Edexcel Maths C3

Topic Questions from Papers

Differentiation

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4. $$f(x) = 3e^x - \tfrac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P. The x-coordinate of P is α.

(b) Show that $\alpha = \tfrac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \tfrac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α.

(c) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)

Leave blank

3. The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x-coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $y = ax + b$, where a and b are constants.

(5)

Leave blank

4. (a) Differentiate with respect to x

(i) x^2e^{3x+2},

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4\sin(2y + 6)$, find $\dfrac{dy}{dx}$ in terms of x.

(5)

Leave blank

2. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(b) $(5 + x^2)^{\frac{3}{2}}$.

(3)

Q2

(Total 6 marks)

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3. The curve C has equation

$$x = 2\sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C.

(1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P.

(4)

(c) Find an equation of the normal to C at P. Give your answer in the form $y = mx + c$, where m and c are exact constants.

(4)

Leave blank

4. (i) The curve C has equation

$$y = \frac{x}{9 + x^2}.$$

Use calculus to find the coordinates of the turning points of C.

(6)

(ii) Given that

$$y = (1 + e^{2x})^{\frac{3}{2}},$$

find the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}\ln 3$.

(5)

Leave blank

Question 4 continued

Leave blank

3. A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation. **(3)**

(b) Hence find the coordinates of the turning points of C. **(3)**

(c) Find $\frac{d^2y}{dx^2}$. **(2)**

(d) Determine the nature of each turning point of the curve C. **(2)**

Leave blank

2. A curve C has equation

$$y = e^{2x} \tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

Leave blank

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \ -\pi \leqslant x \leqslant \pi .$$

The point $A(0, 4)$ lies on C.

(a) Find an equation of the normal to the curve C at A.

(5)

(b) Express y in the form $R\sin(2x+\alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(c) Find the coordinates of the points of intersection of the curve C with the x-axis. Give your answers to 2 decimal places.

(4)

Leave blank

Question 7 continued

Leave blank

1. The point P lies on the curve with equation

$$y = 4e^{2x+1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

N 3 0 7 4 5 A 0 2 2 4

Leave blank

6. (a) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x+2)$.

(3)

Given that $y = \dfrac{3x^2+6x-7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(c) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

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Question 6 continued

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Turn over

Leave blank

1. (a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

Leave blank

4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

Leave blank

5. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)

N 3 5 3 8 1 A 0 1 2 2 8

Leave blank

2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \quad x \neq \frac{5}{3}$$

The point P on C has x-coordinate 2. Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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5.

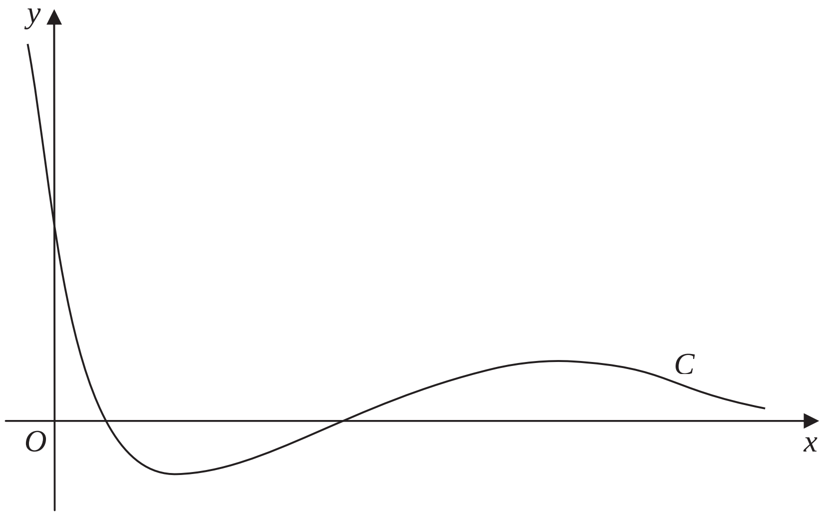

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(a) Find the coordinates of the point where C crosses the y-axis.
(1)

(b) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.
(3)

(c) Find $\frac{dy}{dx}$.
(3)

(d) Hence find the exact coordinates of the turning points of C.
(5)

Leave blank

Question 5 continued

H 3 5 3 8 5 A 0 1 5 2 8

Turn over

Leave blank

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

Leave blank

8. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

H 3 5 4 0 4 R A 0 2 6 2 8

Leave blank

Question 8 continued

Q8

(Total 9 marks)

TOTAL FOR PAPER: 75 MARKS

END

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1. Differentiate with respect to x

(a) $\ln\left(x^2 + 3x + 5\right)$ **(2)**

(b) $\dfrac{\cos x}{x^2}$ **(3)**

Leave blank

7. $$\mathrm{f}(x) = \frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$\mathrm{f}(x) = \frac{5}{(2x+1)(x+3)}$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.

(8)

Leave blank

Question 7 continued

P 3 8 1 5 9 A 0 1 7 2 4

Turn over

Leave blank

8. (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Give your answers to 3 significant figures.

(4)

$$f(x) = e^{2x}\cos 3x$$

(b) Show that $f'(x)$ can be written in the form

$$f'(x) = Re^{2x}\cos(3x + \alpha)$$

where R and α are the constants found in part (a).

(5)

(c) Hence, or otherwise, find the smallest positive value of x for which the curve with equation $y = f(x)$ has a turning point.

(3)

Leave blank

Question 8 continued

Q8

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

P 3 8 1 5 9 A 0 2 3 2 4

Leave blank

1. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\dfrac{\sin 4x}{x^3}$

(5)

Leave blank

4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

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3.

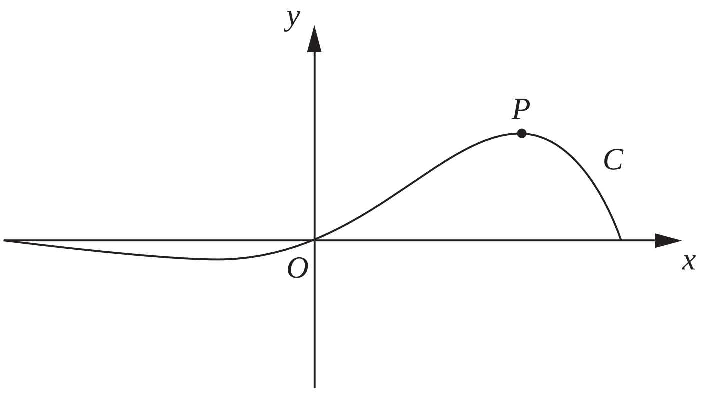

Figure 1

Figure 1 shows a sketch of the curve C which has equation

$$y = e^{x\sqrt{3}} \sin 3x, \quad -\frac{\pi}{3} \leqslant x \leqslant \frac{\pi}{3}$$

(a) Find the x coordinate of the turning point P on C, for which $x > 0$
Give your answer as a multiple of π.

(6)

(b) Find an equation of the normal to C at the point where $x = 0$

(3)

Leave blank

7. (a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}} \ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

Leave blank

1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

Leave blank

5. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{1 + x^2}$

(5)

Leave blank

5. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\dfrac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\dfrac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n - 1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n[2a + (n - 1)d]$