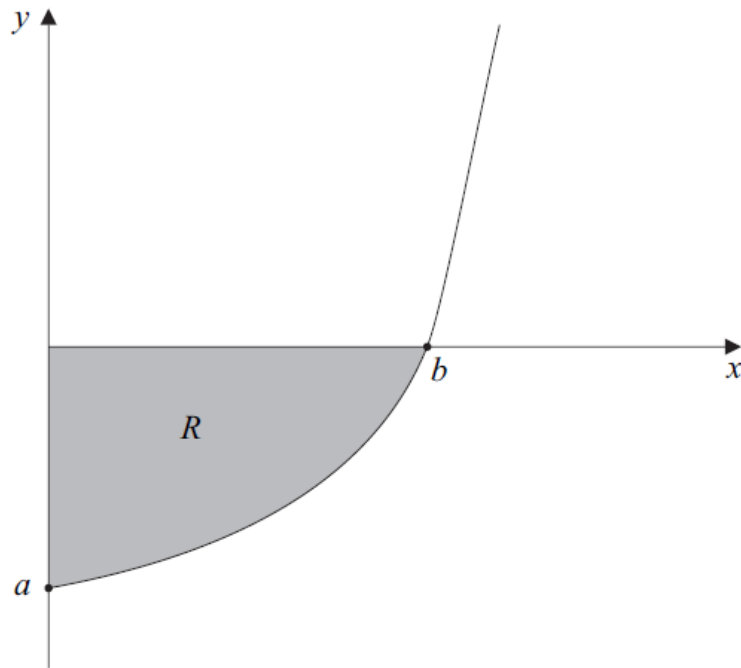


Core 3 Exponentials & Logarithms Questions

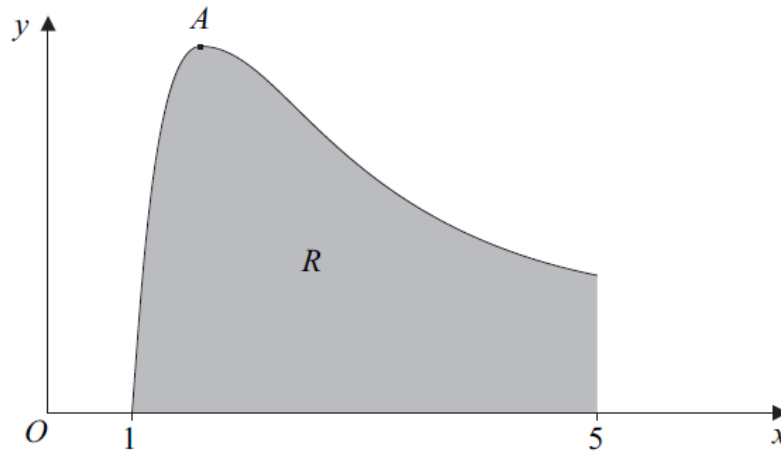
- 5 The diagram shows part of the graph of $y = e^{2x} - 9$. The graph cuts the coordinate axes at $(0, a)$ and $(b, 0)$.



- (a) State the value of a , and show that $b = \ln 3$. (3 marks)
- (b) Show that $y^2 = e^{4x} - 18e^{2x} + 81$. (1 mark)
- (c) The shaded region R is rotated through 360° about the x -axis. Find the volume of the solid formed, giving your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers. (6 marks)
- (d) Sketch the curve with equation $y = |e^{2x} - 9|$ for $x \geq 0$. (2 marks)
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- 9 (a) Given that $y = x^{-2} \ln x$, show that $\frac{dy}{dx} = \frac{1 - 2 \ln x}{x^3}$. (4 marks)

- (c) The sketch shows the graph of $y = x^{-2} \ln x$.



- (i) Using the answer to part (a), find, in terms of e , the x -coordinate of the stationary point A . (2 marks)
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- 5 (a) A curve has equation $y = e^{2x} - 10e^x + 12x$.

(i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Find $\frac{d^2y}{dx^2}$. (1 mark)

- (b) The points P and Q are the stationary points of the curve.

- (i) Show that the x -coordinates of P and Q are given by the solutions of the equation

$$e^{2x} - 5e^x + 6 = 0 \quad (1 \text{ mark})$$

- (ii) By using the substitution $z = e^x$, or otherwise, show that the x -coordinates of P and Q are $\ln 2$ and $\ln 3$. (3 marks)

- (iii) Find the y -coordinates of P and Q , giving each of your answers in the form $m + 12 \ln n$, where m and n are integers. (3 marks)

- (iv) Using the answer to part (a)(ii), determine the nature of each stationary point.
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(b) (i) Given that $y = x \ln x$, find $\frac{dy}{dx}$. (2 marks)

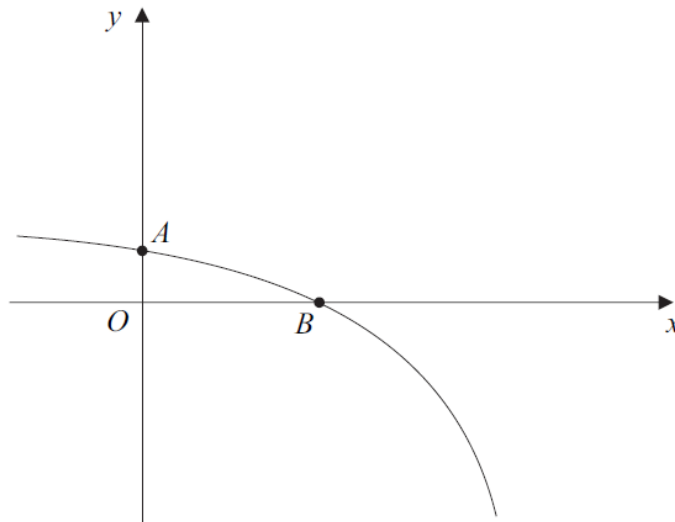
(ii) Hence, or otherwise, find $\int \ln x \, dx$. (2 marks)

(iii) Find the exact value of $\int_1^5 \ln x \, dx$. (2 marks)

(b) (i) Find $\frac{dx}{dy}$ when $x = 2y^3 + \ln y$. (1 mark)

(ii) Hence find an equation of the tangent to the curve $x = 2y^3 + \ln y$ at the point $(2,1)$. (3 marks)

9 The sketch shows the graph of $y = 4 - e^{2x}$. The curve crosses the y -axis at the point A and the x -axis at the point B .



(a) (i) Find $\int (4 - e^{2x}) dx$. (2 marks)

(ii) Hence show that $\int_0^{\ln 2} (4 - e^{2x}) dx = 4 \ln 2 - \frac{3}{2}$. (2 marks)

(b) (i) Write down the y -coordinate of A . (1 mark)

(ii) Show that $x = \ln 2$ at B . (2 marks)

(c) Find the equation of the normal to the curve $y = 4 - e^{2x}$ at the point B . (4 marks)

(d) Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at B and the y -axis. (3 marks)

1 (a) Differentiate $\ln x$ with respect to x . (1 mark)

(b) Given that $y = (x + 1) \ln x$, find $\frac{dy}{dx}$. (2 marks)

(c) Find an equation of the normal to the curve $y = (x + 1) \ln x$ at the point where $x = 1$.

7 (a) A curve has equation $y = (x^2 - 3)e^x$.

(i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Find $\frac{d^2y}{dx^2}$. (2 marks)

(b) (i) Find the x -coordinate of each of the stationary points of the curve. (4 marks)

(ii) Using your answer to part (a)(ii), determine the nature of each of the stationary points. (2 marks)

Core 3 Exponentials & Logarithms Answers

5(a)	$a = -8$ $e^{2x} - 9 = 0$ $e^{2x} = 9$ $2x = \ln 9$ $x = \ln 3$	B1 M1			
	(b) $(e^{2x} - 9)^2 = e^{4x} - 18e^{2x} + 81$	B1	1		AG
	(c) $V = \pi \int y^2 (dx)$ $= (\pi) \int e^{4x} - 18e^{2x} + 81 dx$ $= (\pi) \left[\frac{e^{4x}}{4} - 9e^{2x} + 81x \right]_0^{\ln 3}$	B1 M1 M1 A1			AG Condone verification 1 ST or 2 nd term correct All correct
	$= (\pi) \left[\frac{e^{4x}}{4} - 9e^{2x} + 81x \right]_0^{\ln 3}$ $= (\pi) \left[\left(\frac{e^{\ln 81}}{4} - 9e^{\ln 9} + 81 \ln 3 \right) - \left(\frac{1}{4} - 9 \right) \right]$ $= \pi [81 \ln 3 - 52]$	M1 A1 m1			1 ST or 2 nd term correct All correct Attempt at limits with $\ln 3$
(d)		A1 M1 A1F	6 2		Modulus graph All correct
Total			12		

9(a)	$y = x^{-2} \ln x$ $\frac{dy}{dx} = x^{-2} \frac{1}{x} - 2x^{-3} \ln x$ $= \frac{1 - 2 \ln x}{x^3}$	M1 A1 A1			Use of product or quotient each term Convincing argument $x^{-2} \times \frac{1}{x} = x^{-3}$ AG
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(c)(i)	$\text{At } A, \frac{dy}{dx} = 0$ $1 - 2 \ln x = 0$ $\ln x = \frac{1}{2}$ $x = e^{\frac{1}{2}}$	M1 A1	2	Attempt at $\ln x = k$
5(a)	$y = e^{2x} - 10e^x + 12x$ <p>(i) $\frac{dy}{dx} = 2e^{2x} - 10e^x + 12$</p> <p>(ii) $\frac{d^2y}{dx^2} = 4e^{2x} - 10e^x$</p> <p>(b)(i) $2e^{2x} - 10e^x + 12 = 0$ $e^{2x} - 5e^x + 6 = 0$</p> <p>(ii) $z^2 - 5z + 6 = 0$ $z = 2, 3$ $z = 2, e^x = 2$ $x = \ln 2$ $z = 3, e^x = 3$ $x = \ln 3$</p>	B1 B1 B1F B1 M1 M1 A1	2 1 1 3	$2e^{2x}$ remaining terms correct, no extras ft 1 slip AG (be convinced) use of $z = e^x$ oe finding $e^x =$ their 2,3 all correct AG SC: verification
(iii)	$x = \ln 2 :$ $y = e^{2 \ln 2} - 10e^{\ln 2} + 12 \ln 2$ $\text{or } 2^2 - 10 \times 2 + 12 \ln 2$ $= 4 - 20 + 12 \ln 2$ $= -16 + 12 \ln 2$ $x = \ln 3 :$ $y = e^{2 \ln 3} - 10e^{\ln 3} + 12 \ln 3$ $= 9 - 30 + 12 \ln 3$ $= -21 + 12 \ln 3$	M1 A1 A1	3	$\ln 2$ (B1) $\ln 3$ (B1) either substitution of their $x = \ln 2$ $(e^x = 2)$ or their $x = \ln 3$ $(e^x = 3)$
(iv)	$x = \ln 2 :$ $\frac{d^2y}{dx^2} = 4e^{2 \ln 2} - 10e^{\ln 2}$	M1		use of; in either of their $e^x = 2, 3$ into their $\frac{d^2y}{dx^2}$

$= 16 - 20 = -4$ \therefore maximum $x = \ln 3 :$ $\frac{d^2y}{dx^2} = 4e^{2\ln 3} - 10e^{\ln 3}$ $= 36 - 30 = 6$ \therefore minimum	A1		CSO
	A1	3	CSO
Total		13	

(b)(i) $y = x \ln x$ $\frac{dy}{dx} = x \times \frac{1}{x} + \ln x$ $= \ln x + 1$	M1		use of product rule (only differentiating, 2 terms with + sign)
	A1	2	
(ii) $\int (\ln x + 1) dx = x \ln x$ $\int \ln x dx = x \ln x - x (+c)$	M1		OE; attempt at parts with $u = \ln x$
	A1	2	
(iii) $\int_1^5 \ln x dx = [x \ln x - x]_1^5$ $= (5 \ln 5 - 5) - (1 \ln 1 - 1)$ $5 \ln 5 - 4$	M1		correct substitution of limits into their (ii) provided $\ln x$ is involved ISW
	A1	2	
Total		9	

(b)(i) $x = 2y^3 + \ln y$ $\frac{dx}{dy} = 6y^2 + \frac{1}{y}$	B1	1	
(ii) At (2,1) $\frac{dx}{dy} = 6 + 1 = 7$ $\frac{dy}{dx} = \frac{1}{7}$ $(y-1) = \frac{1}{7}(x-2)$	M1		May be implied
	A1 ✓		
	A1	3	OE

<p>9(a)(i)</p> $\int (4 - e^{2x}) dx$ $= 4x - \frac{1}{2} e^{2x} (+c)$ <p>(ii)</p> $\int_0^{\ln 2} \left[4x - \frac{1}{2} e^{2x} \right] dx$ $= \left[4 \ln 2 - \frac{1}{2} e^{2 \ln 2} \right] - \left[(0) - \frac{1}{2} (e^0) \right]$ $= 4 \ln 2 - 2 + \frac{1}{2}$ $= 4 \ln 2 - \frac{3}{2}$ <p>(b)(i)</p> $x = 0$ $y = 4 - 1 = 3$ <p>(ii)</p> At B, $y = 0$ $4 - e^{2x} = 0$ $e^{2x} = 4$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p>	<p>2</p> <p>2</p> <p>1</p>	<p>4x</p> <p>$-\frac{1}{2} e^{2x}$</p> <p>Substitute both $\ln 2$ and 0 correctly into an integrated expression</p> <p>Convincing</p> <p>AG</p> <p>Or reverse argument</p>
<p>(c)</p> $x = \ln 2$ $\frac{dy}{dx} = -2e^{2x}$ $x = \ln 2$, Gradient $= -2e^{2 \ln 2}$ $= -8$ <p>Gradient normal $= \frac{1}{8} = \frac{1}{2e^{2 \ln 2}}$</p> <p>Equation $y = \frac{1}{8}x - \frac{1}{8} \ln 2$</p> <p>(d)</p> When $x = 0$ $y = -\frac{1}{8} \ln 2$ <p>Area $\Delta = \frac{1}{16} (\ln 2)^2$ condone - ve sign $= 0.03$</p> <p>Total area $= 4 \ln 2 - \frac{3}{2} + \frac{1}{16} (\ln 2)^2 = 1.30$</p> <p style="text-align: right;">AWRT</p> <p style="text-align: center;">Total</p>	<p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1[✓]</p> <p>A1</p>	<p>2</p> <p>4</p> <p>3</p>	<p>AG</p> <p>$x = \ln 2$ into ke^{2x}</p> <p>OE</p> <p>OE</p> <p>Attempt to integrate their line and substitute $x = 0, \ln 2$</p> <p>$\frac{1}{2} (\text{their } y) \times \ln 2$</p> <p>CSO</p>
Total		14	

1(a)	$y = \ln x$ $\frac{dy}{dx} = \frac{1}{x}$	B1	1	penalise + c once on 1(a) or 2(a)
(b)	$y = (x+1)\ln x$ $\frac{dy}{dx} = (x+1) \times \frac{1}{x} + \ln x$	M1 A1	2	product rule
(c)	$y = (x+1)\ln x$ $\frac{dy}{dx} = \frac{1}{x} + 1 + \ln x$ $x = 1: \frac{dy}{dx} = 1 + 1 = 2$ Grad normal = $-\frac{1}{2}$ $y = -\frac{1}{2}(x-1)$	M1 M1 A1 A1	 4	substitute $x = 1$ into their $\frac{dy}{dx}$ use of $m_1 m_2 = -1$ CSO OE
Total			7	

7(a)(i)	$y = (x^2 - 3)e^x$ $\frac{dy}{dx} = (x^2 - 3)e^x + 2xe^x$	M1 A1	2	product rule
(ii)	$\frac{d^2y}{dx^2} = (x^2 - 3)e^x + 2xe^x + 2xe^x + 2e^x$	M1 A1	2	product rule from their $\frac{dy}{dx}$
(b)(i)	$\frac{dy}{dx} = 0$ $\Rightarrow e^x(x^2 + 2x - 3) = 0$ $e^x(x+3)(x-1) = 0$ $\therefore x = -3, 1$	M1 m1 A1 A1	 4	$e^x f(x) = 0$ from $\frac{dy}{dx} = 0$ attempt at factorising or use of formula first correct solution second correct solution, and no others SC No working shown: $x = -3$ B2, $x = 1$ B2 Condone slip
(ii)	$x = -3$ $y'' = -4e^x$ max (-0.2) $x = 1$ $y'' = 4e^x$ min (10.9)	M1 A1	2	
Total			10	