

## Core 3 Integration Questions

3 (a) (i) Given that  $f(x) = x^4 + 2x$ , find  $f'(x)$ . (1 mark)

(ii) Hence, or otherwise, find  $\int \frac{2x^3 + 1}{x^4 + 2x} dx$ . (2 marks)

(b) (i) Use the substitution  $u = 2x + 1$  to show that

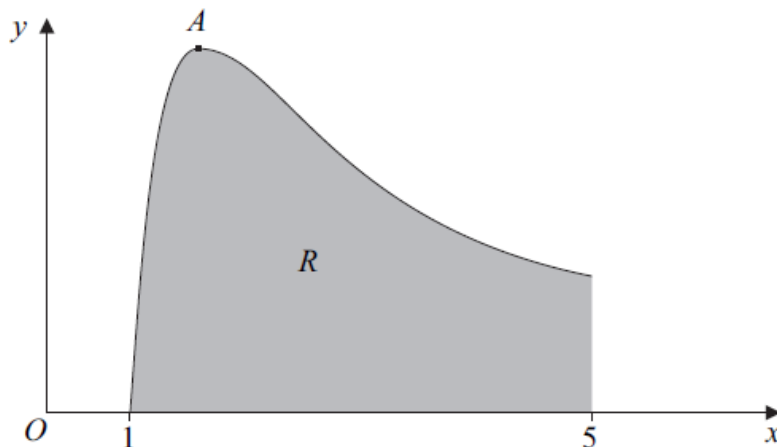
$$\int x\sqrt{2x+1} dx = \frac{1}{4} \int \left( u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du \quad (3 \text{ marks})$$

(ii) Hence show that  $\int_0^4 x\sqrt{2x+1} dx = 19.9$  correct to three significant figures. (4 marks)

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(b) Using integration by parts, find  $\int x^{-2} \ln x dx$ . (4 marks)

(c) The sketch shows the graph of  $y = x^{-2} \ln x$ .



(ii) The region  $R$  is bounded by the curve, the  $x$ -axis and the line  $x = 5$ . Using your answer to part (b), show that the area of  $R$  is

$$\frac{1}{5}(4 - \ln 5) \quad (3 \text{ marks})$$

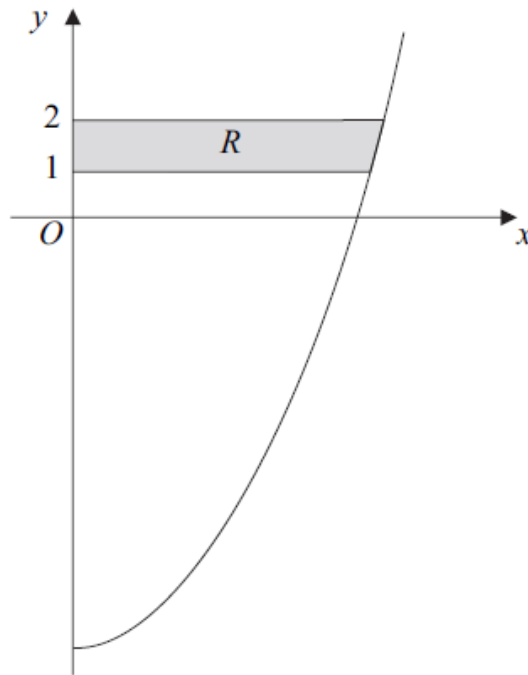
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(b) Use the substitution  $u = 2x + 1$  to find  $\int x(2x + 1)^8 dx$ , giving your answer in terms of  $x$ . (4 marks)

4 (a) Use integration by parts to find  $\int x \sin x \, dx$ . (4 marks)

(b) Using the substitution  $u = x^2 + 5$ , or otherwise, find  $\int x\sqrt{x^2 + 5} \, dx$ . (4 marks)

(c) The diagram shows the curve  $y = x^2 - 9$  for  $x \geq 0$ .



The shaded region  $R$  is bounded by the curve, the lines  $y = 1$  and  $y = 2$ , and the  $y$ -axis.

Find the exact value of the volume of the solid generated when the region  $R$  is rotated through  $360^\circ$  about the  $y$ -axis. (4 marks)

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6 (a) Use integration by parts to find  $\int xe^{5x} \, dx$ . (4 marks)

(b) (i) Use the substitution  $u = \sqrt{x}$  to show that

$$\int \frac{1}{\sqrt{x}(1 + \sqrt{x})} \, dx = \int \frac{2}{1 + u} \, du \quad (2 \text{ marks})$$

(ii) Find the exact value of  $\int_1^9 \frac{1}{\sqrt{x}(1 + \sqrt{x})} \, dx$ . (3 marks)

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## Core 3 Integration Answers

3(a)(i)	$f' = \frac{dy}{dx} = 4x^3 + 2$	B1	1	
(ii)	$\int \frac{2x^3 + 1}{x^4 + 2x} dx$ $= \frac{1}{2} \ln(x^4 + 2x) (+c)$	M1 A1	2	For $k \ln(x^4 + 2x)$ By substitution $k \ln u$ M1 correct A1
(b)(i)	$u = 2x + 1$ $du = 2 dx$ $\int x\sqrt{2x+1} dx =$ $\int \left(\frac{u-1}{2}\right)\sqrt{u} \frac{du}{2}$ $= \frac{1}{4} \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$	B1    M1   A1	       3	Must be in terms of $u$ only incl. $du$       AG
(ii)	$\int_0^4 dx = \int_1^9 du$ $\frac{1}{4} \int u^{\frac{3}{2}} - u^{\frac{1}{2}} = \frac{1}{4} \left[ \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]$ $= \frac{1}{4} \left[ \left( \frac{2}{5} (9)^{\frac{5}{2}} - \frac{2}{3} (9)^{\frac{3}{2}} \right) - \left( \frac{2}{5} - \frac{2}{3} \right) \right]$ $= \frac{1}{4} [79.2 + 0.2\bar{6}]$ $= 19.86$ $= 19.9$	B1  M1 A1      A1	       4	Or changing $u$ 's to $x$ 's at end     } Sight of any of these 3 lines  AG
<b>Total</b>			<b>10</b>	

(b)	$\int x^{-2} \ln x \, dx$	$u = \ln x$	$dv = x^{-2}$	M1	4	Attempt at integration by parts
		$du = \frac{1}{x}$	$v = -x^{-1}$	A1		
		$\int = -\frac{1}{x} \ln x + \int x^{-2} \, dx$		A1		
		$= -\frac{1}{x} \ln x - \frac{1}{x} (+c)$		A1		

(ii)	$R = \left[ -\frac{1}{x} (\ln x + 1) \right]_1^5$	M1		$R = \left[ \text{Their (b)} \right]_1^5$
	$= -\frac{1}{5} (\ln 5 + 1) + (\ln 1 + 1)$	A1		OE
	$= \frac{1}{5} (4 - \ln 5)$	A1	3	convincing argument; AG

(b)	$\int x(2x+1)^8 \, dx$				
	$u = 2x + 1$				
	$du = 2 \, dx$	B1			OE
	$\int = \int \left( \frac{u-1}{2} \right) u^8 \left( \frac{du}{2} \right)$	M1			all in terms of $u$ . Condone omission of $du$
	$= \frac{1}{4} \int u^9 - u^8 \, du$				
	$= \frac{1}{4} \left[ \frac{u^{10}}{10} - \frac{u^9}{9} \right]$	B1			$p \frac{u^{10}}{10} + q \frac{u^9}{9}$
$= \frac{(2x+1)^{10}}{40} - \frac{(2x+1)^9}{36} (+c)$	A1	4		OE; CAO SC: correct answer, no working/parts in $x$ (B1)	

<p>4(a) <math>\int x \sin x \, dx \quad u = x</math></p> $\frac{dv}{dx} = \sin x$ $\frac{du}{dx} = 1 \quad v = -\cos x$ $\int = -x \cos x - \int -\cos x (dx)$ $= -x \cos x + \sin x (+c)$ <p>(b) <math>u = x^2 + 5</math> <math>du = 2x \, dx</math></p> $\int = \int \frac{1}{2} u^{\frac{1}{2}} (du)$ $= \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$	<p>M1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1<sup>✓</sup></p>	<p>4</p>	<p>For differentiating one term and integrating other</p> <p>For correctly substituting their terms into parts formula</p> <p>CSO</p> <p><math>\int k u^{\frac{1}{2}} (du)</math> condone omission of <math>du</math> but M0 if <math>dx</math> <math>k = \frac{1}{2}</math> OE</p> <p>Ft <math>\int k u^{\frac{1}{2}} du</math></p>
<p>(c) <math>= \frac{1}{3} \sqrt{(x^2 + 5)^3} (+c)</math></p> <p><math>y = x^2 - 9</math> <math>x^2 = y + 9</math> <math>V = \pi \int x^2 \, dy</math></p> $= \pi \int (y + 9) \, dy$ $= (\pi) \left[ \frac{y^2}{2} + 9y \right]_1^2 \text{ or } (\pi) \left[ \frac{(y+9)^2}{2} \right]_1^2$ $= (\pi) \left[ 20 - 9\frac{1}{2} \right]$ $= 10\frac{1}{2}\pi$	<p>A1</p> <p>B1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>4</p> <p>4</p>	<p>CSO</p> <p>SC <math>\frac{2}{6} \sqrt{(x^2 + 5)^3}</math> with no working B3</p> <p>Must have <math>\pi</math> and <math>x^2</math>, condone omission of <math>dy</math>, but B0 if <math>dx</math></p> <p><math>\int</math> "their <math>x^2</math>" <math>dy</math> integrated } <math>\pi</math> not necessary Limits 2 and 1 substituted in } correct order including - sign }</p> <p>CSO</p>
<b>Total</b>		<b>12</b>	

6(a)	$\int xe^{5x} dx$ $u = x \quad dv = e^{5x}$ $du = 1 \quad v = \frac{1}{5} e^{5x}$ $\int = \frac{1}{5} xe^{5x} - \int \frac{1}{5} e^{5x} dx$ $= \frac{1}{5} xe^{5x} - \frac{1}{25} e^{5x} (+c)$	M1 A1 A1 A1	4	integrate one term, differentiate one term
(b)(i)	$u = x^{\frac{1}{2}}$ $du = \frac{1}{2} x^{-\frac{1}{2}} dx$ $\int = \int \frac{1}{1+u} \times 2 du$	M1 A1	2	correct with no errors; AG
(ii)	$\int_1^9 dx = \int_1^3 \frac{2}{1+u} du$ $= [2 \ln(1+u)]_1^3$ $= 2 \ln 4 - 2 \ln 2$ $(\ln 4)$	m1 M1 A1	3	correct limits used in correct expression, ignoring $k$ for $k \ln(1+u)$ ISW OE
<b>Total</b>			<b>9</b>	