB AG for fully correct methods finding x -intercept of equation of tangent and equating to $-\ln m$ isw approximations not $e^{-2} \times 1$
	(iv)	Area under curve = $\int_0^1 x e^{-2x} dx$ $u = x, u' = 1, v' = e^{-2x}, v = -\frac{1}{2} e^{-2x}$ $= \left[-\frac{1}{2} x e^{-2x} \right]_0^1 + \int_0^1 \frac{1}{2} e^{-2x} dx$ $= \left[-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} \right]_0^1$ $= (-\frac{1}{2} e^{-2} - \frac{1}{4} e^{-2}) - (0 - \frac{1}{4} e^0)$ $[= \frac{1}{4} - \frac{3}{4} e^{-2}]$ Area of triangle = $\frac{1}{2} \text{ base} \times \text{height}$ $= \frac{1}{2} \times 1 \times e^{-2}$ So area enclosed = $\frac{1}{4} - 5e^{-2}/4$	M1 A1ft A1 A1 M1 A1 Alcao [7]	parts, condone $v = k e^{-2x}$, provided it is used consistently in their parts formula ft their v $-\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}$ o.e correct expression ft their 1, e^{-2} or $[e^{-2} x^2/2]$ o.e. must be exact, two terms only ignore limits until 3 rd A1 need not be simplified o.e. using isosceles triangle M1 may be implied from 0.067... isw

Question	Answer	Marks	Gui
5	$\int_0^{\pi/2} \frac{\sin 2x}{3 + \cos 2x} dx = \left[-\frac{1}{2} \ln(3 + \cos 2x) \right]_0^{\pi/2}$ <p>or $u = 3 + \cos 2x$, $du = -2\sin 2x dx$</p> $\int_0^{\pi/2} \frac{\sin 2x}{3 + \cos 2x} dx = \int_4^2 -\frac{1}{2u} du$ $= \left[-\frac{1}{2} \ln u \right]_4^2$ $= -\frac{1}{2} \ln 2 + \frac{1}{2} \ln 4$ $= \frac{1}{2} \ln (4/2)$ $= \frac{1}{2} \ln 2 *$	M1 A2 M1 A1 A1 A1 A1 A1 [5]	$k \ln(3 + \cos 2x)$ $-\frac{1}{2} \ln(3 + \cos 2x)$ o.e. e.g. $du/dx = -2\sin 2x$ or if $v = \cos 2x$, $dv = -2\sin 2x dx$ o.e. condone $2\sin 2x dx$ $\int -\frac{1}{2u} du$, or if $v = \cos 2x$, $\int -\frac{1}{2(3+v)} dv$ $-\frac{1}{2} \ln u]$ or $[-\frac{1}{2} \ln(3+v)]$ ignore incorrect limits from correct working o.e. e.g. $-\frac{1}{2} \ln(3+\cos(2.\pi/2)) + \frac{1}{2} \ln(3 + \cos(2.0))$ o.e. required step for final A1, must have evaluated to 4 and 2 at this stage NB AG