1. (a) Express $3 \cos \theta + 4 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence find the maximum value of $3 \cos \theta + 4 \sin \theta$ and the smallest positive value of θ for which this maximum occurs.

(3)

The temperature, f (t), of a warehouse is modelled using the equation

$$f(t) = 10 + 3 \cos(15t)° + 4 \sin(15t)°,$$

where t is the time in hours from midday and $0 \le t < 24$.

(c) Calculate the minimum temperature of the warehouse as given by this model.

(2)

(d) Find the value of t when this minimum temperature occurs.

(3)
(Total 12 marks)

2. (a) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(b) (i) Prove that

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} \equiv 2 \sec x, \;\; x \neq (2n + 1)\frac{\pi}{2}.$$

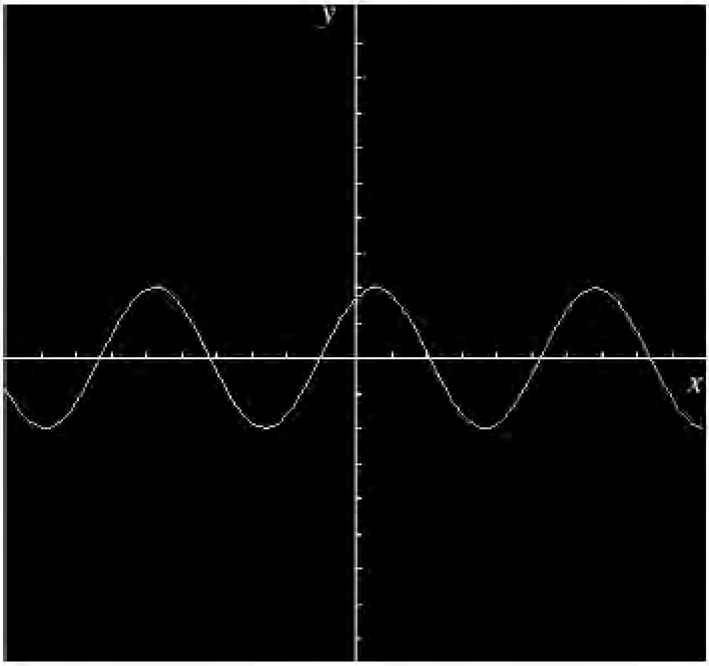
(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)
(Total 11 marks)

3.



The diagram above shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3}\cos x + \sin x.$$

(a) Express the equation of the curve in the form $y = R\sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(b) Find the values of x, $0 \le x < 2\pi$, for which $y = 1$.

(4)
(Total 8 marks)

4. (a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\text{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\text{cosec}^4\theta - \cot^4\theta \equiv \text{cosec}^2\theta + \cot^2\theta.$$

(2)

(c) Solve, for $90° < \theta < 180°$,

$$\text{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)
(Total 10 marks)

5. (a) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$

(3)

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(ii) show that $\frac{dy}{dx} = \sin 2x$

(4)
(Total 12 marks)

6. (a) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \qquad x \neq (n - \tfrac{1}{4})\pi,\ n \in \mathbb{Z}$

(2)

(ii) $\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$

(3)

(b) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)
(Total 12 marks)

7. (a) Differentiate with respect to x

(i) $x^2 e^{3x+2}$,

(4)

(ii) $\dfrac{\cos(2x^3)}{3x}$.

(4)

(b) Given that $x = 4\sin(2y + 6)$, find $\frac{dy}{dx}$ in terms of x.

(5)
(Total 13 marks)

8. $f(x) = 12\cos x - 4\sin x$.

Given that $f(x) = R\cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leq x \leq 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12\cos x - 4\sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)
(Total 12 marks)

9. (a) Given that $2\sin(\theta + 30)° = \cos(\theta + 60)°$, find the exact value of $\tan\theta°$.

(5)

(b) (i) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(ii) Hence solve, for $0 \leq x < 2\pi$,

$$\cos 2x = \sin x,$$

giving your answers in terms of π.

(5)

(iii) Show that $\sin 2y \tan y + \cos 2y \equiv 1$, for $0 \leq y < \frac{1}{2}\pi$.

(3)
(Total 15 marks)

10. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(b) Solve, for $0 \leq \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)
(Total 8 marks)

11. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(b) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(c) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(d) Hence, for $0 \leq \theta < \pi$, solve

$$2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)
(Total 15 marks)

12. (a) Sketch, on the same axes, in the interval $0 \leq x \leq 180$, the graphs of

$y = \tan x°$ and $y = 2 \cos x°$,

showing clearly the coordinates of the points at which the graphs meet the axes.

(4)

(b) Show that $\tan x° = 2 \cos x°$ can be written as

$2 \sin^2 x° + \sin x° - 2 = 0$.

(3)

(c) Hence find the values of x, in the interval $0 \leq x \leq 180$, for which $\tan x° = 2 \cos x°$.

(4)
(Total 11 marks)

13. (i) Given that $\sin x = \frac{3}{5}$, use an appropriate double angle formula to find the exact value of $\sec 2x$.

(4)

(ii) Prove that

$$\cot 2x + \operatorname{cosec} 2x \equiv \cot x, \qquad \left(x \neq \frac{n\pi}{2}, n \in \mathbb{Z}\right).$$

(4)
(Total 8 marks)

14. (i) (a) Express $(12 \cos \theta - 5 \sin \theta)$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 4,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place.

(3)

(ii) Solve

$$8 \cot \theta - 3 \tan \theta = 2,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place.

(5)
(Total 12 marks)

15. (a) Prove that

$$\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \equiv \cos 2\theta.$$

(4)

(b) Hence, or otherwise, prove

$$\tan^2 \frac{\pi}{8} = 3 - 2\sqrt{2}.$$

(5)
(Total 9 marks)

16. (i) Given that $\cos(x + 30)° = 3\cos(x - 30)°$, prove that $\tan x° = -\frac{\sqrt{3}}{2}$.

(5)

(ii) (a) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$.

(3)

(b) Verify that $\theta = 180°$ is a solution of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

(1)

(c) Using the result in part (a), or otherwise, find the other two solutions, $0 < \theta < 360°$, of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

(4)
(Total 13 marks)

17. (a) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Show that the equation $\sec x + \sqrt{3}\,\text{cosec}\, x = 4$ can be written in the form

$$\sin x + \sqrt{3}\cos x = 2\sin 2x.$$

(3)

(c) Deduce from parts (a) and (b) that $\sec x + \sqrt{3}\,\text{cosec}\, x = 4$ can be written in the form

$$\sin 2x - \sin(x + 60°) = 0.$$

(1)

(d) Hence, using the identity $\sin X - \sin Y = 2\cos\frac{X+Y}{2}\sin\frac{X-Y}{2}$, or otherwise, find the values of x in the interval $0 \le x \le 180°$, for which $\sec x + \sqrt{3}\,\text{cosec}\, x = 4$.

(5)
(Total 13 marks)

18. On separate diagrams, sketch the curves with equations

(a) $y = \arcsin x, \quad -1 \leq x \leq 1,$

(b) $y = \sec x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$, stating the coordinates of the end points of your curves in each case.

(4)

Use the trapezium rule with five equally spaced ordinates to estimate the area of the region bounded by the curve with equation $y = \sec x$, the x-axis and the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, giving your answer to two decimal places.

(4)
(Total 8 marks)

19. (a) Prove that for all values of x,

$$\sin x + \sin (60° - x) \equiv \sin (60° + x).$$

(4)

(b) Given that $\sin 84° - \sin 36° = \sin \alpha°$, deduce the exact value of the acute angle α.

(2)

(c) Solve the equation

$$4 \sin 2x + \sin (60° - 2x) = \sin (60° + 2x) - 1$$

for values of x in the interval $0 \leq x < 360°$, giving your answers to one decimal place.

(5)
(Total 11 marks)

20. (a) Using the formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B,$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

show that

(i) $\sin (A + B) - \sin (A - B) = 2 \cos A \sin B,$

(2)

(ii) $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$.

(2)

(b) Use the above results to show that

$$\frac{\sin(A + B) - \sin(A - B)}{\cos(A - B) - \cos(A + B)} = \cot A.$$

(3)

Using the result of part (b) and the exact values of sin 60° and cos 60°,

(c) find an exact value for cot 75° in its simplest form.

(4)
(Total 11 marks)

21. In a particular circuit the current, I amperes, is given by

$$I = 4 \sin \theta - 3 \cos \theta, \qquad \theta > 0,$$

where θ is an angle related to the voltage.

Given that $I = R \sin(\theta - \alpha)$, where $R > 0$ and $0 \le \alpha < 360°$,

(a) find the value of R, and the value of α to 1 decimal place.

(4)

(b) Hence solve the equation $4 \sin \theta - 3 \cos \theta = 3$ to find the values of θ between 0 and 360°.

(5)

(c) Write down the greatest value for I.

(1)

(d) Find the value of θ between 0 and 360° at which the greatest value of I occurs.

(2)
(Total 12 marks)

1.	(a)	$R^2 = 3^2 + 4^2$		M1	
		$R = 5$		A1	
		$\tan \alpha = \dfrac{4}{3}$		M1	
		$\alpha = 53...°$	awrt 53°	A1	4
	(b)	Maximum value is 5	ft their R	B1 ft	
		At the maximum, $\cos(\theta - \alpha) = 1$ *or* $\theta - \alpha = 0$		M1	
		$\theta = \alpha = 53\ ...°$	ft their α	A1 ft	3
	(c)	$f(t) = 10 + 5\cos(15t - \alpha)°$			
		Minimum occurs when $\cos(15t - \alpha)° = -1$		M1	
		The minimum temperature is $(10 - 5)° = 5°$		A1 ft	2
	(d)	$15t - \alpha = 180$		M1	
		$t = 15.5$	awrt 15.5	M1 A1	3

[12]

2.	(a)	$\cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$		M1	
		$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$		M1	
		$= (2\cos^2 x - 1)\cos x - 2(1 - \cos^2 x)\cos x$	any correct expression	A1	
		$= 4\cos^3 x - 3\cos x$		A1	4
	(b)	(i) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$		M1	
		$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$		A1	
		$= \dfrac{2(1+\sin x)}{(1+\sin x)\cos x}$		M1	
		$= \dfrac{2}{\cos x} = 2\sec x$ (*)	cso	A1	4
	(c)	$\sec x = 2$ or $\cos x = \dfrac{1}{2}$		M1	
		$x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$	accept awrt 1.05, 5.24	A1, A1	3

[11]

3.	(a)	$R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$		M1A1	
		$\tan \alpha = \sqrt{3} \Rightarrow \alpha = \dfrac{\pi}{3}$	accept awrt 1.05	M1 A1	4
	(b)	$\sin(x + \text{their } \alpha) = \dfrac{1}{2}$		M1	
		$x + \text{their } \alpha = \dfrac{\pi}{6}\left(\dfrac{5\pi}{6}, \dfrac{13\pi}{6}\right)$		A1	
		$xi = \dfrac{\pi}{2}, \dfrac{11\pi}{6}$	accept awrt 1.57, 5.76	M1 A1	4

The use of degrees loses only one mark in this question.
Penalise the first time it occurs in an answer and then ignore.

[8]

4.	(a)	Dividing $\sin^2 \theta + \cos^2 \theta \equiv 1$ by $\sin^2 \theta$ to give $\dfrac{\sin^2 \theta}{\sin^2 \theta} + \dfrac{\cos^2 \theta}{\sin^2 \theta} \equiv \dfrac{1}{\sin^2 \theta}$	M1	
		Completion: $1 + \cot^2 \theta = \text{cosec}^2 \Rightarrow \text{cosec}^2\theta - \cot^2 \theta \equiv 1$ AG	A1*	2
	(b)	$\text{cosec}^4\theta - \cot^4 \theta \equiv (\text{cosec}^2\theta - \cot^2 \theta)(\text{cosec}^2\theta + \cot^2 \theta)$	M1	
		$\equiv (\text{cosec}^2\theta + \cot^2\theta)$ using (a) AG	A1*	2

*Using LHS = $(1 + \cot^2 \theta)^2 - \cot^4 \theta$, using (a) & elim. $\cot^4\theta$ M1, conclusion {using (a) again} A1**

Conversion to sines and cosines: needs $\dfrac{(1-\cos^2 \theta)(1+\cos^2 \theta)}{\sin^4 \theta}$ for M1

(c)	Using (b) to form $\text{cosec}^2\theta + \cot^2 \theta \equiv 2 - \cot\theta$	M1	
	Forming quadratic in $\cot\theta$	M1	
	$\Rightarrow 1 + \cot^2 \theta + \cot^2 \theta \equiv 2 - \cot\theta$ {using (a)}	M1	
	$2\cot^2\theta + \cot\theta - 1 = 0$	A1	
	Solving: $(2 \cot \theta - 1)(\cot\theta + 1 = 0)$ to $\cot \theta =$	M1	
	$\left(\cot \theta = \dfrac{1}{2}\right)$ or $\cot \theta = -1$	A1	
	$\theta = 135°$ (or correct value(s) for candidate dep. on 3Ms)	A1ft	6

Ignore solutions outside range
Extra "solutions" in range loses A1ft, but candidate may possibly have more than one "correct" solution.

[10]

5. (a) Method for finding $\sin A$ M1

$\sin A = -\frac{\sqrt{7}}{4}$ A1 A1

First A1 for $\frac{\sqrt{7}}{4}$*, exact*

Second A1 for sign (even if dec. answer given)

Use of $\sin 2A \equiv 2 \sin A \cos A$ M1

$\sin 2A = \frac{3\sqrt{7}}{8}$ or equivalent exact A1ft 5

± f.t. Requires exact value, dependent on 2nd M

(b) (i) $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right)$

$\equiv \cos 2x \cos \frac{\pi}{3} - \sin 2x \sin \frac{\pi}{3} + \cos 2x \cos \frac{\pi}{3} + \sin 2x \sin \frac{\pi}{3}$ M1

$\equiv 2 \cos x \cos \frac{\pi}{3}$ A1

[This can be just written down (using factor formulae) for M1 A1]

$= \cos 2x$ AG A1* 3

M1 A1 earned, if $\equiv 2 \cos 2x \cos \frac{\pi}{3}$ *just written down, using factor theorem*

Final A1 requires some working after first result.*

(b) (ii) $\frac{dy}{dx} = 6 \sin x \cos x - 2 \sin 2x$ B1 B1

or $6 \sin x \cos x - 2\sin\left(2x + \frac{\pi}{3}\right) - 2\sin\left(2x - \frac{\pi}{3}\right)$

$= 3\sin 2x - 2\sin 2x$ M1

$= \sin 2x$ AG A1 4

First B1 for 6 sin x cos x ; second B1 for remaining term(s)

[12]

6.	(a)	(i)	$\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x}$	M1	
			$= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x + \sin x}$		
			$= \cos x - \sin x$ **AG**	A1	2
		(ii)	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x)$	M1, M1	
			$= \cos^2 x - \frac{1}{2} - \sin x \cos x$ **AG**	A1	3
	(b)		$\cos\theta\left(\dfrac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \dfrac{1}{2}$		
			$\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$	M1	
			$\cos^2\theta - \cos\theta \sin\theta = \frac{1}{2}$		
			$\frac{1}{2}(\cos 2\theta + 1) - \frac{1}{2}\sin 2\theta = \frac{1}{2}$	M1	
			$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$		
			$\sin 2\theta = \cos 2\theta$ **AG**	A1	3
	(c)		$\sin 2\theta = \cos 2\theta$		
			$\tan 2\theta = 1$	M1	
			$2\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$	A1 for 1	
			$\theta = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$	M1 (4 solns)	
				A1	4

[12]

7	(a)	(i)	$\dfrac{d}{dx}(e^{3x+2}) = 3e^{3x+2}$ (or $3e^2e^{3x}$)	At any stage	B1	
			$\dfrac{dy}{dx} = 3x^2 e^{3x+2} + 2xe^{3x+2}$	Or equivalent	M1 A1+A1	4

(ii) $\dfrac{d}{dx}(\cos(2x^3)) = -6x^2 \sin(2x^3)$ At any stage M1 A1

$\dfrac{dy}{dx} = \dfrac{-18x^3 \sin(2x^3) - 3\cos(2x^3)}{9x^2}$ M1 A1 4

Alternatively using the product rule for second M1 A1

$y = (3x)^{-1} \cos(2x^3)$

$\dfrac{dy}{dx} = -3(3x)^{-2} \cos(2x^3) - 6x^2 (3x)^{-1} \sin(2x^3)$

Accept equivalent unsimplified forms

(b) $1 = 8\cos(2y+6)\dfrac{dy}{dx}$ or $\dfrac{dx}{dy} = 8\cos(2y+6)$ M1

$\dfrac{dy}{dx} = \dfrac{1}{8\cos(2y+6)}$ M1 A1

$\dfrac{dy}{dx} = \dfrac{1}{8\cos\left(\arcsin\left(\dfrac{x}{4}\right)\right)}$ $\left(= (\pm)\dfrac{1}{2\sqrt{(16-x^2)}}\right)$ M1 A1 5

[13]

8. (a) $R\cos\alpha = 12$, $R\sin\alpha = 4$

$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6 M1 A1

$\tan\alpha = \dfrac{4}{12}$, $\Rightarrow$ $\alpha \approx 18.43°$ awrt 18.4° M1, A1 4

(b) $\cos(x + \text{their } \alpha) = \dfrac{7}{\text{their } R}$ (≈ 0.5534) M1

$x + \text{their } \alpha = 56.4°$ awrt 56° A1

$= \ ...\ , 303.6°$ 360° – their principal value M1

$x = °38.0, 285.2°$ Ignore solutions out of range A1, A1 5

If answers given to more than 1 dp, penalise first time then accept awrt above.

(c) (i) minimum value is $-\sqrt{160}$ ft their R B1ft

		(ii)	$\cos(x + \text{their } \alpha) = -1$		M1	
			$x \approx 161.57°$	cao	A1	3

[12]

9.	(a)		$2\sin(\theta + 30)° = \cos(\theta + 60)°$ $2\sin\theta°\cos30° + 2\cos\theta°\sin30° = \cos\theta°\cos60° - \sin\theta°\sin60°$		B1B1	
			$\frac{2\sqrt{3}}{2}\sin\theta° + \frac{2}{2}\cos\theta° = \frac{1}{2}\cos\theta° - \frac{\sqrt{3}}{2}\sin\theta°$		M1	
			Finding $\tan\theta°$, $\tan\theta° = -\frac{1}{3\sqrt{3}}$ or equiva. Exact		M1,A1	5
	(b)	(i)	Setting $A = B$ to give $\cos 2A = \cos^2 A - \sin^2 A$		M1	
			Correct completion:$= (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$		A1	2
			Need to see intermediate step above for A1			
		(ii)	Forming quadratic in $\sin x$ $[2\sin^2 x + \sin x - 1 = 0]$		M1	
			Solving $[(2\sin x - 1)(\sin x + 1) = 0$ or formula] $[\sin\theta = ½$ or $\sin\theta = -1]$		M1	
			$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$;		A1,A1ft	
			A1ft for $\pi - $ "α"			
			$\theta = \frac{3\pi}{2}$		A1	5
		(iii)	LHS $= 2\sin y\cos y\,\frac{\sin y}{\cos y} + (1 - 2\sin^2 y)$		B1M1	
			B1 use of $\tan y = \frac{\sin y}{\cos y}$, *M1 forming expression in* $\sin y$, $\cos y$ *only*			
			Completion: $= 2\sin^2 y + (1 - 2\sin^2 y) = 1$ AG		A1	3
			[Alternative: LHS $= \frac{\sin 2y\sin y + \cos 2y\cos y}{\cos y}$ *B1M1* $= \frac{\cos(2y - y)}{\cos y} = 1$ *A1]*			

[15]

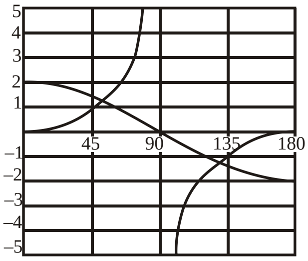
10.	(a)	Dividing by $\cos^2\theta$: $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$	M1	
		Completion: $1 + \tan^2\theta \equiv \sec^2\theta$	A1	2
		(no errors seen)		
	(b)	use of $1 + \tan^2\theta = \sec^2\theta$: $2(\sec^2\theta - 1) + \sec\theta = 1$	M1	
		$[2\sec^2\theta + \sec\theta - 3 = 0]$		
		Factorising or solving: $(2\sec\theta + 3)(\sec\theta - 1) = 0$		
		$[\sec\theta = -\frac{3}{2}$ or $\sec\theta = 1]$		
		$\theta = 0$	B1	
		$\cos\theta = -\frac{2}{3}$; $\theta_1 = 131.8°$	M1 A1	
		$\theta_2 = 228.2°$	A1 ft	6
		[A1ft for $\theta_2 = 360° - \theta_1$]		

[8]

11.	(a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1	
		$= (1 - \sin^2 A)$; $-\sin^2 A = 1 - 2\sin^2 A$ (*)	A1	2
	(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta$; $-3(1 - 2\sin^2\theta) - 3\sin\theta + 3$	B1; M1	
		$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1	
		$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1	4
	(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$		
		Complete method for R (may be implied by correct answer)		
		$[R^2 = 4^2 + 6^2$, $R\sin\alpha = 4$, $R\cos\alpha = 6]$	M1	
		$R = \sqrt{52}$ or 7.21	A1	
		Complete method for α; $\alpha = 0.588$	M1 A1	4
		(allow 33.7°)		
	(d)	$\sin\theta(4\cos\theta + 6\sin\theta - 3) = 0$	M1	
		$\theta = 0$	B1	
		$\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160..$ (24.6°)	M1	
		$\theta + 0.588 = (0.4291), 2.7125$ [or $\theta + 33.7° = (24.6°), 155.4°$]	dM1	
		$\theta = 2.12$	cao A1	5

[15]

12. (a)



Tangent graph shape	M1	
180 indicated	A1	
Cosine graph shape	M1	
2 and 90 indicated	A1	4

Allow separate sketches.

(b)

Using $\tan x = \dfrac{\sin x}{\cos x}$ and multiplying both sides by $\cos x$. ($\sin x = 2\cos^2 x$)	M1	
Using $\sin^2 x + \cos^2 x = 1$	M1	
$2\sin^2 x + \sin x - 2 = 0$ (*)	A1	3

(c)

Solving quadratic: $\sin x = \dfrac{-1 \pm \sqrt{17}}{4}$ (or equiv.)		M1 A1	
$x = 51.3$	(3 s.f. or better, 51.33…) α	A1	
$x = 128.7$ (accept 129)	(3 s.f. or better) $180 - \alpha$ ($\alpha \neq 90n$)	B1ft	4

[11]

13. (i)

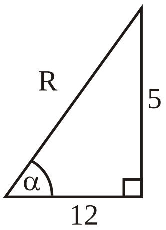
A correct form of $\cos 2x$ used		M1	
$1 - 2\left(\dfrac{3}{5}\right)^2$ or $\left(\dfrac{4}{5}\right)^2 - \left(\dfrac{3}{5}\right)^2$ or $2\left(\dfrac{4}{5}\right)^2 - 1$	$\left\{\dfrac{7}{25}\right\}$	A1	
$\sec 2x = \dfrac{1}{\cos 2x}$; $= \dfrac{25}{7}$ or $3\dfrac{4}{7}$		M1A1	4

(ii)

(a) $\dfrac{\cos 2x}{\sin 2x} + \dfrac{1}{\sin 2x}$ or (b) $\dfrac{1}{\tan 2x} + \dfrac{1}{\sin 2x}$	M1	
Forming single fraction (or ** multiplying both sides by $\sin 2x$)	M1	
Use of correct trig. formulae throughout and producing expression in terms of $\sin x$ and $\cos x$	M1	
Completion (cso) e.g. $\dfrac{2\cos^2 x}{2\sin x\cos x} = \dfrac{\cos x}{\sin x} = \cot x$ (*)	A1	4

[8]

14. (a) (i) $12\cos\theta - 5\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha.$



$R^2 = 5^2 + 12^2, \Rightarrow \underline{R = 13}$ M1, A1

$\tan\,\alpha = \dfrac{5}{12}, \Rightarrow \alpha = 22.6°$ (AWRT 22.6)

or 0.39^C (AWRT 0.39^C) M1, A1 4

M1 for correct expression for R or R^2

M1 for correct trig expression for α

(b) (i) $\cos(\theta + 22.6) = \dfrac{4}{13}$ M1

$\theta + 22.6 = 72.1,$ M1

$\underline{\theta = 49.5}$ (only) A1 3

M1 $\cos(\theta + \alpha) = \dfrac{4}{R}$

M1 $\theta + \alpha = \ldots$ *ft their R*

(ii) $\dfrac{8}{\tan\theta} - 3\tan\theta = 2$ M1

i.e. $0 = 3\tan^2\theta + 2\tan\theta - 8$ M1

$0 = (3\tan\theta - 4)(\tan\theta + 2)$ M1

$\tan\theta = \dfrac{4}{3}$ or -2

$\tan\theta = \dfrac{4}{3} \Rightarrow \underline{\theta = 53.1}$ A1

[*ignore θ not in range e.g. θ = 116.6*] A1 5

M1 Use of cot $\theta = \dfrac{1}{\tan\theta}$

M1 3TQ in tan θ = 0

M1 Attempt to solve 3TQ = 0

A1 For Final A mark must deal with tan θ = – 2

[12]

15. (a) $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}} \left(\text{or } \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta} \text{ or equivalent} \right)$ — $\dfrac{\text{M1}}{\text{M1}}$

$\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} = \dfrac{\cos 2\theta}{1} = \cos 2\theta\ (*)$ — M1 A1 — 4

cso

(b) $\theta = \dfrac{\pi}{8}$, $\cos 2\theta = \dfrac{1}{\sqrt{2}}$ — M1

$\dfrac{1-t^2}{1+t^2} = \dfrac{1}{\sqrt{2}}$ — M1

$t^2 = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ — M1

$= \dfrac{\sqrt{2}-1}{\sqrt{2}+1}.\dfrac{\sqrt{2}-1}{\sqrt{2}-1} = 3 - 2\sqrt{2}\ (*)$ — M1 A1 — 5

cso

[9]

Alternative to (b)

$\dfrac{2t}{1-t^2} = \tan 2\theta = 1$ — M1

$t^2 + 2t - 1 = 0$ — M1

$t = \sqrt{2} - 1$ — M1

$t^2 = (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}\ (*)$ — M1 A1 — 5

cso

16. (i) $\cos x \cos 30 - \sin x \sin 30 = 3(\cos x \cos 30 + \sin x \sin 30)$ — M1

Correct use of cos(x ± 30)

$\Rightarrow \sqrt{3}\cos x - \sin x = 3\sqrt{3}\cos x + 3\sin x$ — M1, A1

Sub. for sin 30 etc
decimals M1, surds A1

i.e. $-4\sin x = 2\sqrt{3}\cos x \rightarrow \tan x = -\dfrac{\sqrt{3}}{2}\ (*)$ — M1, A1cso — 5

Collect terms and use $\tan x = \dfrac{\sin x}{\cos x}$

(ii) (a)	LHS = $\dfrac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$	M1; A1	
	Use of cos 2A or sin 2A; both correct		
	= $\dfrac{\sin\theta}{\cos\theta}$ = $\underline{\tan\theta}$ (*)	A1 cso	3
(b)	Verifying: 0 = 2 – 2 (since sin 360 = 0, cos 360 = 1)	B1 cso	1
(c)	Equation $\rightarrow$ 1 = $\dfrac{2(1-\cos 2\theta)}{\sin 2\theta}$	M1	
	Rearrange to form $\dfrac{1-\cos 2\theta}{\sin 2\theta}$		
	$\Rightarrow \tan\theta = \frac{1}{2}$ or $\cot\theta = 2$	A1	
	i.e. θ = (26.6° or 206.6°) AWRT $\underline{27°, 207°}$		
	1st solution	M1	
	must be $\tan\theta = \pm\frac{1}{2}$ *or 2*		
	(both)	A1	4

[13]

Alt 1

(c)	$2\sin\theta\cos\theta = 2 - 2(1 - 2\sin^2\theta)$	M1
	Use of cos 2A <u>and</u> sin 2A	
	$0 = 2\sin\theta(2\sin\theta - \cos\theta)$	
	$\Rightarrow (\sin\theta = 0)\ \tan\theta = \frac{1}{2}$ etc, as in scheme	A1

Alt 2

(c) $2\cos 2\theta + \sin 2\theta = 2 \Rightarrow \cos(2\theta - \alpha) = \frac{2}{\sqrt{5}}$ M1

$\alpha = 22.6$ (or 27) A1

$2\theta = 2\alpha, 360, 360 + 2\alpha$
$\theta = \alpha, 180 + \alpha$ i.e. $\theta = 27°$ or $207°$ (or 1 dp)

$\theta = \alpha$ or $180 + \alpha$ M1
A1 both

17. (a) $\sin x + \sqrt{3}\cos x = R\sin(x + \alpha)$

$= R(\sin x\cos\alpha + \cos x\sin\alpha)$ M1

$R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ A1

Method for R or α, e.g. $R = \sqrt{(1 + 3)}$ or $\tan\alpha = \sqrt{3}$ M1

Both $R = 2$ and $\alpha = 60$ A1 4

(b) $\sec x + \sqrt{3}\operatorname{cosec} x = 4 \Rightarrow \frac{1}{\cos x} + \frac{\sqrt{3}}{\sin x} = 4$ B1

$\Rightarrow \sin x + \sqrt{3}\cos x = 4\sin x\cos x$ M1

$= 2\sin 2x$ (*) M1 3

(c) Clearly producing $2\sin 2x = 2\sin(x + 60)$ A1 1

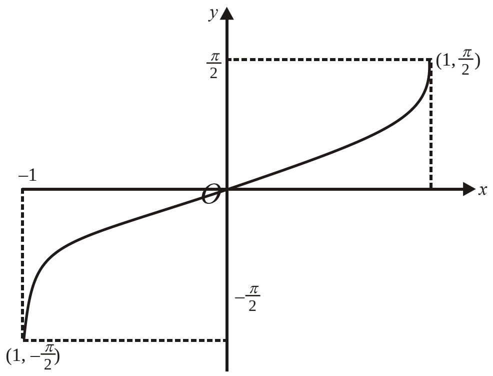
(d) $\sin 2x - \sin(x + 60) = 0 \Rightarrow \cos\frac{3x+60}{2}\sin\frac{x-60}{2} = 0$ M1

$\cos\frac{3x+60}{2} = 0 \Rightarrow x = 40°, 160°$ M1 A1 A1 ft

$\sin\frac{x-60}{2} = 0 \Rightarrow x = 60°$ B1 5

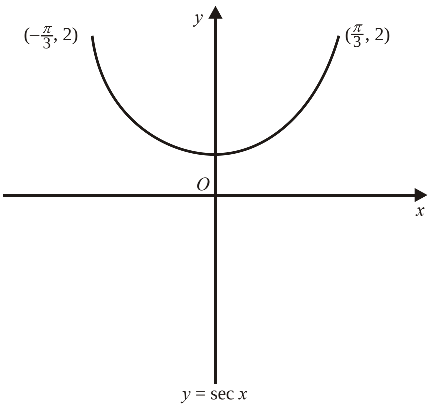
[13]

18. (a)



y = arcsin x

(a) Shape correct
passing through O: G1;
end-points: G1 2



(b)

Shape correct,
symmetry in Oy: G1
end-points: G1 2

(c)

x	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
sec x	2	1.155	1	1.155	2

Area estimate = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x \, \mathrm{d}x = \frac{\pi}{6}\left[\frac{2+2}{2} + 1.155 + 1 + 1.155\right]$ M1 A1 A1

= 2.78 (2 d.p.) A1 4

[8]

19.	(a)	LHS = $\sin x + \sin 60° - \cos 60° \sin x$	M1	
		$= \sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$	A1	
		RHS = $\sin 60° \cos x + \cos 60° \sin x$	M1	
		$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$ = LHS	A1	4
	(b)	From (*a*), $\sin(60° + x) - \sin(60° - x) = \sin x$		
		$x = 24° \Rightarrow \sin 84° - \sin 36° = \sin 24°$	M1	
		$\Rightarrow \alpha = 24°$	A1	2
	(c)	$3\sin 2x + \sin 2x + \sin(60° - 2x) = \sin(60° + 2x) - 1$	M1	
		Using (*a*), $3\sin 2x = -1$	A1	
		$2x = 199.47°$ or $340.53°$	M1	
		$x = 99.7°, 170.3°$	A1	
		or $279.7°, 350.3°$	A1 ft	5

[11]

20.	(a)	(i) $\sin(A + B) - \sin(A - B)$		
		$= \sin A \cos B + \sin B \cos A - \sin A \cos B + \sin B \cos A$	M1	
		$= 2\sin B \cos A$ **(*)**	A1 cso	2
		(ii) $\cos(A - B) - \cos(A + B)$		
		$= \cos A \cos B + \sin A \sin B - \cos A \cos B + \sin A \sin B$	M1	
		$= 2\sin A \sin B$ **(*)**	A1 cso	2
	(b)	$\frac{\sin(A+B) - \sin(A-B)}{\cos(A-B) - \sin(A+B)} = \frac{2\sin B \cos A}{2\sin A \sin B}$	M1	
		$= \frac{\cos A}{\sin A}$	A1	
		$= \cot A$ **(*)**	A1 cso	3

(c)	Let $A = 75°$ and $B = 15°$	B1	
	$\dfrac{\sin 90^{\circ} - \sin 60^{\circ}}{\cos 60^{\circ} - \cos 90^{\circ}} = \cot 75^{\circ}$	M1	
	$\cot 75° = \dfrac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2} - 0} = 2 - \sqrt{3}$	M1 A1	4

[11]

21.	(a)	$4 \sin \theta - 3 \cos \theta = R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$ $\sin \theta$ terms give $4 = R \cos \alpha$ $\cos \theta$ terms give $3 = R \sin \alpha$		
		$\tan \alpha = 0.75$	M1	
		$\alpha = 36.9^{\circ}$	A1	
		$R^2 = 4^2 + 3^2 = 25 \Rightarrow R = 5$	M1 A1	4
	(b)	$5 \sin (\theta - 36.9^{\circ}) = 3$		
		$\sin (\theta - 36.9^{\circ}) = 0.6$	M1	
		$\theta - 36.9^{\circ} = 36.9^{\circ}, 143.1$	A1 M1	
		$\theta = 73.7^{\circ}, 180°$		
		awrt 74°	A1 A15	
	(c)	Max value 5	B1	1
	(d)	$\sin (\theta - 36.9^{\circ}) = 1$	M1	
		$\theta - 36.9° = 90°$		
		$\theta = 90° + 36.9^{\circ} = 126.9^{\circ}$	A1	2

[12]

1. Part (a) was very well done and the majority gained full marks. A few candidates found the complementary angle or gave their answer in radians. There was some evidence of candidates running out of time in part (b) but, given part (a), it was possible just to write down the answers to this part and many were able to do this. Some very basic manipulation errors were seen; for example, proceeding from $5\cos(\theta - 53.1°) = 5$ to $\cos(\theta - 53.1°) = 0$. Those who were able to interpret the model in parts (c) and (d) and see the connection with part (a) frequently gained full marks very quickly. However the majority of candidates failed to spot any connection with part (a) and frequently just substituted $t = 0$. Fortunately very few candidates attempted calculus and almost none of those who did recognised that differentiating functions in degrees is not a straightforward process.

2. Part (a) proved to be the most difficult part of this question. Those who had the foresight to choose the correct identities, produced the correct answer of $4\cos^3 x - 3\cos x$ quickly. Unfortunately many candidates were unable to produce correct double angle identities, or chose an ill advised version, making little headway as a result.

Part (b) produced better attempts with most candidates scoring some marks on this question. It was perhaps disturbing that the error $(1 + \sin x)^2 = 1 + \sin^2 x$ was not infrequently seen, resulting in the loss of 3 of the 4 marks in this part. Part (c) produced a great many correct solutions, the only consistent errors being a lack of knowledge of radians or the inability to find the angle in the fourth quadrant.

3. There were a minority who left this question blank, showing an incomplete knowledge of the syllabus. However, on the whole, this was the best tackled question on the paper and the great majority could obtain values for R and α and use these values to demonstrate a valid method to solve part (b). A few did get their value for tan α inverted and obtained $\alpha = \frac{\pi}{6}$. The mark scheme allowed these candidates to gain 3 of the 4 marks in part (b). The wording of the question does imply that α is in radians and those who gave the answer as 60° lost one mark. Such errors are only penalised once in a question and if the candidate carried on into part (b) in degrees, they were allowed full marks there if their solution was otherwise correct. Part (b) was well done but the second solution $x = \frac{11\pi}{6}$ was often overlooked. A few candidates attempted part (b) by squaring. This comes out quite well but introduces a incorrect solution within the specified range and full marks were only given if this solution was rejected.

4. In part (a) most candidates took the given identity, divided by $\sin^2 \theta$ and correctly manipulated their equation to obtain the required result. Correct solutions were also given by those who started with the expression $\text{cosec}\,^2\theta – \cot^2 \theta$ and used the given identity to show that this expression came to 1. However, those candidates who assumed the result (i.e.$\text{cosec}\,^2\theta – \cot^2 \theta \equiv 1$) and manipulated this to obtain the given identity were not given the final mark unless they drew at least a minimal conclusion from this (e.g. hence result). Candidates who understood the link between parts (a) and (b) and used the difference of two squares completed part (b) easily. Other, more lengthy, solutions were also seen. Weaker candidates tended to produce circular arguments or use incorrect statements such as $\text{cosec}\,^2\theta – \cot^2 \theta = 1 \Rightarrow \text{cosec}\,^4\theta – \cot^4 \theta = 1$. The first method mark for linking parts (b) and (c) was gained by most candidates. Many were also able to use the result in part (a) to obtain a quadratic in cotθ Candidates who did not spot these links were usually unsuccessful. For those candidates who obtained a quadratic equation, factorising was generally done well although less proficiency was seen in giving solutions to the resulting trigonometric equations in the correct range.

5. Marks were only given in part (a) if a method was seen, so no credit was given to answers obtained entirely from a calculator. Many candidates were able to find the exact value of the sine of an acute angle whose cosine is $\frac{3}{4}$ and were also able to use the double angle formula for sin 2θ. However, candidates found it a greater challenge to work out in which quadrant 2A would appear and relate this to the correct sign. Candidates who incorrectly used a 3, 4, 5 triangle seemed unperturbed by producing a value for 2 sin$A$ outside the range $-1 \leq 2 \sin A \leq 1$. Many correct solutions were seen to part (b)(i) by candidates who used either of the relevant trigonometric identities given in the formulae book. A few candidates spent unnecessary time deriving results which are given in the formulae book and some were not able to evaluate $\cos\frac{\pi}{3}$. Weaker candidates tended to ignore trigonometric identities and write incorrect statements of the form $\cos(A + B) = \cos A + \cos B$. In part (b)(ii), those candidates who attempted to find $\frac{dy}{dx}$ from the form given in the question were rarely able to continue beyond the differential to find the given answer. However, it was often done successfully by those candidates who used the link between parts (i) and (ii). Some candidates used various trigonometric identities to rearrange their expression for y before differentiating. Although these solutions were sometimes long-winded, they were quite often successful. This was another occasion on which candidates needed to be careful to show all the steps in their work to reach a convincing conclusion to a given answer. As in similar questions, some candidates tried to fool the examiner by inserting the given answer following several previous lines of incorrect working.

6. **Pure Mathematics P2**

In (ai) most candidates identified the appropriate form of the double angle formula to substitute in the left hand side of this identity, but the following cancelling was frequently incorrect. The most popular (and incorrect) answer was:
$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{\cos^2 x}{\cos x} - \frac{\sin^2 x}{\sin x} = \cos x - \sin x$. Some candidates started by multiplying both sides of the identity by ($\cos x + \sin x$) and multiplying out the difference of two squares. This method was acceptable provided that the candidate clearly picked out this form of the double angle formula as a known result and completed their argument correctly. The response to (aii) was usually much more successful, but many candidates clearly have problems over the distinctions between $\cos^2 x$, $\cos 2x$ and $2\cos x$.

In (b) most candidates started by applying (ai), but usually went on to pick out the double angle formulae in their working rather than apply (aii).

In the final part, some candidates expended considerable time and effort in trying to expand both sides of this equation to obtain an equation in a single trig.function. This invariably involved a false step of the form $\sin 2x = 2\sin x$. Some tried squaring both sides of the equation which allowed them an equation in $\sin 2x$ or $\cos 2x$, but also introduced false solutions that were not discarded. Those candidates who did realise that this equation is equivalent to an equation in $\tan 2x$ often went on to obtain the correct answers, but common errors here involved answers in degrees, radians but not expressed as multiples of π, or an incomplete set of solutions. Some candidates had difficulty in rewriting the equation and arrived at the false equation $\tan 2x = 0$.

Core Mathematics C3

Candidates who gained both the marks in part (a)(i) were in the minority. Most realised that the identity $\cos 2x \equiv \cos^2 x - \sin^2 x$ was the appropriate identity to use but the false working $\frac{\cos^2 x - \sin^2 x}{\cancel{\cos x} + \cancel{\sin x}} = \cos x - \sin x$ was as common as the correct working using the difference of two squares. Part (a)(ii) was better done and many gained full marks here. An unexpected aspect of many of the proofs seen here, and in part (b), was that the formula $\cos 2x \equiv \cos^2 x - \sin^2 x$ seemed to be much better known than $\cos 2x \equiv \cos^2 x - x - 1$. Many produced correct proofs using the former of these versions of the double angle formulae and $\cos 2x + \sin^2 x = 1$. This was, of course, accepted for full marks but did waste a little time and these times can mount up in the course of a paper. There were many correct proofs to part (b). The majority started using part (a)(i) but then finished their demonstrations using double angle formulae rather than part (a)(ii). Part (d) was clearly unexpected in this context. The topic is in the C2 specification and may be tested on this paper. Among those who did realise they could divide by $\cos 2\theta$, the error $\tan 2\theta = 0$ was common. Some very complicated methods of solution were seen but these were rarely successful. An error of logic was frequently. On reaching an equation of the form $f(\theta)\, g(\theta) = A$. where A is a non-zero constant, candidates proceeded to deduce that either $f(\theta) = A$ or $g(\theta) = A$. A few candidates drew diagrams showing clearly the symmetrical nature of $\cos 2\theta$ and $\sin 2\theta$ and deduced the four solutions from this. Such an approach is sound and was awarded full marks.

7. Part (a)(i) was generally well done. In part (a)(ii), many had difficulty in differentiating $\cos(2x^3)$ and $-\sin(6x^2)$ was commonly seen. When the quotient rule was applied, it was often very unclear if candidates were using a correct version of the rule and candidates should be encouraged to quote formulae they are using. Notational carelessness often loses marks in questions of this kind. If $\cos(2x^3)$ is differentiated and the expression $6x^2 - \sin(2x^3)$ results, the examiner cannot interpret this as $6x^2 \times (-\sin(2x^3))$ unless there is some evidence that the candidate interprets it this way. A substantial proportion of those who wrote down $6x^2 - \sin(2x^3)$ showed in their later work that it had been misinterpreted. Similarly in the denominator of the quotient rule $3x^2$, as opposed to $(3x)^2$ or $9x^2$, cannot be awarded the appropriate accuracy mark unless a correct expression appears at some point. The use of the product rule in such questions is a disadvantage to all but the ablest candidates. In this case, few who attempted the question this way could handle the 3 correctly and the negative indices defeated many.

Part (b) was clearly unexpected by many candidates and some very lengthy attempts began by expanding sin $(2y + 6)$ as $\sin 2y \cos 6 + \cos 2y \sin 6$. This led to attempts to use the product rule and errors like $\frac{\mathrm{d}}{\mathrm{d}x}(\sin 6) = \cos 6$ were frequent. Many could, however, get the first step $\frac{\mathrm{d}x}{\mathrm{d}y} = 8\cos(2y+6)$ but on inverting to get $\frac{\mathrm{d}x}{\mathrm{d}y}$ simply turned the y into an x. Those reaching the correct $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{\cos(2y+6)}$ usually stopped there and the correct solution, in terms of x, was achieved by less than 10% of candidates. The answer $\frac{1}{8\cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$ was accepted for full marks.

8. Part (a) was well done. Such errors as were seen arose from $\alpha = \pm 3$ or $\tan\alpha = -\frac{1}{3}$. In part (b), the majority could find the answer 38.0 although a substantial minority, with the correct method, failed to obtain this result through premature approximation. If an answer is required to one decimal place the candidate must work to at least 2 decimal places. The second answer proved more difficult. Many candidates produced their "secondary value" at the wrong place in their solution, giving the value of $360^\circ - \left(\arccos\frac{7}{\sqrt{160}} - \alpha\right) \approx 322.0^\circ$ instead of

$$360^\circ - \arccos\frac{7}{\sqrt{160}} - \alpha \approx 285.2^\circ .$$

Part (c) was not well done. In (c)(i) 12 or 0 were often given as the minimum value and few realised that the answer to (c)(ii) was the solution of $\cos(x + a) = -1$. Many produced solutions involving R. A few tried differentiation and this was rarely successful.

9. A few candidates offered no attempt at this question, but those who did attempt it seemed to score well.

(a) Many managed to expand the compound angles. Most seemed to be comfortable working with surds, and realised that tan□□θ was obtained by dividing □sinθ□by cosθ. A lot did get to a correct form for the answer, although there were many sign slips and errors in combining √3 and √3/2.

(b) (i) Many candidates produced very quick answers, although some could not progress beyond the initial statement $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(ii) Most candidates made the link with the previous part and went on to form the required quadratic equation. Many seemed comfortable working in radians, although some worked in degrees and then converted their answers. Solutions outside the range were quite often seen, with $-\pi/2$ being particularly common. Several candidates reached the stage $1-2\sin^2 x = \sin x$, but could make no further progress. There were also several spurious attempts to solve this equation in this form. Some candidates ignored the instruction "hence". A few of these were able to deduce the correct values for x by considering sketches of the functions.

(iii) Many candidates produced very quick answers by looking at what they had shown in (b)(i), although in some cases after a page of work candidates had horrific expressions involving $\cot y$ and $\sec^2 y$. A number of candidates were not able to make progress because they did not attempt to rewrite $\tan y$. Some candidates resorted to showing that the relation was true for one of two specific values of y that they chose. There was a common misread in this part, with many candidates seeing $2y$ in place of $\sin 2y$.

10. (a) A variety of methods were used and most candidates were able to prove the identity. Many divided the given statement $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$. Others started with the answer, replacing tan with sin/cos and sec with 1/cos. (Where an answer is printed it is important that each step of the argument is clearly shown to gain full credit.)

(b) Most candidates solved the trigonometric equation confidently, but many used sines and cosines rather than take the lead from the question and using sec. Factorisation was good and the solution $\theta = 0$ was usually found (but also 360, which was outside the range). For those who used the correct equation, the other two angles were found correctly. A number of candidates changed sec into $\sec^2$ at the start of this question, changing the question quite radically. There were not many who needed the final follow through mark, since most who reached that stage had the correct value for θ.

11. This question yielded the most variety of solutions: some showed good quality mathematics with concise methods and accurate answers.

(a) This was generally accessible to the candidates, but some could not move past the given identity.

(b) There were many correct answers to this part, but candidates did not always show sufficient working which is crucial when answers are given on the paper. There were some sign errors in the processing steps. Most candidates worked from left to right.

(c) The method here was generally well known but inaccuracies occurred and e.g. $R\sin\alpha = 6$ was too often seen. Some candidates failed to give their answers in this part correct to at least 3sf.

(d) Many candidates failed to notice the connection between b) and d). Not all candidates were able to solve the trig equation successfully, with sinθ sometimes being lost rather than giving the solution θ = 0. Those that did go on to solve the remaining equation were frequently unable to proceed beyond inverse sin to the correct answer 2.12. Some candidates attempted a squaring approach but generally ended up with excess solutions. Candidates did not always work in radians to 3 sf.

12. Graph sketches in part (a) were often disappointing, with the tangent graph in particular proving difficult for many candidates. While some indicated the asymptote clearly, others seemed unsure of the increasing nature of the function or of the existence of a separate branch. Sketches of $y = 2\cos x$ were generally better, although some confused this with $y = \cos 2x$.

Part (b) was usually well done, with most candidates being aware of the required identities $\tan x = \dfrac{\sin x}{\cos x}$ and $\sin^2 x + \cos^2 x = 1$.

Although in part (c) some candidates produced a factorisation of the quadratic function, the majority used the quadratic formula correctly to solve the equation.

A few the omitted the second solution, but apart from this the main mistake was to approximate prematurely and then to give the answer to an inappropriate degree of accuracy (e.g. sin x = 0.78, therefore x = 51.26).

13. Only the more able candidates produced concise correct solutions to this question. In part (i) candidates were required to use an appropriate double angle formula; finding x from the calculator and then substituting the result in sec $2x$ = $\dfrac{1}{\cos 2x}$ only gained one mark. The other most common error seen was to evaluate $\left\{1 - 2\sin^2\left(\frac{3}{5}\right)\right\}$ instead of $\left\{1 - 2\left(\frac{3}{5}\right)^2\right\}$ for cos2x, but $\dfrac{1}{1 - 2\sin^2 x}$ becoming $1 - ½\sin^2 x$ was also noticed too often In part (ii) candidates who rewrote cot $2x$ as $\dfrac{\cos 2x}{\sin 2x}$, rather than $\dfrac{1}{\tan 2x}$ and then $\dfrac{1 - \tan^2 x}{2\tan x}$, made the most progress but it was disappointing to see the problems this part caused and the amount of extra space that many candidates required.

14. While many candidates demonstrated a clear understanding of the method required for part (i)(a), there was widespread confusion over what to do with the minus sign, and far too many candidates who were happy with statements such as $\sin\alpha = 5$ and $\cos\alpha = 12$.

Those who obtained 13 and 22.6 in part (a) usually went on to solve part (b) successfully. In (ii) most knew a correct definition for $\cot\theta$, but usually only those who formed a quadratic in $\tan\theta$ were able to solve the equation successfully. Here, and in part (b), the instruction to give answers to 1 decimal place was occasionally ignored.

There was some use of graphical calculators to solve part (ii). An answer of 53.1 can be obtained quite quickly this way, but this does not show that this is the only solution in the range and so no credit was given. Whilst candidates can use these calculators in P2 they should be encouraged to show their working in questions such as this if they wish to gain full credit.

15. This proved the most testing question on the paper. In part (a) many were able to make a sensible start on the left hand side of the identity by using either (or both) of $\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$ or $\tan^2\theta = \sec^2\theta - 1$ in the numerator and denominator. However, simplifying the resulting expression to the point where a double angle formula could be used proved too difficult for the majority of students. Part (b) proved beyond all but a minority of candidates. Very few candidates were able to make the connection between the two parts of the question.

16. The examiners were pleased with the quality of many responses to this question, in the past the trigonometry topic has not been answered well.

Most candidates were able to start part (i) correctly, and usually they then substituted for sin 30 and cos 30. There were a number of processing errors, lost minus signs or 2s, in the final stage to obtain $\tan x$ from $\dfrac{\sin x}{\cos x}$, but many fully correct solutions were seen.

In part (ii)(a) most knew the formula for $\sin 2x$ but those who started with $\cos 2x = \cos^2 x - \sin^2 x$ often failed to use brackets carefully and were unable to establish the result accurately. In part (b) most used the simple substitution as expected but some candidates tried to rearrange the equation and often this led to errors creeping in. The final part of the question discriminated well. The most successful solutions usually saw the link with part (a) and used $\dfrac{2(1-\cos 2\theta)}{\sin 2\theta} = 2\tan\theta$ but often this was put equal to 0, rather than 1, and no further progress was made. The stronger candidates sailed through all the parts of this question and produced some impressive solutions.

17. This question tested several aspects of this section of the syllabus and so it was good to see so many candidates confidently progress through the question until part (d).

In part (d) many candidates who correctly reached the stage $2\cos\left(\frac{3x+60}{2}\right)\sin\left(\frac{x-60}{2}\right) = 0$ seemed unable to continue, and others expanded at length producing copious amounts of irrelevant work. The important result that, if $pq = 0$ then either $p = 0$ or $q = 0$ seemed to be unfamiliar in this context. The minority of candidates who were familiar with the method went on to gain marks but it was rare to see all 5 marks earned in this part.

18. No Report available for this question.

19. No Report available for this question.

20. No Report available for this question.

21. No Report available for this question.