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Exercise A, Question 1

Question:

A student makes the mistake of thinking that $\sin(A+B) \equiv \sin A + \sin B$. Choose non-zero values of A and B to show that this statement is not true for all values of A and B.

Solution:

Example: Take $A = 30°$, $B = 60°$

$\sin A = \frac{1}{2}$

$\sin B = \frac{\sqrt{3}}{2}$

$\sin A + \sin B \neq 1$

but $\sin(A+B) = \sin 90° = 1$.

This proves that $\sin(A+B) = \sin A + \sin B$ is *not* true for all values. There will be many values of A and B for which it is true, e.g. $A = -30°$ and $B = +30°$, and that is the danger of trying to prove a statement by taking particular examples. To prove a statement requires a sound argument; to disprove only requires one example.

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Exercise A, Question 2

Question:

Using the expansion of $\cos(A - B)$ with $A = B = \theta$, show that $\sin^2\theta + \cos^2\theta \equiv 1$.

Solution:

$\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$
Set $A = \theta$, $B = \theta$

$\Rightarrow \quad \cos(\theta - \theta) \equiv \cos\theta\cos\theta + \sin\theta\sin\theta$

$\Rightarrow \quad \cos 0 \equiv \cos^2\theta + \sin^2\theta$

So $\cos^2\theta + \sin^2\theta \equiv 1 \qquad (\text{since } \cos 0 = 1)$

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Exercise A, Question 3

Question:

(a) Use the expansion of $\sin(A-B)$ to show that $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$.

(b) Use the expansion of $\cos(A-B)$ to show that $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$.

Solution:

(a) $\sin(A-B) \equiv \sin A\cos B - \cos A\sin B$

Set $A=\frac{\pi}{2}$, $B=\theta$

$$\Rightarrow \quad \sin\left(\frac{\pi}{2}-\theta\right) \equiv \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$$

$$\Rightarrow \quad \sin\left(\frac{\pi}{2}-\theta\right) \equiv \cos\theta \qquad \left(\text{since } \sin\frac{\pi}{2}=1,\ \cos\frac{\pi}{2}=0\right)$$

(b) $\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$

Set $A=\frac{\pi}{2}$, $B=\theta$

$$\Rightarrow \quad \cos\left(\frac{\pi}{2}-\theta\right) \equiv \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta$$

$$\Rightarrow \quad \cos\left(\frac{\pi}{2}-\theta\right) \equiv \sin\theta \qquad \left(\text{since } \cos\frac{\pi}{2}=0,\ \sin\frac{\pi}{2}=1\right)$$

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Exercise A, Question 4

Question:

Express the following as a single sine, cosine or tangent:

(a) $\sin 15° \cos 20° + \cos 15° \sin 20°$

(b) $\sin 58° \cos 23° - \cos 58° \sin 23°$

(c) $\cos 130° \cos 80° - \sin 130° \sin 80°$

(d) $\frac{\tan 76° - \tan 45°}{1 + \tan 76° \tan 45°}$

(e) $\cos 2\theta \cos \theta + \sin 2\theta \sin \theta$

(f) $\cos 4\theta \cos 3\theta - \sin 4\theta \sin 3\theta$

(g) $\sin \frac{1}{2}\theta \cos 2\frac{1}{2}\theta + \cos \frac{1}{2}\theta \sin 2\frac{1}{2}\theta$

(h) $\frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta}$

(i) $\sin (A + B) \cos B - \cos (A + B) \sin B$

(j) $\cos \left(\frac{3x + 2y}{2} \right) \cos \left(\frac{3x - 2y}{2} \right) - \sin \left(\frac{3x + 2y}{2} \right) \sin \left(\frac{3x - 2y}{2} \right)$

Solution:

(a) Using $\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$
$\sin 15° \cos 20° + \cos 15° \sin 20° \equiv \sin (15° + 20°) \equiv \sin 35°$

(b) Using $\sin (A - B) \equiv \sin A \cos B - \cos A \sin B$
$\sin 58° \cos 23° - \cos 58° \sin 23° \equiv \sin (58° - 23°) \equiv \sin 35°$

(c) Using $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$
$\cos 130° \cos 80° - \sin 130° \sin 80° \equiv \cos (130° + 80°) \equiv \cos 210°$

(d) Using $\tan (A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\frac{\tan 76° - \tan 45°}{1 + \tan 76° \tan 45°} \equiv \tan(76° - 45°) \equiv \tan 31°$$

(e) Using $\cos(A - B) \equiv \cos A \cos B + \sin A \sin B$

$\cos 2\theta \cos\theta + \sin 2\theta \sin\theta \equiv \cos(2\theta - \theta) \equiv \cos\theta$

(f) Using $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

$\cos 4\theta \cos 3\theta - \sin 4\theta \sin 3\theta \equiv \cos(4\theta + 3\theta) \equiv \cos 7\theta$

(g) Using $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$

$$\sin \tfrac{1}{2}\theta \cos 2\tfrac{1}{2}\theta + \cos \tfrac{1}{2}\theta \sin 2\tfrac{1}{2}\theta \equiv \sin\left(\tfrac{1}{2}\theta + 2\tfrac{1}{2}\theta\right) \equiv \sin 3\theta$$

(h) Using $\tan(A + B) \equiv \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\frac{\tan 2\theta + \tan 3\theta}{1 - \tan 2\theta \tan 3\theta} \equiv \tan(2\theta + 3\theta) \equiv \tan 5\theta$$

(i) Using $\sin(P - Q) \equiv \sin P \cos Q - \cos P \sin Q$

$\sin(A + B)\cos B - \cos(A + B)\sin B \equiv \sin[(A + B) - B] \equiv \sin A$

(j) Using $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$

$$\cos\left(\frac{3x + 2y}{2}\right)\cos\left(\frac{3x - 2y}{2}\right) - \sin\left(\frac{3x + 2y}{2}\right)\sin\left(\frac{3x - 2y}{2}\right)$$

$$\equiv \cos\left[\left(\frac{3x + 2y}{2}\right) + \left(\frac{3x - 2y}{2}\right)\right] \equiv \cos\left(\frac{6x}{2}\right) \equiv \cos 3x$$

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Exercise A, Question 5

Question:

Calculate, without using your calculator, the exact value of:

(a) $\sin 30° \cos 60° + \cos 30° \sin 60°$

(b) $\cos 110° \cos 20° + \sin 110° \sin 20°$

(c) $\sin 33° \cos 27° + \cos 33° \sin 27°$

(d) $\cos \frac{\pi}{8} \cos \frac{\pi}{8} - \sin \frac{\pi}{8} \sin \frac{\pi}{8}$

(e) $\sin 60° \cos 15° - \cos 60° \sin 15°$

(f) $\cos 70° (\cos 50° - \tan 70° \sin 50°)$

(g) $\frac{\tan 45° + \tan 15°}{1 - \tan 45° \tan 15°}$

(h) $\frac{1 - \tan 15°}{1 + \tan 15°}$

(i) $\frac{\tan (\frac{7\pi}{12}) - \tan (\frac{\pi}{3})}{1 + \tan (\frac{7\pi}{12}) \tan (\frac{\pi}{3})}$

(j) $\sqrt{3} \cos 15° - \sin 15°$

Solution:

(a) Using $\sin (A + B)$ expansion

$\sin 30° \cos 60° + \cos 30° \sin 60° = \sin (30 + 60)° = \sin 90° = 1$

(b) $\cos 110° \cos 20° + \sin 110° \sin 20° = \cos (110 - 20)° = \cos 90° = 0$

(c) $\sin 33° \cos 27° + \cos 33° \sin 27° = \sin (33 + 27)° = \sin 60° = \frac{\sqrt{3}}{2}$

(d) $\cos \frac{\pi}{8} \cos \frac{\pi}{8} - \sin \frac{\pi}{8} \sin \frac{\pi}{8} = \cos \left(\frac{\pi}{8} + \frac{\pi}{8} \right) = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

(e) $\sin 60° \cos 15° - \cos 60° \sin 15° = \sin (60 - 15)° = \sin 45° = \frac{\sqrt{2}}{2}$

(f) $\cos 70° \cos 50° - \cos 70° \tan 70° \sin 50°$
$= \cos 70° \cos 50° - \sin 70° \sin 50°$

$$\left(\cos\theta \times \tan\theta = \cancel{\cos\theta} \times \frac{\sin\theta}{\cancel{\cos\theta}} = \sin\theta\right)$$

$= \cos(70 + 50)$
$= \cos 120°$
$= -\cos 60° = -\frac{1}{2}$

(g) $\frac{\tan 45° + \tan 15°}{1 - \tan 45° \tan 15°} = \tan(45 + 15)° = \tan 60° = \sqrt{3}$

(h) $\frac{1 - \tan 15°}{1 + \tan 15°} = \frac{\tan 45° - \tan 15°}{1 + \tan 45° \tan 15°}$ (using $\tan 45° = 1$)

$= \tan(45 - 15)° = \tan 30° = \frac{\sqrt{3}}{3}$

(i) $\frac{\tan\left(\frac{7\pi}{12}\right) - \tan\left(\frac{1}{3}\pi\right)}{1 + \tan\left(\frac{7\pi}{12}\right)\tan\left(\frac{1}{3}\pi\right)} = \tan\left(\frac{7\pi}{12} - \frac{\pi}{3}\right) = \tan\frac{3\pi}{12} = \tan\frac{\pi}{4} = 1$

(j) This is very similar to part (e) but you need to rewrite it as $2\left(\frac{\sqrt{3}}{2}\cos 15° - \frac{1}{2}\sin 15°\right)$ to appreciate it!

$\sqrt{3}\cos 15° - \sin 15° \equiv 2\left(\frac{\sqrt{3}}{2}\cos 15° - \frac{1}{2}\sin 15°\right)$
$\equiv 2(\sin 60° \cos 15° - \cos 60° \sin 15°)$
$\equiv 2\sin(60 - 15)°$
$\equiv 2\sin 45°$
$= \sqrt{2}$

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Exercise A, Question 6

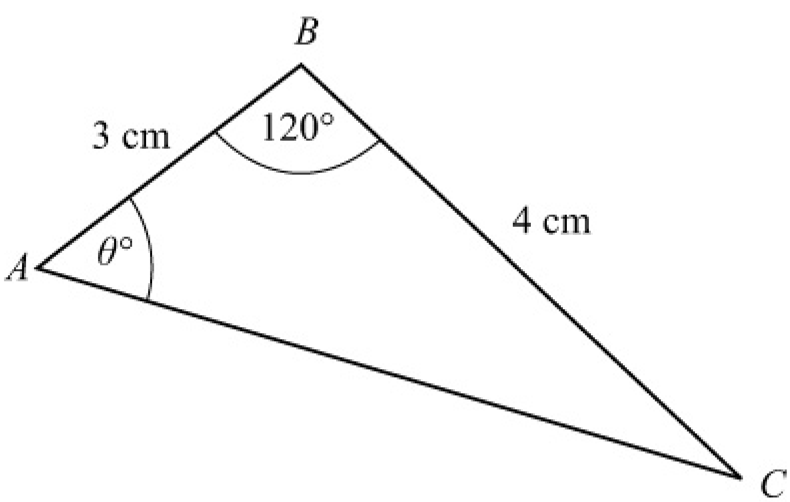
Question:

Triangle ABC is such that $AB = 3\text{cm}$, $BC = 4\text{cm}$, $\angle ABC = 120°$ and $\angle BAC = \theta°$.

(a) Write down, in terms of θ, an expression for $\angle ACB$.

(b) Using the sine rule, or otherwise, show that $\tan\theta° = \frac{2\sqrt{3}}{5}$.

Solution:



(a) $\angle \text{ACB} = 180° - 120° - \theta° = (60 - \theta)°$

(b) Using sine rule: $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\Rightarrow \quad \frac{\sin(60-\theta)°}{3} = \frac{\sin\theta°}{4}$

$\Rightarrow \quad 4\sin(60-\theta)° = 3\sin\theta°$

$\Rightarrow \quad 4\sin 60°\cos\theta° - 4\cos 60°\sin\theta° = 3\sin\theta°$

$\Rightarrow \quad 2\sqrt{3}\cos\theta° - 2\sin\theta° = 3\sin\theta° \qquad \left(\sin 60° = \frac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad 5\sin\theta° = 2\sqrt{3}\cos\theta°$

$$\Rightarrow \quad \frac{\sin\theta°}{\cos\theta°} = \frac{2\sqrt{3}}{5}$$

$$\Rightarrow \quad \tan\theta° = \frac{2\sqrt{3}}{5}$$

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Exercise A, Question 7

Question:

Prove the identities

(a) $\sin(A+60°) + \sin(A-60°) \equiv \sin A$

(b) $\frac{\cos A}{\sin B} - \frac{\sin A}{\cos B} \equiv \frac{\cos(A+B)}{\sin B\cos B}$

(c) $\frac{\sin(x+y)}{\cos x\cos y} \equiv \tan x + \tan y$

(d) $\frac{\cos(x+y)}{\sin x\sin y} + 1 \equiv \cot x\cot y$

(e) $\cos\left(\theta + \frac{\pi}{3}\right) + \sqrt{3}\sin\theta \equiv \sin\left(\theta + \frac{\pi}{6}\right)$

(f) $\cot(A+B) \equiv \frac{\cot A\cot B - 1}{\cot A + \cot B}$

(g) $\sin^2(45+\theta)° + \sin^2(45-\theta)° \equiv 1$

(h) $\cos(A+B)\cos(A-B) \equiv \cos^2 A - \sin^2 B$

Solution:

(a) L.H.S. $\equiv \sin(A+60°) + \sin(A-60°)$

$\equiv \sin A\cos 60° + \cos A\sin 60° + \sin A\cos 60° - \cos A\sin 60°$

$\equiv 2\sin A\cos 60°$

$\equiv \sin A$ (since $\cos 60° = \frac{1}{2}$)

$\equiv$ R.H.S.

(b) L.H.S. $\equiv \frac{\cos A}{\sin B} - \frac{\sin A}{\cos B}$

$\equiv \frac{\cos A\cos B - \sin A\sin B}{\sin B\cos B}$

$\equiv \frac{\cos(A+B)}{\sin B\cos B}$

$\equiv$ R.H.S.

(c) L.H.S. $\equiv \frac{\sin(x+y)}{\cos x\cos y}$

$$\equiv \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}$$

$$\equiv \frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}$$

$$\equiv \frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}$$

$$\equiv \tan x + \tan y$$

$\equiv$ R.H.S.

(d) L.H.S. $\equiv \frac{\cos (x + y)}{\sin x \sin y} + 1$

$$\equiv \frac{\cos (x + y) + \sin x \sin y}{\sin x \sin y}$$

$$\equiv \frac{\cos x \cos y - \sin x \sin y + \sin x \sin y}{\sin x \sin y}$$

$$\equiv \frac{\cos x \cos y}{\sin x \sin y}$$

$$\equiv \cot x \cot y$$

$\equiv$ R.H.S.

(e) L.H.S. $\equiv \cos \left(\theta + \frac{\pi}{3} \right) + \sqrt{3} \sin \theta$

$$\equiv \cos \theta \cos \frac{\pi}{3} - \sin \theta \sin \frac{\pi}{3} + \sqrt{3} \sin \theta$$

$$\equiv \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta + \sqrt{3} \sin \theta$$

$$\equiv \frac{\sqrt{3}}{2} \sin \theta + \frac{1}{2} \cos \theta$$

$$\equiv \sin \theta \cos \frac{\pi}{6} + \cos \theta \sin \frac{\pi}{6} \qquad \left(\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} , \sin \frac{\pi}{6} = \frac{1}{2} \right)$$

$$\equiv \sin \left(\theta + \frac{\pi}{6} \right) \qquad [\sin (A + B)]$$

$\equiv$ R.H.S.

(f)

$$
\begin{aligned}
\text{R.H.S.} &\equiv \frac{\cot A \cot B - 1}{\cot A + \cot B} \\
&\equiv \frac{\frac{1}{\tan A} \times \frac{1}{\tan B} - 1}{\frac{1}{\tan A} + \frac{1}{\tan B}} \\
&\equiv \frac{\frac{1 - \tan A \tan B}{\tan A \tan B}}{\frac{\tan B - \tan A}{\tan A \tan B}} \\
&\equiv \frac{1 - \tan A \tan B}{\tan A \tan B} \times \frac{\tan A \tan B}{\tan A + \tan B} \\
&\equiv \frac{1 - \tan A \tan B}{\tan A + \tan B} \\
&\equiv \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}} \\
&\equiv \frac{1}{\tan(A+B)} \\
&\equiv \cot(A+B) \\
&\equiv \text{L.H.S.}
\end{aligned}
$$

(g) L.H.S. $\equiv \sin^2(45+\theta)^\circ + \sin^2(45-\theta)^\circ$

$\equiv (\sin 45^\circ \cos\theta^\circ + \cos 45^\circ \sin\theta^\circ)^2 + (\sin 45^\circ \cos\theta^\circ - \cos 45^\circ \sin\theta^\circ)^2$

As $\sin 45^\circ = \cos 45^\circ$ it is easier to take out as a common factor.

$\equiv (\sin 45^\circ)^2 [(\cos\theta^\circ + \sin\theta^\circ)^2 + (\cos\theta^\circ - \sin\theta^\circ)^2]$

$\equiv \frac{1}{2}\left(\cos^2\theta^\circ + 2\sin\theta^\circ\cos\theta^\circ + \sin^2\theta^\circ + \cos^2\theta^\circ - 2\sin\theta^\circ\cos\theta^\circ + \sin^2\theta^\circ\right)$

$\equiv \frac{1}{2}\left[2\left(\sin^2\theta^\circ + \cos^2\theta^\circ\right)\right]$

$\equiv \frac{1}{2} \times 2 \quad (\sin^2\theta^\circ + \cos^2\theta^\circ \equiv 1)$

$\equiv 1$

$\equiv$ R.H.S.

Alternatively:
as $\sin(90^\circ - x^\circ) \equiv \cos x^\circ$,
if $x = 45^\circ + \theta^\circ$ then $\sin(45^\circ - \theta^\circ) \equiv \cos(45^\circ + \theta^\circ)$
and original L.H.S. becomes $\sin^2(45+\theta)^\circ + \cos^2(45+\theta)^\circ$
which is identically = 1

(h) L.H.S. $\equiv \cos(A+B)\cos(A-B)$

$\equiv (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$\equiv \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$$\equiv \cos^2 A\,(1 - \sin^2 B) - (1 - \cos^2 A)\sin^2 B$$
$$\equiv \cos^2 A - \cos^2 A\sin^2 B - \sin^2 B + \cos^2 A\sin^2 B$$
$$\equiv \cos^2 A - \sin^2 B$$
$$\equiv \text{R.H.S.}$$

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Exercise A, Question 8

Question:

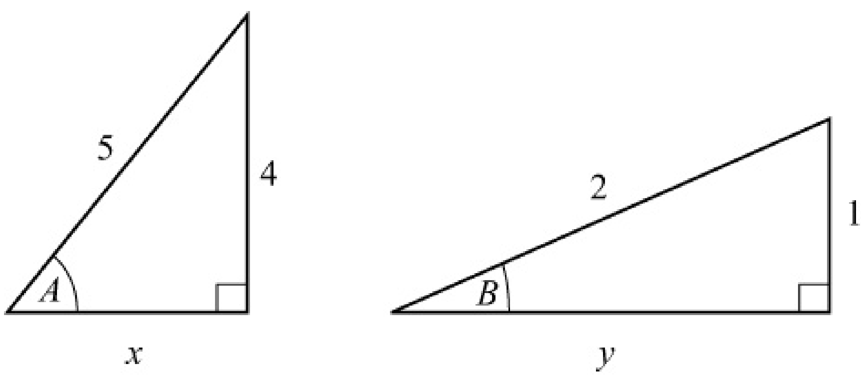
Given that $\sin A = \frac{4}{5}$ and $\sin B = \frac{1}{2}$, where A and B are both acute angles, calculate the exact values of

(a) $\sin (A + B)$

(b) $\cos (A - B)$

(c) $\sec (A - B)$

Solution:



Using Pythagoras' theorem $x = 3$ and $y = \sqrt{3}$

(a) $\sin (A + B) = \sin A \cos B + \cos A \sin B = \frac{4}{5} \times \frac{\sqrt{3}}{2} + \frac{3}{5} \times \frac{1}{2} = \frac{4\sqrt{3} + 3}{10}$

(b) $\cos (A - B) = \cos A \cos B + \sin A \sin B = \frac{3}{5} \times \frac{\sqrt{3}}{2} + \frac{4}{5} \times \frac{1}{2} = \frac{3\sqrt{3} + 4}{10}$

(c) $\sec (A - B) = \frac{1}{\cos (A - B)} = \frac{10}{3\sqrt{3} + 4}$

$$= \frac{10 (3\sqrt{3} - 4)}{(3\sqrt{3} + 4) (3\sqrt{3} - 4)}$$

$$= \frac{10 (3\sqrt{3} - 4)}{27 - 16}$$

$$= \frac{10 (3\sqrt{3} - 4)}{11}$$

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Exercise A, Question 9

Question:

Given that $\cos A = -\frac{4}{5}$, and A is an obtuse angle measured in radians, find the exact value of

(a) $\sin A$

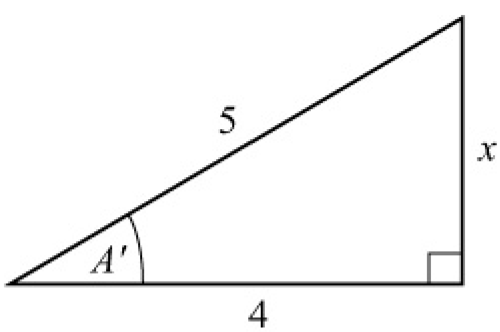
(b) $\cos(\pi + A)$

(c) $\sin\left(\frac{\pi}{3} + A\right)$

(d) $\tan\left(\frac{\pi}{4} + A\right)$

Solution:

Draw a right-angled triangle where $\cos A' = \frac{4}{5}$



Using Pythagoras' theorem $x = 3$
So $\sin A' = \frac{3}{5}$, $\tan A' = \frac{3}{4}$

(a) As A is in the 2nd quadrant, $\sin A = \sin A'$
$\sin A = \frac{3}{5}$

(b) $\cos(\pi + A) = \cos\pi\cos A - \sin\pi\sin A = -\cos A$ $(\cos\pi = -1,$
$\sin\pi = 0)$

$$\cos(\pi + A) = +\frac{4}{5}$$

(c) $\sin\left(\frac{\pi}{3} + A\right) = \sin\frac{\pi}{3}\cos A + \cos\frac{\pi}{3}\sin A$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{4}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{5}\right)$$

$$= \frac{3 - 4\sqrt{3}}{10}$$

(d) As A is in 2nd quadrant, $\tan A = -\tan A' = -\frac{3}{4}$

$$\tan\left(\frac{\pi}{4} + A\right) = \frac{\tan\frac{\pi}{4} + \tan A}{1 - \tan\frac{\pi}{4}\tan A} = \frac{1 + \tan A}{1 - \tan A} = \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{1}{7}$$

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Exercise A, Question 10

Question:

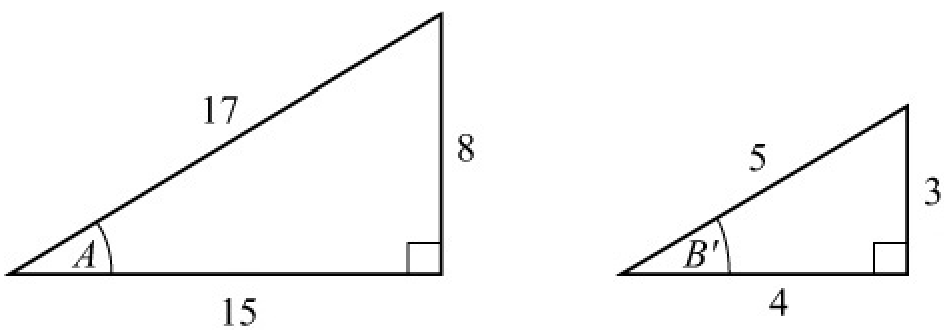
Given that $\sin A = \frac{8}{17}$, where A is acute, and $\cos B = -\frac{4}{5}$, where B is obtuse, calculate the exact value of

(a) $\sin(A - B)$

(b) $\cos(A - B)$

(c) $\cot(A - B)$

Solution:



$\sin B = \sin B'$, $\tan B = -\tan B'$

By Pythagoras' theorem, the remaining sides are 15 and 3.

So $\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$

and $\sin B = \frac{3}{5}$, $\cos B = -\frac{4}{5}$, $\tan B = -\frac{3}{4}$

(a) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$$= \left(\frac{8}{17}\right)\left(-\frac{4}{5}\right) - \left(\frac{15}{17}\right)\left(\frac{3}{5}\right)$$

$$= \frac{-32 - 45}{85} = -\frac{77}{85}$$

(b) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$= \left(\frac{15}{17}\right)\left(-\frac{4}{5}\right) + \left(\frac{8}{17}\right)\left(\frac{3}{5}\right)$$

$$= \frac{-60+24}{85} = -\frac{36}{85}$$

(c) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B} = \dfrac{\frac{8}{15}+\frac{3}{4}}{1-\frac{24}{60}} = \dfrac{\frac{77}{60}}{\frac{36}{60}} = \dfrac{77}{36}$

So $\cot(A-B) = \dfrac{1}{\tan(A-B)} = \dfrac{36}{77}$

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Exercise A, Question 11

Question:

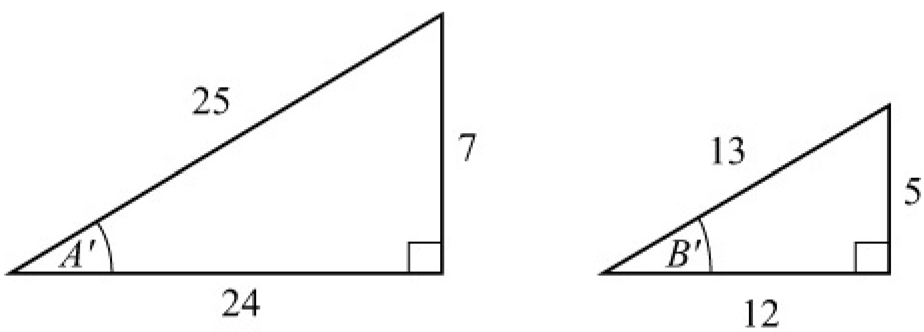
Given that $\tan A = \frac{7}{24}$, where A is reflex, and $\sin B = \frac{5}{13}$, where B is obtuse, calculate the exact value of

(a) $\sin (A + B)$

(b) $\tan (A - B)$

(c) $\text{cosec} (A + B)$

Solution:



Using Pythagoras' theorem, the remaining sides are 25 and 12.
As A is in the 3rd quadrant ($\tan A$ is +ve, and A is reflex),
$\sin A = - \sin A'$, $\cos A = - \cos A'$

So $\sin A = - \frac{7}{25}$, $\cos A = - \frac{24}{25}$, $\tan A = \frac{7}{24}$

As B is in the 2nd quadrant,
$\cos B = - \cos B'$, $\tan B = - \tan B'$

So $\sin B = \frac{5}{13}$, $\cos B = - \frac{12}{13}$, $\tan B = - \frac{5}{12}$

(a)

$$\begin{aligned}\sin (A + B) &= \sin A \cos B + \cos A \sin B \\ &= \left(- \frac{7}{25} \right) \left(- \frac{12}{13} \right) + \left(- \frac{24}{25} \right) \left(\frac{5}{13} \right) \\ &= \frac{84 - 120}{325} = - \frac{36}{325}\end{aligned}$$

(b) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\frac{7}{24} + \frac{5}{12}}{1 - \left(\frac{7}{24}\right)\left(\frac{5}{12}\right)} = \frac{\frac{17}{24}}{\frac{253}{288}} = \frac{204}{253}$

(c) $\operatorname{cosec}(A + B) = \frac{1}{\sin(A + B)} = -\frac{325}{36}$

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Exercise A, Question 12

Question:

Write the following as a single trigonometric function, assuming that θ is measured in radians:

(a) $\cos^2 \theta - \sin^2 \theta$

(b) $2 \sin 4\theta \cos 4\theta$

(c) $\frac{1 + \tan\theta}{1 - \tan\theta}$

(d) $\frac{1}{\sqrt{2}} (\sin\theta + \cos\theta)$

Solution:

(a) $\cos^2 \theta - \sin^2 \theta = \cos\theta \cos\theta - \sin\theta \sin\theta = \cos (\theta + \theta) = \cos 2\theta$

(b) $2 \sin 4\theta \cos 4\theta = \sin 4\theta \cos 4\theta + \sin 4\theta \cos 4\theta$
$= \sin 4\theta \cos 4\theta + \cos 4\theta \sin 4\theta$
$= \sin (4\theta + 4\theta)$
$= \sin 8\theta$

(c) $\frac{1 + \tan\theta}{1 - \tan\theta} = \frac{\tan \frac{\pi}{4} + \tan\theta}{1 - \tan \frac{\pi}{4} \tan\theta}$ (as $\tan \frac{\pi}{4} = 1$)

$= \tan \left(\frac{\pi}{4} + \theta \right)$

(d) $\frac{1}{\sqrt{2}} \sin\theta + \frac{1}{\sqrt{2}} \cos\theta = \sin\theta \cos \frac{\pi}{4} + \cos\theta \sin \frac{\pi}{4}$ (as $\frac{1}{\sqrt{2}} = \cos \frac{\pi}{4} = \sin \frac{\pi}{4}$)

$= \sin \left(\theta + \frac{\pi}{4} \right)$

[**Note:** (d) could be $\cos\left(\theta - \frac{\pi}{4}\right)$]

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Exercise A, Question 13

Question:

Solve, in the interval $0° \leq \theta < 360°$, the following equations. Give answers to the nearest 0.1°.

(a) $3\cos\theta = 2\sin(\theta + 60°)$

(b) $\sin(\theta + 30°) + 2\sin\theta = 0$

(c) $\cos(\theta + 25°) + \sin(\theta + 65°) = 1$

(d) $\cos\theta = \cos(\theta + 60°)$

(e) $\tan(\theta - 45°) = 6\tan\theta$

(f) $\sin\theta + \cos\theta = 1$

Solution:

(a) $3\cos\theta = 2\sin(\theta + 60°)$

$\Rightarrow \quad 3\cos\theta = 2(\sin\theta\cos 60° + \cos\theta\sin 60°)$

$\Rightarrow \quad 3\cos\theta = 2\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = \sin\theta + \sqrt{3}\cos\theta$

$\Rightarrow \quad (3 - \sqrt{3})\cos\theta = \sin\theta$

$\Rightarrow \quad \tan\theta = 3 - \sqrt{3} \qquad \left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

As tan θ is +ve, θ is in 1st and 3rd quadrants.

$\theta = \tan^{-1}(3 - \sqrt{3}), 180° + \tan^{-1}(3 - \sqrt{3}) = 51.7°, 231.7°$

(b) $\sin(\theta + 30°) + 2\sin\theta = 0$

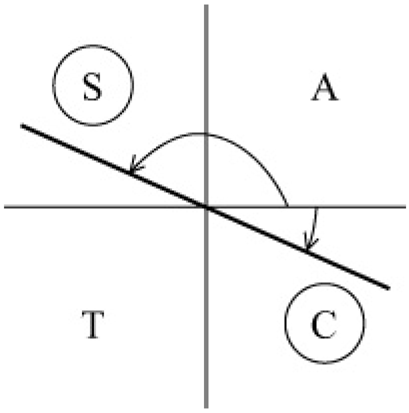
$\Rightarrow \quad \sin\theta\cos 30° + \cos\theta\sin 30° + 2\sin\theta = 0$

$\Rightarrow \quad \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + 2\sin\theta = 0$

$\Rightarrow \quad (4 + \sqrt{3})\sin\theta = -\cos\theta$

$\Rightarrow \quad \tan\theta = -\frac{1}{4 + \sqrt{3}}$

As tan θ is −ve, θ is in the 2nd and 4th quadrants.



$\theta = \tan^{-1}\left(-\frac{1}{4+\sqrt{3}}\right) + 180°, \tan^{-1}\left(-\frac{1}{4+\sqrt{3}}\right) + 360°$

$= 170.1°, 350.1°.$

(c) $\cos(\theta + 25°) + \sin(\theta + 65°) = 1$

$\Rightarrow \quad \cos\theta\cos 25° - \sin\theta\sin 25° + \sin\theta\cos 65° + \cos\theta\sin 65° = 1$

As $\sin(90 - x)° = \cos x°$ and $\cos(90 - x)° = \sin x°$

$\cos 25° = \sin 65°$ and $\sin 25° = \cos 65°$

So $\cos\theta\sin 65° - \sin\theta\cos 65° + \sin\theta\cos 65° + \cos\theta\sin 65° = 1$

$\Rightarrow \quad 2\cos\theta\sin 65° = 1$

$\Rightarrow \quad \cos\theta = \frac{1}{2\sin 65°} = 0.55169$

$\theta = \cos^{-1}(0.55169), 360° - \cos^{-1}(0.55169) = 56.5°, 303.5°$

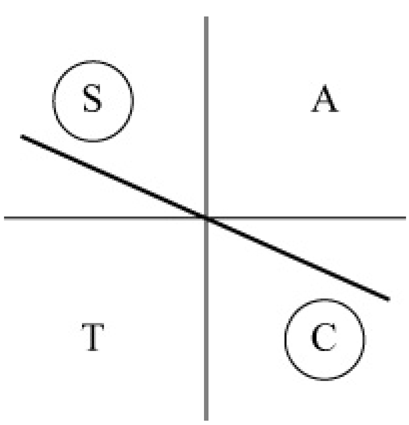
(d) $\cos\theta = \cos(\theta + 60°)$

$\Rightarrow \quad \cos\theta = \cos\theta\cos 60° - \sin\theta\sin 60° = \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$

$\Rightarrow \quad \cos\theta = -\sqrt{3}\sin\theta$

$\Rightarrow \quad \tan\theta = -\frac{1}{\sqrt{3}} \quad \left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

θ is in the 2nd and 4th quadrants.



$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is not in given interval.

$\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + 180°, \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + 360° = 150°,$ 330°

(e) $\tan(\theta - 45°) = 6\tan\theta$

$\Rightarrow \quad \frac{\tan\theta - \tan 45°}{1 + \tan\theta\tan 45°} = 6\tan\theta$

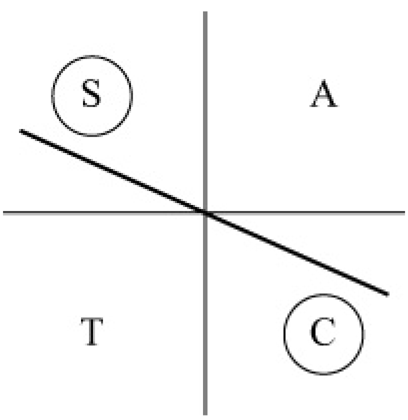
$\Rightarrow \quad \frac{\tan\theta - 1}{1 + \tan\theta} = 6\tan\theta$

$\Rightarrow \quad \tan\theta - 1 = 6\tan\theta + 6\tan^2\theta$

$\Rightarrow \quad 6\tan^2\theta + 5\tan\theta + 1 = 0$

$\Rightarrow \quad (3\tan\theta + 1)(2\tan\theta + 1) = 0$

$\Rightarrow \quad \tan\theta = -\frac{1}{3}$ or $\tan\theta = -\frac{1}{2}$



$\tan\theta = -\frac{1}{3} \quad \Rightarrow \quad \theta = \tan^{-1}\left(-\frac{1}{3}\right) + 180°, \tan^{-1}\left(-\frac{1}{3}\right) + 360° = 161.6°, 341.6°$

$\tan\theta = -\frac{1}{2} \quad \Rightarrow \quad \theta = \tan^{-1}\left(-\frac{1}{2}\right) + 180°, \tan^{-1}\left(-\frac{1}{2}\right) + 360° = 153.4°, 333.4°$

Set of solutions: 153.4°, 161.6°, 333.4°, 341.6°

(f) $\sin\theta + \cos\theta = 1$

$\Rightarrow \quad \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \cos 45° \sin\theta + \sin 45° \cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin(\theta + 45°) = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta + 45° = 45°, 135°$

$\Rightarrow \quad \theta = 0°, 90°$

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Exercise A, Question 14

Question:

(a) Solve the equation $\cos\theta\cos 30° - \sin\theta\sin 30° = 0.5$, for $0 \leq \theta \leq 360°$.

(b) Hence write down, in the same interval, the solutions of $\sqrt{3}\cos\theta - \sin\theta = 1$.

Solution:

(a) $\cos\theta\cos 30° - \sin\theta\sin 30° = 0.5$

$\Rightarrow \quad \cos(\theta + 30°) = 0.5$

$\Rightarrow \quad \theta + 30° = 60°, 300°$

$\Rightarrow \quad \theta = 30°, 270°$

(b) $\cos\theta\cos 30° - \sin\theta\sin 30° \equiv \cos\theta \times \frac{\sqrt{3}}{2} - \sin\theta \times \frac{1}{2}$

So $\cos\theta\cos 30° - \sin\theta\sin 30° = \frac{1}{2}$

is identical to $\sqrt{3}\cos\theta - \sin\theta = 1$

Solutions are same as (a), i.e. 30°, 270°

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Exercise A, Question 15

Question:

(a) Express $\tan(45+30)^\circ$ in terms of $\tan 45°$ and $\tan 30°$.

(b) Hence show that $\tan 75° = 2 + \sqrt{3}$.

Solution:

(a) $\tan(45+30)^\circ = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

(b)

$$\tan 75° = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{4+2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3}$$

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Exercise A, Question 16

Question:

Show that $\sec 105^\circ = -\sqrt{2}(1+\sqrt{3})$

Solution:

$$\cos(60+45)^\circ = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

So $\sec 105^\circ$
$$= \frac{1}{\cos 105^\circ} = \frac{2\sqrt{2}}{1-\sqrt{3}}$$
$$= \frac{2\sqrt{2}(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$$
$$= \frac{2\sqrt{2}(1+\sqrt{3})}{-2}$$
$$= -\sqrt{2}(1+\sqrt{3})$$

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Exercise A, Question 17

Question:

Calculate the exact values of

(a) $\cos 15°$

(b) $\sin 75°$

(c) $\sin (120 + 45)°$

(d) $\tan 165°$

Solution:

(a) $\cos 15° = \cos (45 - 30)°$
$= \cos 45° \cos 30° + \sin 45° \sin 30°$
$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$
$= \frac{\sqrt{2}}{4} (\sqrt{3} + 1)$

(b) $\sin 75° = \sin (45 + 30)°$
$= \sin 45° \cos 30° + \cos 45° \sin 30°$
$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$
$= \frac{\sqrt{2}}{4} (\sqrt{3} + 1)$
$[\sin 75° = \cos (90 - 75°) = \cos 15°]$

(c) $\sin (120 + 45)° = \sin 120° \cos 45° + \cos 120° \sin 45°$
$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \left(- \frac{1}{2} \right) \frac{\sqrt{2}}{2}$
$= \frac{\sqrt{2}}{4} (\sqrt{3} - 1)$

(d) $\tan 165° = \tan (120 + 45)° = \frac{\tan 120° + \tan 45°}{1 - \tan 120° \tan 45°}$ $(\tan 120° = -\sqrt{3})$

$$= \frac{-\sqrt{3}+1}{1+\sqrt{3}}$$

$$= \frac{(-\sqrt{3}+1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{-4+2\sqrt{3}}{2}$$

$$= -2+\sqrt{3}$$

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Exercise A, Question 18

Question:

(a) Given that $3\sin(x-y)-\sin(x+y)=0$, show that $\tan x=2\tan y$.

(b) Solve $3\sin(x-45°)-\sin(x+45°)=0$, for $0 \le x \le 360°$.

Solution:

(a) $3\sin(x-y)-\sin(x+y)=0$

$\Rightarrow \quad 3\sin x\cos y-3\cos x\sin y-\sin x\cos y-\cos x\sin y=0$

$\Rightarrow \quad 2\sin x\cos y=4\cos x\sin y$

$\Rightarrow \quad \dfrac{2\sin x\cos y}{\cos x\cos y}=\dfrac{4\cos x\sin y}{\cos x\cos y}$

$\Rightarrow \quad \dfrac{2\sin x}{\cos x}=\dfrac{4\sin y}{\cos y}$

$\Rightarrow \quad 2\tan x=4\tan y$

So $\tan x=2\tan y$

(b) Put $y=45° \quad \Rightarrow \quad \tan x=2$

So $x=\tan^{-1}2,\ 180°+\tan^{-1}2=63.4°,\ 243.4°$

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Exercise A, Question 19

Question:

Given that $\sin x\left(\cos y + 2\sin y\right) = \cos x\left(2\cos y - \sin y\right)$, find the value of $\tan\left(x + y\right)$.

Solution:

$$\sin x\left(\cos y + 2\sin y\right) = \cos x\left(2\cos y - \sin y\right)$$

$\Rightarrow \quad \sin x\cos y + 2\sin x\sin y = 2\cos x\cos y - \cos x\sin y$

$\Rightarrow \quad \sin x\cos y + \cos x\sin y = 2\left(\cos x\cos y - \sin x\sin y\right)$

$\Rightarrow \quad \sin\left(x + y\right) = 2\cos\left(x + y\right)$

$\Rightarrow \quad \dfrac{\sin\left(x + y\right)}{\cos\left(x + y\right)} = 2$

$\Rightarrow \quad \tan\left(x + y\right) = 2$

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Exercise A, Question 20

Question:

Given that $\tan(x-y)=3$, express $\tan y$ in terms of $\tan x$.

Solution:

As $\tan(x-y)=3$

so $\dfrac{\tan x-\tan y}{1+\tan x\tan y}=3$

$\Rightarrow \quad \tan x-\tan y=3+3\tan x\tan y$

$\Rightarrow \quad 3\tan x\tan y+\tan y=\tan x-3$

$\Rightarrow \quad \tan y(3\tan x+1)=\tan x-3$

$\Rightarrow \quad \tan y=\dfrac{\tan x-3}{3\tan x+1}$

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Exercise A, Question 21

Question:

In each of the following, calculate the exact value of $\tan x°$.

(a) $\tan(x-45)° = \frac{1}{4}$

(b) $\sin(x-60)° = 3\cos(x+30)°$

(c) $\tan(x-60)° = 2$

Solution:

(a) $\tan(x-45)° = \frac{1}{4}$

$\Rightarrow \quad \frac{\tan x° - \tan 45°}{1+\tan x° \tan 45°} = \frac{1}{4}$

$\Rightarrow \quad 4\tan x° - 4 = 1 + \tan x° \qquad (\tan 45° = 1)$

$\Rightarrow \quad 3\tan x° = 5$

$\Rightarrow \quad \tan x° = \frac{5}{3}$

(b) $\sin(x-60)° = 3\cos(x+30)°$

$\Rightarrow \quad \sin x° \cos 60° - \cos x° \sin 60° = 3\cos x° \cos 30° - 3\sin x° \sin 30°$

$\Rightarrow \quad \sin x° \times \frac{1}{2} - \cos x° \times \frac{\sqrt{3}}{2} = 3\cos x° \times \frac{\sqrt{3}}{2} - 3\sin x° \times \frac{1}{2}$

$\Rightarrow \quad 4\sin x° = 4\sqrt{3}\cos x°$

$\Rightarrow \quad \frac{\sin x°}{\cos x°} = \frac{4\sqrt{3}}{4}$

$\Rightarrow \quad \tan x° = \sqrt{3}$

(c) $\tan(x-60)° = 2$

$\Rightarrow \quad \frac{\tan x° - \tan 60°}{1+\tan x° \tan 60°} = 2$

$$\Rightarrow \quad \frac{\tan x^\circ - \sqrt{3}}{1 + \sqrt{3}\tan x^\circ} = 2$$

$$\Rightarrow \quad \tan x^\circ - \sqrt{3} = 2 + 2\sqrt{3}\tan x^\circ$$

$$\Rightarrow \quad (2\sqrt{3} - 1)\tan x^\circ = -(2 + \sqrt{3})$$

$$\Rightarrow \quad \tan x^\circ = -\frac{(2 + \sqrt{3})}{2\sqrt{3} - 1} = -\frac{(2 + \sqrt{3})(2\sqrt{3} + 1)}{(2\sqrt{3} - 1)(2\sqrt{3} + 1)} = -\frac{8 + 5\sqrt{3}}{11}$$

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Exercise A, Question 22

Question:

Given that $\tan A° = \frac{1}{5}$ and $\tan B° = \frac{2}{3}$, calculate, without using your calculator, the value of $A + B$,

(a) where A and B are both acute,

(b) where A is reflex and B is acute.

Solution:

(a) $\tan(A° + B°) = \frac{\tan A° + \tan B°}{1 - \tan A° \tan B°}$

$$= \frac{\frac{1}{5} + \frac{2}{3}}{1 - \frac{1}{5} \times \frac{2}{3}} = \frac{\frac{13}{15}}{\frac{15-2}{15}} = \frac{\frac{13}{15}}{\frac{13}{15}} = 1$$

As $\tan(A + B)°$ is +ve, $A + B$ is in the 1st or 3rd quadrants, but as they are both acute $A + B$ cannot be in the 3rd quadrant.
So $(A + B)° = \tan^{-1} 1 = 45°$
i.e. $A + B = 45$

(b) A is reflex but $\tan A°$ is +ve, so A is in 3rd quadrant,
i.e. $180° < A° < 270°$
and $0° < B° < 90°$
$(A + B)°$ must be in the 3rd quadrant as $\tan(A + B)°$ is +ve.
So $A + B = 225$

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Exercise A, Question 23

Question:

Given that $\cos y = \sin(x + y)$, show that $\tan y = \sec x - \tan x$.

Solution:

$\cos y = \sin(x + y)$

$\Rightarrow \quad \cos y = \sin x \cos y + \cos x \sin y$

Divide throughout by $\cos x \cos y$

$$\frac{\cos y}{\cos x \cos y} = \frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}$$

$\Rightarrow \quad \sec x = \tan x + \tan y$

$\Rightarrow \quad \tan y = \sec x - \tan x$

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Exercise A, Question 24

Question:

Given that $\cot A = \frac{1}{4}$ and $\cot (A + B) = 2$, find the value of $\cot B$.

Solution:

$\cot (A + B) = 2$

$\Rightarrow \quad \tan (A + B) = \frac{1}{2}$

$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1}{2}$

But as $\cot A = \frac{1}{4}$, then $\tan A = 4$.

So $\frac{4 + \tan B}{1 - 4 \tan B} = \frac{1}{2}$

$\Rightarrow \quad 8 + 2 \tan B = 1 - 4 \tan B$

$\Rightarrow \quad 6 \tan B = - 7$

$\Rightarrow \quad \tan B = - \frac{7}{6}$

So $\cot B = \frac{1}{\tan B} = - \frac{6}{7}$

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Exercise A, Question 25

Question:

Given that $\tan\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$, show that $\tan x = 8 - 5\sqrt{3}$.

Solution:

$$\tan\left(x+\frac{\pi}{3}\right)=\frac{1}{2}$$

$$\Rightarrow \frac{\tan x + \tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}=\frac{1}{2}$$

$$\Rightarrow \frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x}=\frac{1}{2} \qquad \left(\tan\frac{\pi}{3}=\sqrt{3}\right)$$

$$\Rightarrow 2\tan x + 2\sqrt{3} = 1 - \sqrt{3}\tan x$$

$$\Rightarrow (2+\sqrt{3})\tan x = 1 - 2\sqrt{3}$$

$$\Rightarrow \tan x = \frac{1-2\sqrt{3}}{2+\sqrt{3}} = \frac{(1-2\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{2-4\sqrt{3}-\sqrt{3}+6}{1} = 8-5\sqrt{3}$$

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Exercise B, Question 1

Question:

Write the following expressions as a single trigonometric ratio:

(a) $2\sin 10° \cos 10°$

(b) $1 - 2\sin^2 25°$

(c) $\cos^2 40° - \sin^2 40°$

(d) $\frac{2\tan 5°}{1 - \tan^2 5°}$

(e) $\frac{1}{2\sin\left(24\frac{1}{2}\right)° \cos\left(24\frac{1}{2}\right)°}$

(f) $6\cos^2 30° - 3$

(g) $\frac{\sin 8°}{\sec 8°}$

(h) $\cos^2 \frac{\pi}{16} - \sin^2 \frac{\pi}{16}$

Solution:

(a) $2\sin 10° \cos 10° = \sin 20°$ (using $2\sin A\cos A \equiv \sin 2A$)

(b) $1 - 2\sin^2 25° = \cos 50°$ (using $\cos 2A \equiv 1 - 2\sin^2 A$)

(c) $\cos^2 40° - \sin^2 40° = \cos 80°$ (using $\cos 2A \equiv \cos^2 A - \sin^2 A$)

(d) $\frac{2\tan 5°}{1 - \tan^2 5°} = \tan 10°$ (using $\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$)

(e) $\dfrac{1}{2\sin\left(24\frac{1}{2}\right)^\circ\cos\left(24\frac{1}{2}\right)^\circ} = \dfrac{1}{\sin 49^\circ} = \operatorname{cosec} 49^\circ$

(f) $6\cos^2 30^\circ - 3 = 3(2\cos^2 30^\circ - 1) = 3\cos 60^\circ$

(g) $\dfrac{\sin 8^\circ}{\sec 8^\circ} = \sin 8^\circ \cos 8^\circ = \dfrac{1}{2}(2\sin 8^\circ \cos 8^\circ) = \dfrac{1}{2}\sin 16^\circ$

(h) $\cos^2 \dfrac{\pi}{16} - \sin^2 \dfrac{\pi}{16} = \cos \dfrac{2\pi}{16} = \cos \dfrac{\pi}{8}$

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Exercise B, Question 2

Question:

Without using your calculator find the exact values of:

(a) $2\sin\left(22\frac{1}{2}\right)^\circ\cos\left(22\frac{1}{2}\right)^\circ$

(b) $2\cos^2 15^\circ - 1$

(c) $(\sin 75^\circ - \cos 75^\circ)^2$

(d) $\dfrac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

Solution:

(a) $2\sin\left(22\frac{1}{2}\right)^\circ\cos\left(22\frac{1}{2}\right)^\circ = \sin 2\times 22\frac{1}{2}^\circ = \sin 45^\circ = \frac{\sqrt{2}}{2}$

(b) $2\cos^2 15^\circ - 1 = \cos(2\times 15^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

(c)
$$\begin{aligned}(\sin 75^\circ - \cos 75^\circ)^2 &= \sin^2 75^\circ + \cos^2 75^\circ - 2\sin 75^\circ\cos 75^\circ\\ &= 1 - \sin(2\times 75^\circ) \qquad (\sin^2 75^\circ + \cos^2 75^\circ = 1)\\ &= 1 - \sin 150^\circ\\ &= 1 - \frac{1}{2}\\ &= \frac{1}{2}\end{aligned}$$

(d) $\dfrac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} = \tan\left(2\times\frac{\pi}{8}\right) = \tan\frac{\pi}{4} = 1$

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Exercise B, Question 3

Question:

Write the following in their simplest form, involving only one trigonometric function:

(a) $\cos^2 3\theta - \sin^2 3\theta$

(b) $6 \sin 2\theta \cos 2\theta$

(c) $\dfrac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$

(d) $2 - 4 \sin^2 \frac{\theta}{2}$

(e) $\sqrt{1 + \cos 2\theta}$

(f) $\sin^2 \theta \cos^2 \theta$

(g) $4 \sin \theta \cos \theta \cos 2\theta$

(h) $\dfrac{\tan \theta}{\sec^2 \theta - 2}$

(i) $\sin^4 \theta - 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

Solution:

(a) $\cos^2 3\theta - \sin^2 3\theta = \cos (2 \times 3\theta) = \cos 6\theta$

(b) $6 \sin 2\theta \cos 2\theta = 3 (2 \sin 2\theta \cos 2\theta) = 3 \sin (2 \times 2\theta) = 3 \sin 4\theta$

(c) $\dfrac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \tan \left(2 \times \frac{\theta}{2} \right) = \tan \theta$

(d) $2 - 4\sin^2\left(\frac{1}{2}\theta\right) = 2\left[1 - 2\sin^2\left(\frac{1}{2}\theta\right)\right] = 2\cos\left(2 \times \frac{1}{2}\theta\right)$

$= 2\cos\theta$

(e) $\sqrt{1 + \cos 2\theta} = \sqrt{1 + (2\cos^2\theta - 1)} = \sqrt{2\cos^2\theta} = \sqrt{2}\cos\theta$

(f) $\sin^2\theta\cos^2\theta = \frac{1}{4}(4\sin^2\theta\cos^2\theta) = \frac{1}{4}(2\sin\theta\cos\theta)^2 = \frac{1}{4}\sin^2 2\theta$

(g) $4\sin\theta\cos\theta\cos 2\theta$

$= 2(2\sin\theta\cos\theta)\cos 2\theta$

$= 2\sin 2\theta\cos 2\theta$

$= \sin 4\theta \qquad (\sin 2A = 2\sin A\cos A \text{ with } A = 2\theta)$

(h) $\frac{\tan\theta}{\sec^2\theta - 2} = \frac{\tan\theta}{(1 + \tan^2\theta) - 2}$

$= \frac{\tan\theta}{\tan^2\theta - 1}$

$= -\frac{\tan\theta}{1 - \tan^2\theta}$

$= -\frac{1}{2}\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)$

$= -\frac{1}{2}\tan 2\theta$

(i) $\cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta = (\cos^2\theta - \sin^2\theta)^2 = \cos^2 2\theta$

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Exercise B, Question 4

Question:

Given that $\cos x = \frac{1}{4}$, find the exact value of $\cos 2x$.

Solution:

$$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{4}\right)^2 - 1 = \frac{1}{8} - 1 = -\frac{7}{8}$$

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Exercise B, Question 5

Question:

Find the possible values of $\sin\theta$ when $\cos 2\theta = \frac{23}{25}$.

Solution:

$\cos 2\theta = 1 - 2\sin^2\theta$

So $\frac{23}{25} = 1 - 2\sin^2\theta$

$\Rightarrow \quad 2\sin^2\theta = 1 - \frac{23}{25} = \frac{2}{25}$

$\Rightarrow \quad \sin^2\theta = \frac{1}{25}$

$\Rightarrow \quad \sin\theta = \pm\frac{1}{5}$

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Exercise B, Question 6

Question:

Given that $\cos x + \sin x = m$ and $\cos x - \sin x = n$, where m and n are constants, write down, in terms of m and n, the value of $\cos 2x$.

Solution:

$\cos x + \sin x = m$
$\cos x - \sin x = n$
Multiply the equations:
$(\cos x + \sin x)(\cos x - \sin x) = mn$
$\Rightarrow \quad \cos^2 x - \sin^2 x = mn$
$\Rightarrow \quad \cos 2x = mn$

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Exercise B, Question 7

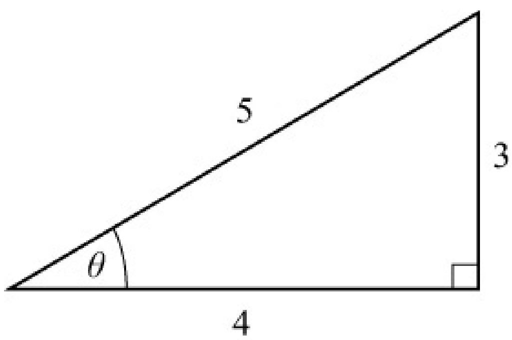
Question:

Given that $\tan\theta = \frac{3}{4}$, and that θ is acute:

(a) Find the exact value of
(i) $\tan 2\theta$
(ii) $\sin 2\theta$
(iii) $\cos 2\theta$

(b) Deduce the value of $\sin 4\theta$.

Solution:



The hypotenuse is 5,
so $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

(a) (i) $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{\frac{3}{2}}{1 - \frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \times \frac{16}{7} = \frac{24}{7}$

(ii) $\sin 2\theta = 2\sin\theta\cos\theta = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

(iii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

(b) $\sin 4\theta = 2\sin 2\theta\cos 2\theta = 2 \times \frac{24}{25} \times \frac{7}{25} = \frac{336}{625}$

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Exercise B, Question 8

Question:

Given that $\cos A = -\frac{1}{3}$, and that A is obtuse:

(a) Find the exact value of
(i) $\cos 2A$
(ii) $\sin A$
(iii) $\operatorname{cosec} 2A$

(b) Show that $\tan 2A = \frac{4\sqrt{2}}{7}$.

Solution:

(a) (i) $\cos 2A = 2\cos^2 A - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = \frac{2}{9} - 1 = -\frac{7}{9}$

(ii) $\cos 2A = 1 - 2\sin^2 A$

$\Rightarrow \quad -\frac{7}{9} = 1 - 2\sin^2 A$

$\Rightarrow \quad 2\sin^2 A = 1 + \frac{7}{9} = \frac{16}{9}$

$\Rightarrow \quad \sin^2 A = \frac{8}{9}$

$\Rightarrow \quad \sin A = \pm\frac{2\sqrt{2}}{3} \qquad (\sqrt{8} = 2\sqrt{2})$

but A is in 2nd quadrant $\Rightarrow$ $\sin A$ is +ve.

So $\sin A = \frac{2\sqrt{2}}{3}$

(iii) $\operatorname{cosec} 2A = \frac{1}{\sin 2A} = \frac{1}{2\sin A\cos A} = \frac{1}{2 \times \frac{2\sqrt{2}}{3} \times \left(-\frac{1}{3}\right)} = -\frac{9}{4\sqrt{2}} = -\frac{9\sqrt{2}}{8}$

(b) $$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{-\frac{4\sqrt{2}}{9}}{-\frac{7}{9}} = \frac{-4\sqrt{2}}{9} \times \frac{-9}{7} = \frac{4\sqrt{2}}{7}$$

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Exercise B, Question 9

Question:

Given that $\pi < \theta < \frac{3\pi}{2}$, find the value of $\tan\frac{\theta}{2}$ when $\tan\theta = \frac{3}{4}$.

Solution:

Using $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$

$$\Rightarrow \quad \frac{3}{4} = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$$

$$\Rightarrow \quad 3 - 3\tan^2\frac{\theta}{2} = 8\tan\frac{\theta}{2}$$

$$\Rightarrow \quad 3\tan^2\frac{\theta}{2} + 8\tan\frac{\theta}{2} - 3 = 0$$

$$\Rightarrow \quad \left(3\tan\frac{\theta}{2} - 1\right)\left(\tan\frac{\theta}{2} + 3\right) = 0$$

so $\tan\frac{\theta}{2} = \frac{1}{3}$ or $\tan\frac{\theta}{2} = -3$

but $\pi < \theta < \frac{3\pi}{2}$

so $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$

i.e. $\frac{\theta}{2}$ is in the 2nd quadrant

So $\tan\frac{\theta}{2}$ is −ve.

$$\Rightarrow \quad \tan\frac{\theta}{2} = -3$$

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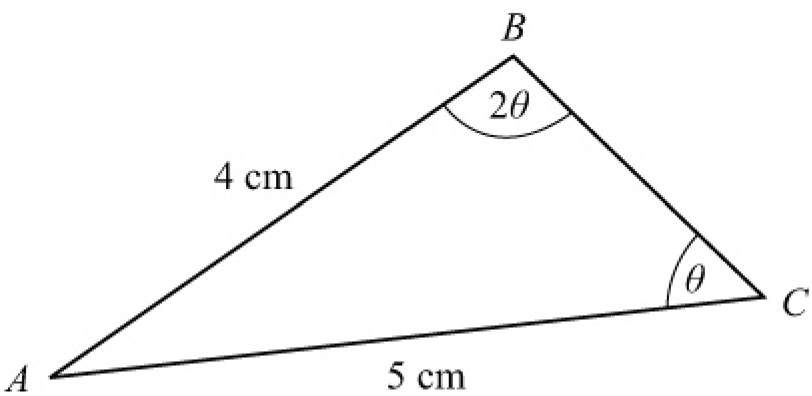
Exercise B, Question 10

Question:

In $\triangle ABC$, AB = 4 cm, AC = 5 cm, $\angle$ ABC = 2θ and $\angle$ ACB = θ. Find the value of θ, giving your answer, in degrees, to 1 decimal place.

Solution:



Using sine rule with $\frac{\sin B}{b} = \frac{\sin C}{c}$

$$\Rightarrow \quad \frac{\sin 2\theta}{5} = \frac{\sin \theta}{4}$$

$$\Rightarrow \quad \frac{2\sin\theta\cos\theta}{5} = \frac{\sin\theta}{4}$$

Cancel $\sin\theta$ as $\theta \neq 0°$ or $180°$

So $2\cos\theta = \frac{5}{4}$

$$\Rightarrow \quad \cos\theta = \frac{5}{8}$$

So $\theta = \cos^{-1}\frac{5}{8} = 51.3°$

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Exercise B, Question 11

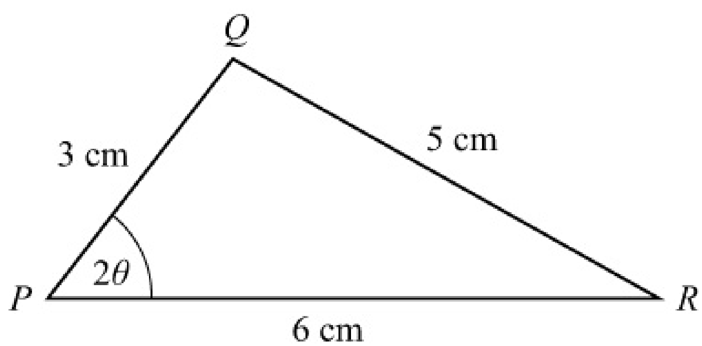
Question:

In $\triangle PQR$, PQ = 3 cm, PR = 6 cm, QR = 5 cm and $\angle$ QPR = 2θ.

(a) Use the cosine rule to show that $\cos 2\theta = \frac{5}{9}$.

(b) Hence find the exact value of $\sin \theta$.

Solution:



(a) Using cosine rule with $\cos P = \frac{q^2 + r^2 - p^2}{2qr}$

$$\cos 2\theta = \frac{36 + 9 - 25}{2 \times 6 \times 3} = \frac{20}{36} = \frac{5}{9}$$

(b) Using $\cos 2\theta = 1 - 2\sin^2 \theta$

$$\frac{5}{9} = 1 - 2\sin^2 \theta$$

$$\Rightarrow \quad 2\sin^2 \theta = 1 - \frac{5}{9} = \frac{4}{9}$$

$$\Rightarrow \quad \sin^2 \theta = \frac{2}{9}$$

$$\Rightarrow \quad \sin\theta = \pm \frac{\sqrt{2}}{3}$$

but $\sin \theta$ cannot be negative for θ in a triangle

so $\sin\theta = \frac{\sqrt{2}}{3}$

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Exercise B, Question 12

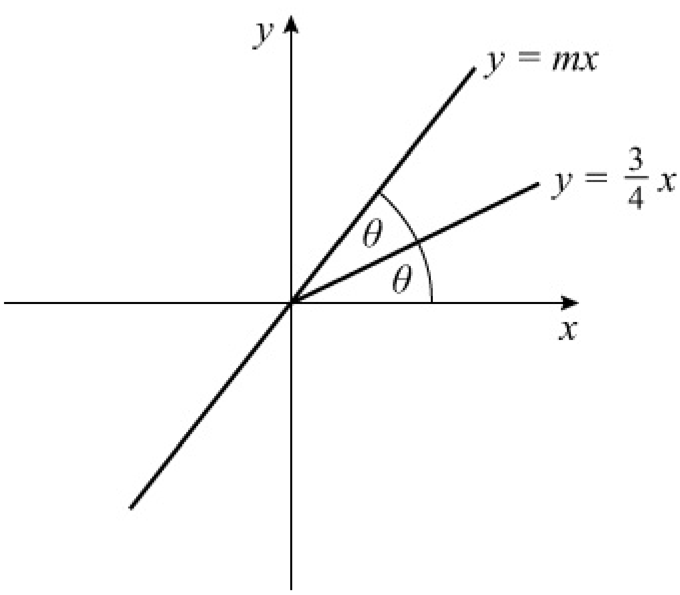
Question:

The line *l*, with equation $y = \frac{3}{4}x$, bisects the angle between the *x*-axis and the line $y = \text{mx}$, $m > 0$. Given that the scales on each axis are the same, and that *l* makes an angle θ with the *x*-axis,

(a) write down the value of $\tan\theta$.

(b) Show that $m = \frac{24}{7}$.

Solution:



(a) The gradient of line *l* is $\frac{3}{4}$, which is $\tan\theta$.

So $\tan\theta = \frac{3}{4}$

(b) The gradient of $y = \text{mx}$ is m, and as $y = \frac{3}{4}x$ bisects the angle between $y = \text{mx}$ and *x*-axis

$$m = \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2} \times \frac{16}{7} = \frac{24}{7}$$

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Exercise C, Question 1

Question:

Prove the following identities:

(a) $\frac{\cos 2A}{\cos A + \sin A} \equiv \cos A - \sin A$

(b) $\frac{\sin B}{\sin A} - \frac{\cos B}{\cos A} \equiv 2 \operatorname{cosec} 2A \sin (B - A)$

(c) $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$

(d) $\frac{\sec^2 \theta}{1 - \tan^2 \theta} \equiv \sec 2\theta$

(e) $2 (\sin^3 \theta \cos \theta + \cos^3 \theta \sin \theta) \equiv \sin 2\theta$

(f) $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} \equiv 2$

(g) $\operatorname{cosec} \theta - 2 \cot 2\theta \cos \theta \equiv 2 \sin \theta$

(h) $\frac{\sec \theta - 1}{\sec \theta + 1} \equiv \tan^2 \frac{\theta}{2}$

(i) $\tan \left(\frac{\pi}{4} - x \right) \equiv \frac{1 - \sin 2x}{\cos 2x}$

Solution:

(a) L.H.S. $\equiv \frac{\cos 2A}{\cos A + \sin A}$

$$\equiv \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$$

$$\equiv \frac{(\cos A + \sin A) (\cos A - \sin A)}{\cos A + \sin A}$$

$$\equiv \cos A - \sin A \equiv \text{R.H.S.}$$

(b) L.H.S. $\equiv \frac{\sin B}{\sin A} - \frac{\cos B}{\cos A}$

$$\equiv \frac{\sin B \cos A - \cos B \sin A}{\sin A \cos A}$$

$$\equiv \frac{\sin(B-A)}{\frac{1}{2}(2\sin A\cos A)}$$

$$\equiv \frac{2\sin(B-A)}{\sin 2A}$$

$$\equiv 2\operatorname{cosec} 2A\sin(B-A) \equiv \text{R.H.S.}$$

(c) L.H.S. $\equiv \frac{1-\cos 2\theta}{\sin 2\theta}$

$$\equiv \frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$$

$$\equiv \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$$

$$\equiv \frac{\sin\theta}{\cos\theta}$$

$$\equiv \tan\theta \equiv \text{R.H.S.}$$

(d) L.H.S. $\equiv \frac{\sec^2\theta}{1-\tan^2\theta}$

$$\equiv \frac{1}{\cos^2\theta(1-\tan^2\theta)}$$

$$\equiv \frac{1}{\cos^2\theta-\sin^2\theta} \quad \left(\text{as } \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\right)$$

$$\equiv \frac{1}{\cos 2\theta}$$

$$\equiv \sec 2\theta \equiv \text{R.H.S.}$$

(e) L.H.S. $\equiv 2(\sin^3\theta\cos\theta + \cos^3\theta\sin\theta)$

$$\equiv 2\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)$$

$$\equiv \sin 2\theta \quad (\text{since } \sin^2\theta+\cos^2\theta\equiv 1)$$

$$\equiv \text{R.H.S.}$$

(f) L.H.S. $\equiv \frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$

$$\equiv \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$\equiv \frac{\sin(3\theta-\theta)}{\frac{1}{2}\sin 2\theta}$$

$$\equiv \frac{\sin 2\theta}{\frac{1}{2}\sin 2\theta}$$

$$\equiv 2 \equiv \text{ R.H.S.}$$

(g) $\text{L.H.S.} \equiv \operatorname{cosec}\theta - 2\cot 2\theta\cos\theta$

$$\equiv \operatorname{cosec}\theta - 2\frac{\cos 2\theta}{\sin 2\theta}\cos\theta$$

$$\equiv \operatorname{cosec}\theta - \frac{2\cos 2\theta\cos\theta}{2\sin\theta\cos\theta}$$

$$\equiv \frac{1}{\sin\theta} - \frac{\cos 2\theta}{\sin\theta}$$

$$\equiv \frac{1-\cos 2\theta}{\sin\theta}$$

$$\equiv \frac{1-(1-2\sin^2\theta)}{\sin\theta}$$

$$\equiv \frac{2\sin^2\theta}{\sin\theta}$$

$$\equiv 2\sin\theta \equiv \text{ R.H.S.}$$

(h)

$$\text{L.H.S.} \equiv \frac{\sec\theta - 1}{\sec\theta + 1}$$

$$\equiv \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$$

$$\equiv \frac{1-\cos\theta}{1+\cos\theta}$$

$$\equiv \frac{1-\left(1-2\sin^2\frac{\theta}{2}\right)}{1+\left(2\cos^2\frac{\theta}{2}-1\right)}$$

$$\equiv \frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$

$$\equiv \tan^2\frac{\theta}{2} \equiv \text{R.H.S.}$$

(i)

$$\text{L.H.S.} \equiv \tan\left(\frac{\pi}{4} - x\right)$$

$$\equiv \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}$$

$$\equiv \frac{1 - \tan x}{1 + \tan x}$$

$$\equiv \frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}}$$

$$\equiv \frac{\cos x - \sin x}{\cos x + \sin x}$$

Multiply 'top and bottom' by $\cos x - \sin x$

$$\equiv \frac{\cos^2 x + \sin^2 x - 2\sin x\cos x}{\cos^2 x - \sin^2 x}$$

$$\equiv \frac{1 - \sin 2x}{\cos 2x} \equiv \text{R.H.S.}$$

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Exercise C, Question 2

Question:

(a) Show that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$.

(b) Hence find the value of $\tan 75° + \cot 75°$.

Solution:

(a) L.H.S. $\equiv \tan\theta + \cot\theta$

$$\equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\equiv \frac{2}{2\sin\theta\cos\theta} \qquad (\sin^2\theta + \cos^2\theta \equiv 1)$$

$$\equiv \frac{2}{\sin 2\theta}$$

$$\equiv 2\operatorname{cosec} 2\theta \equiv \text{R.H.S.}$$

(b) Use $\theta = 75°$

$$\Rightarrow \quad \tan 75° + \cot 75° = 2\operatorname{cosec} 150° = 2 \times \frac{1}{\sin 150°} = 2 \times \frac{1}{\frac{1}{2}} = 4$$

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Exercise C, Question 3

Question:

Solve the following equations, in the interval shown in brackets. Give answers to 1 decimal place where appropriate.

(a) $\sin 2\theta = \sin\theta \quad \{ 0 \le \theta \le 2\pi \{$

(b) $\cos 2\theta = 1 - \cos\theta \quad \{ -180° < \theta \le 180° \{$

(c) $3\cos 2\theta = 2\cos^2\theta \quad \{ 0 \le \theta < 360° \{$

(d) $\sin 4\theta = \cos 2\theta \quad \{ 0 \le \theta \le \pi \{$

(e) $2\tan 2y \tan y = 3 \quad \{ 0 \le y < 360° \{$

(f) $3\cos\theta - \sin\frac{\theta}{2} - 1 = 0 \quad \left\{ 0 \le \theta < 720° \right\}$

(g) $\cos^2\theta - \sin 2\theta = \sin^2\theta \quad \{ 0 \le \theta \le \pi \{$

(h) $2\sin\theta = \sec\theta \quad \{ 0 \le \theta \le 2\pi \{$

(i) $2\sin 2\theta = 3\tan\theta \quad \{ 0 \le \theta < 360° \{$

(j) $2\tan\theta = \sqrt{3}(1 - \tan\theta)(1 + \tan\theta) \quad \{ 0 \le \theta \le 2\pi \{$

(k) $5\sin 2\theta + 4\sin\theta = 0 \quad \{ -180° < \theta \le 180° \{$

(l) $\sin^2\theta = 2\sin 2\theta \quad \{ -180° < \theta \le 180° \{$

(m) $4\tan\theta = \tan 2\theta \quad \{ 0 \le \theta < 360° \{$

Solution:

(a) $\sin 2\theta = \sin\theta, 0 \le \theta \le 2\pi$

$\Rightarrow \quad 2\sin\theta\cos\theta = \sin\theta$

$\Rightarrow \quad 2\sin\theta\cos\theta - \sin\theta = 0$

$\Rightarrow \quad \sin\theta(2\cos\theta - 1) = 0$

$\Rightarrow \quad \sin\theta = 0 \text{ or } \cos\theta = \frac{1}{2}$

Solution set: $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, 2\pi$

(b) $\cos 2\theta = 1 - \cos\theta, \ -180° < \theta \leq 180°$

$\Rightarrow \quad 2\cos^2\theta - 1 = 1 - \cos\theta$

$\Rightarrow \quad 2\cos^2\theta + \cos\theta - 2 = 0$

$\Rightarrow \quad \cos\theta = \frac{-1 \pm \sqrt{17}}{4}$

As $\frac{-1-\sqrt{17}}{4} < -1$, $\cos\theta = \frac{-1+\sqrt{17}}{4}$

As $\cos\theta$ is +ve, θ is in 1st and 4th quadrants.

Calculator solution is $\cos^{-1}\left(\frac{-1+\sqrt{17}}{4}\right) = 38.7°$.

Solutions are $\pm 38.7°$

(c) $3\cos 2\theta = 2\cos^2\theta, 0 \leq \theta < 360°$

$\Rightarrow \quad 3(2\cos^2\theta - 1) = 2\cos^2\theta$

$\Rightarrow \quad 6\cos^2\theta - 3 = 2\cos^2\theta$

$\Rightarrow \quad 4\cos^2\theta = 3$

$\Rightarrow \quad \cos^2\theta = \frac{3}{4}$

$\Rightarrow \quad \cos\theta = \pm\frac{\sqrt{3}}{2}$

θ will be in all four quadrants.
Solution set: 30°, 150°, 210°, 330°

(d) $\sin 4\theta = \cos 2\theta, 0 \leq \theta \leq \pi$

$\Rightarrow \quad 2\sin 2\theta\cos 2\theta = \cos 2\theta$

$\Rightarrow \quad \cos 2\theta(2\sin 2\theta - 1) = 0$

$\Rightarrow \quad \cos 2\theta = 0 \text{ or } \sin 2\theta = \frac{1}{2}$

$\cos 2\theta = 0$ in $0 \leq 2\theta \leq 2\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\sin 2\theta = \frac{1}{2}$ in $0 \leq 2\theta \leq 2\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}$

Solution set: $\frac{\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}, \frac{3\pi}{4}$

(e) $2\tan 2y \tan y = 3,\ 0 \leq y < 360°$

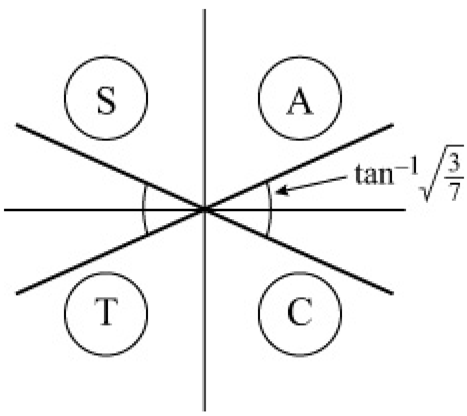
$\Rightarrow \quad \frac{4\tan y}{1 - \tan^2 y}\tan y = 3$

$\Rightarrow \quad 4\tan^2 y = 3 - 3\tan^2 y$

$\Rightarrow \quad 7\tan^2 y = 3$

$\Rightarrow \quad \tan^2 y = \frac{3}{7}$

$\Rightarrow \quad \tan y = \pm\sqrt{\frac{3}{7}}$



y is in all four quadrants.

$y = \tan^{-1}\sqrt{\frac{3}{7}}, 180° + \tan^{-1}\left(-\sqrt{\frac{3}{7}}\right), 180° + \tan^{-1}\sqrt{\frac{3}{7}}, 360°$
$+ \tan^{-1}\left(-\sqrt{\frac{3}{7}}\right)$

$y = 33.2°, 146.8°, 213.2°, 326.8°$

(f) $3\cos\theta - \sin\frac{\theta}{2} - 1 = 0,\ 0 \leq \theta \leq 720°$

$\Rightarrow \quad 3\left(1 - 2\sin^2\frac{\theta}{2}\right) - \sin\frac{\theta}{2} - 1 = 0$

$\Rightarrow \quad 6\sin^2\frac{\theta}{2} + \sin\frac{\theta}{2} - 2 = 0$

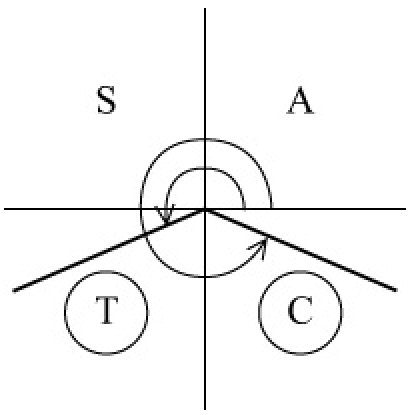
$\Rightarrow \quad \left(3\sin\frac{\theta}{2} + 2\right)\left(2\sin\frac{\theta}{2} - 1\right) = 0$

$\Rightarrow \quad \sin\frac{\theta}{2} = -\frac{2}{3}$ or $\sin\frac{\theta}{2} = \frac{1}{2}$

$\sin\frac{\theta}{2} = \frac{1}{2}$ in $0 \leq \frac{\theta}{2} \leq 360°$

$\Rightarrow \quad \frac{\theta}{2} = 30°, 150° \quad \Rightarrow \quad \theta = 60°, 300°$

$\sin\frac{\theta}{2} = -\frac{2}{3}$ in $0 \leq \frac{\theta}{2} \leq 360°$



$\Rightarrow \quad \frac{\theta}{2} = 180° - \sin^{-1}\left(-\frac{2}{3}\right), 360° + \sin^{-1}\left(-\frac{2}{3}\right) = 221.8°,$
$318.2°$

$\Rightarrow \quad \theta = 443.6°, 636.4°$

Solution set: 60°, 300°, 443.6°, 636.4°

(g) $\cos^2\theta - \sin 2\theta = \sin^2\theta, 0 \leq \theta \leq \pi$

$\Rightarrow \quad \cos^2\theta - \sin^2\theta = \sin 2\theta$

$\Rightarrow \quad \cos 2\theta = \sin 2\theta$

$\Rightarrow \quad \tan 2\theta = 1$ (divide both sides by $\cos 2\theta$)

$\tan 2\theta = 1$ in $0 \leq 2\theta \leq 2\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$\Rightarrow \quad \theta = \frac{\pi}{8}, \frac{5\pi}{8}$

(h) $2\sin\theta = \sec\theta, 0 \leq \theta \leq 2\pi$

$\Rightarrow \quad 2\sin\theta = \frac{1}{\cos\theta}$

$\Rightarrow \quad 2\sin\theta\cos\theta = 1$

$\Rightarrow \quad \sin 2\theta = 1$

$\sin 2\theta = 1$ in $0 \leq 2\theta \leq 4\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{2}, \frac{5\pi}{2}$ (see graph)

$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{5\pi}{4}$

(i) $2\sin 2\theta = 3\tan\theta, 0 \leq \theta < 360°$

$\Rightarrow \quad 4\sin\theta\cos\theta = \frac{3\sin\theta}{\cos\theta}$

$\Rightarrow \quad 4\sin\theta\cos^2\theta = 3\sin\theta$

$\Rightarrow \quad \sin\theta(4\cos^2\theta - 3) = 0$

$\Rightarrow \quad \sin\theta = 0$ or $\cos^2\theta = \frac{3}{4}$

$\sin\theta = 0 \quad \Rightarrow \quad \theta = 0°, 180°$

$\cos^2\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \pm\frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = 30°, 150°, 210°, 330°$

Solution set: 0°, 30°, 150°, 180°, 210°, 330°

(j) $2\tan\theta = \sqrt{3}(1 - \tan\theta)(1 + \tan\theta), 0 \leq \theta \leq 2\pi$

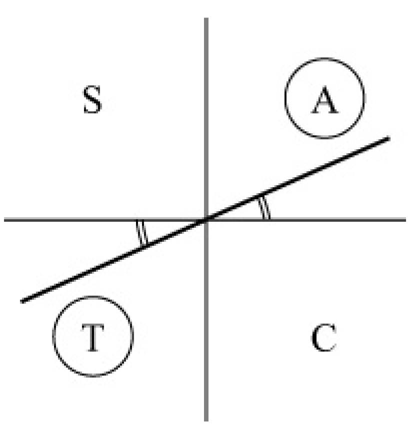
$\Rightarrow \quad 2\tan\theta = \sqrt{3}(1 - \tan^2\theta)$

$\Rightarrow \quad \sqrt{3}\tan^2\theta + 2\tan\theta - \sqrt{3} = 0$

$\Rightarrow \quad (\sqrt{3}\tan\theta - 1)(\tan\theta + \sqrt{3}) = 0$

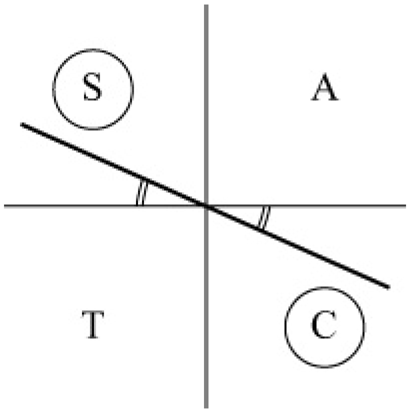
$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$ or $\tan\theta = -\sqrt{3}$

$\tan\theta = \frac{1}{\sqrt{3}}, 0 \leq \theta \leq 2\pi$



$\Rightarrow \quad \theta = \tan^{-1}\frac{1}{\sqrt{3}}, \pi + \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}, \frac{7\pi}{6}$

$\tan\theta = -\sqrt{3}, 0 \leq \theta \leq 2\pi$



$\Rightarrow \quad \theta = \pi + \tan^{-1}(-\sqrt{3}),\ 2\pi + \tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}, \frac{5\pi}{3}$

Solution set: $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$

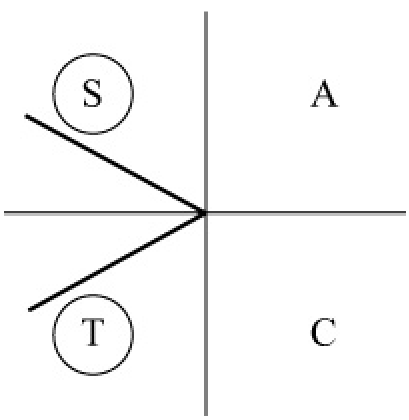
(k) $5\sin 2\theta + 4\sin\theta = 0,\ -180° < \theta \leq 180°$

$\Rightarrow \quad 10\sin\theta\cos\theta + 4\sin\theta = 0$

$\Rightarrow \quad 2\sin\theta(5\cos\theta + 2) = 0$

$\Rightarrow \quad \sin\theta = 0 \text{ or } \cos\theta = -\frac{2}{5}$

$\sin\theta = 0 \quad \Rightarrow \quad \theta = 0°, 180°$ (from graph)



Calculator value for $\cos^{-1}\left(-\frac{2}{5}\right)$ is 113.6°

$\Rightarrow \quad \theta = \pm 113.6°$

Solution set: −113.6°, 0°, 113.6°, 180°

(l) $\sin^2\theta = 2\sin 2\theta,\ -180° < \theta \leq 180°$

$\Rightarrow \quad \sin^2\theta = 4\sin\theta\cos\theta$

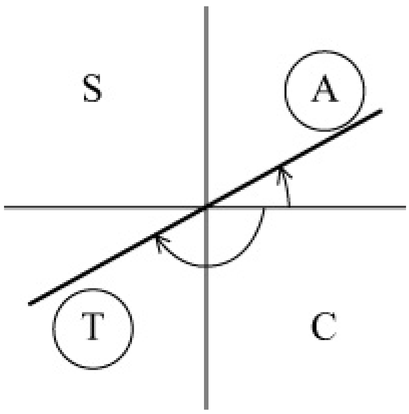
$\Rightarrow \quad \sin\theta(\sin\theta - 4\cos\theta) = 0$

$\Rightarrow \quad \sin\theta = 0$ or $\sin\theta = 4\cos\theta$

$\Rightarrow \quad \sin\theta = 0$ or $\tan\theta = 4$

$\sin\theta = 0 \quad \Rightarrow \quad \theta = 0°, 180°$

$\tan\theta = 4 \quad \Rightarrow \quad \theta = \tan^{-1} 4, \ -180° + \tan^{-1} 4 = 76.0°, -104.0°$



Solution set: −104.0°, 0°, 76.0°, 180°

(m) $4\tan\theta = \tan 2\theta, \ 0 \le \theta < 360°$

$\Rightarrow \quad 4\tan\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$\Rightarrow \quad 2\tan\theta(1 - \tan^2\theta) = \tan\theta$

$\Rightarrow \quad \tan\theta(2 - 2\tan^2\theta - 1) = 0$

$\Rightarrow \quad \tan\theta(1 - 2\tan^2\theta) = 0$

$\Rightarrow \quad \tan\theta = 0$ or $\tan\theta = \pm\sqrt{\frac{1}{2}}$

$\tan\theta = 0 \quad \Rightarrow \quad \theta = 0°, 180°$

$\tan\theta = \pm\sqrt{\frac{1}{2}} \quad \Rightarrow \quad \theta = 35.3°, 144.7°, 215.3°, 324.7°$

Solution set: 0°, 35.3°, 144.7°, 180°, 215.3°, 324.7°

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Exercise C, Question 4

Question:

Given that $p = 2\cos\theta$ and $q = \cos 2\theta$, express q in terms of p.

Solution:

$p = 2\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{p}{2}$

$\cos 2\theta = q$

Using $\cos 2\theta = 2\cos^2\theta - 1$

$q = 2\left(\frac{p}{2}\right)^2 - 1$

$\Rightarrow \quad q = \frac{p^2}{2} - 1$

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Exercise C, Question 5

Question:

Eliminate θ from the following pairs of equations:

(a) $x = \cos^2 \theta, y = 1 - \cos 2\theta$

(b) $x = \tan\theta, y = \cot 2\theta$

(c) $x = \sin\theta, y = \sin 2\theta$

(d) $x = 3\cos 2\theta + 1, y = 2\sin\theta$

Solution:

(a) $\cos^2 \theta = x, \cos 2\theta = 1 - y$

Using $\cos 2\theta = 2\cos^2 \theta - 1$

$\Rightarrow \quad 1 - y = 2x - 1$

$\Rightarrow \quad y = 2 - 2x = 2(1 - x)$ (any form)

(b) $y = \cot 2\theta \quad \Rightarrow \quad \tan 2\theta = \frac{1}{y}$

$x = \tan\theta$

Using $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2 \theta}$

$\Rightarrow \quad \frac{1}{y} = \frac{2x}{1 - x^2}$

$\Rightarrow \quad 2xy = 1 - x^2$ (any form)

(c) $x = \sin\theta, y = 2\sin\theta\cos\theta$

$\Rightarrow \quad y = 2x\cos\theta$

$\Rightarrow \quad \cos\theta = \frac{y}{2x}$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$

$\Rightarrow \quad x^2 + \frac{y^2}{4x^2} = 1$

$\Rightarrow \quad 4x^4 + y^2 = 4x^2$ or $y^2 = 4x^2(1 - x^2)$ (any form)

(d) $x = 3\cos 2\theta + 1 \quad \Rightarrow \quad \cos 2\theta = \frac{x-1}{3}$

$y = 2\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{y}{2}$

Using $\cos 2\theta = 1 - 2\sin^2\theta$

$\Rightarrow \quad \frac{x-1}{3} = 1 - \frac{2y^2}{4} = 1 - \frac{y^2}{2}$

$\Rightarrow \quad 2(x-1) = 6 - 3y^2 \qquad (\times 6)$

$\Rightarrow \quad 3y^2 = 6 - 2(x-1) = 8 - 2x$

$\Rightarrow \quad y^2 = \frac{2(4-x)}{3}$ (any form)

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Exercise C, Question 6

Question:

(a) Prove that $(\cos 2\theta - \sin 2\theta)^2 \equiv 1 - \sin 4\theta$.

(b) Use the result to solve, for $0 \le \theta < \pi$, the equation $\cos 2\theta - \sin 2\theta = \frac{1}{\sqrt{2}}$.

Give your answers in terms of π.

Solution:

(a) L.H.S. $\equiv (\cos 2\theta - \sin 2\theta)^2$

$\equiv \cos^2 2\theta - 2\sin 2\theta \cos 2\theta + \sin^2 2\theta$

$\equiv (\cos^2 2\theta + \sin^2 2\theta) - (2\sin 2\theta \cos 2\theta)$

$\equiv 1 - \sin 4\theta \qquad (\sin^2 A + \cos^2 A \equiv 1, \sin 2A \equiv 2\sin A \cos A)$

$\equiv$ R.H.S.

(b) You can use $(\cos 2\theta - \sin 2\theta)^2 = \frac{1}{2}$

but this also solves $\cos 2\theta - \sin 2\theta = -\frac{1}{\sqrt{2}}$

so you need to check your final answers.

As $(\cos 2\theta - \sin 2\theta)^2 \equiv 1 - \sin 4\theta$

$\Rightarrow \quad \frac{1}{2} = 1 - \sin 4\theta$

$\Rightarrow \quad \sin 4\theta = \frac{1}{2}$

$0 \le \theta < \pi$, so $0 \le 4\theta < 4\pi$

$\Rightarrow \quad 4\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{24}, \frac{5\pi}{24}, \frac{13\pi}{24}, \frac{17\pi}{24}$

Checking these values in $\cos 2\theta - \sin 2\theta = \frac{1}{\sqrt{2}}$

eliminates $\frac{5\pi}{24}, \frac{13\pi}{24}$ which apply to $\cos 2\theta - \sin 2\theta = -\frac{1}{\sqrt{2}}$

Solutions are $\frac{\pi}{24}, \frac{17\pi}{24}$

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Exercise C, Question 7

Question:

(a) Show that:

(i) $\sin\theta \equiv \dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

(ii) $\cos\theta \equiv \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

(b) By writing the following equations as quadratics in $\tan\frac{\theta}{2}$, solve, in the interval $0 \le \theta \le 360°$:

(i) $\sin\theta + 2\cos\theta = 1$ (ii) $3\cos\theta - 4\sin\theta = 2$

Give answers to 1 decimal place.

Solution:

(a) (i) R.H.S. $\equiv \dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

$$\equiv \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$\equiv \frac{2\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \times \cos^2\frac{\theta}{2}$$

$$\equiv 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$\equiv \sin\theta \quad (\sin 2A = 2\sin A\cos A)$

$\equiv$ L.H.S.

(ii) R.H.S. $\equiv \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

$\equiv \dfrac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$

$\equiv \cos^2\frac{\theta}{2}\left(1-\tan^2\frac{\theta}{2}\right)$

$\equiv \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} \qquad \left(\tan^2\frac{\theta}{2} = \dfrac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}\right)$

$\equiv \cos\theta \qquad (\cos 2A = \cos^2 A - \sin^2 A)$

$\equiv$ L.H.S.

(b) Let $\tan\frac{\theta}{2} = t$

(i) $\sin\theta + 2\cos\theta = 1$

$\Rightarrow \quad \dfrac{2t}{1+t^2} + \dfrac{2(1-t^2)}{1+t^2} = 1$

$\Rightarrow \quad 2t + 2 - 2t^2 = 1 + t^2$

$\Rightarrow \quad 3t^2 - 2t - 1 = 0$

$\Rightarrow \quad (3t+1)(t-1) = 0$

$\Rightarrow \quad \tan\frac{\theta}{2} = -\frac{1}{3}$ or $\tan\frac{\theta}{2} = 1 \qquad 0 \le \frac{\theta}{2} \le 180°$

$\tan\frac{\theta}{2} = 1 \quad \Rightarrow \quad \frac{\theta}{2} = 45° \quad \Rightarrow \quad \theta = 90°$

$\tan\frac{\theta}{2} = -\frac{1}{3} \quad \Rightarrow \quad \frac{\theta}{2} = 161.56° \quad \Rightarrow \quad \theta = 323.1°$

Solution set: 90°, 323.1°

(ii) $3\cos\theta - 4\sin\theta = 2$

$\Rightarrow \quad \dfrac{3(1-t^2)}{1+t^2} - \dfrac{4\times 2t}{1+t^2} = 2$

$\Rightarrow \quad 3(1-t^2) - 8t = 2(1+t^2)$

$\Rightarrow \quad 5t^2 + 8t - 1 = 0$

$\Rightarrow \quad t = \dfrac{-8 \pm \sqrt{84}}{10}$

For $\tan \dfrac{\theta}{2} = \dfrac{-8 + \sqrt{84}}{10} \quad 0 \leq \dfrac{\theta}{2} \leq 180°$

$\dfrac{\theta}{2} = 6.65° \quad \Rightarrow \quad \theta = 13.3°$

For $\tan \dfrac{\theta}{2} = \dfrac{-8 - \sqrt{84}}{10} \quad 0 \leq \dfrac{\theta}{2} \leq 180°$

$\dfrac{\theta}{2} = 120.2° \quad \Rightarrow \quad \theta = 240.4°$

Solution set: 13.3°, 240.4°

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Exercise C, Question 8

Question:

(a) Using $\cos 2A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$, show that:

(i) $\cos^2 \frac{x}{2} \equiv \frac{1 + \cos x}{2}$

(ii) $\sin^2 \frac{x}{2} \equiv \frac{1 - \cos x}{2}$

(b) Given that $\cos\theta = 0.6$, and that θ is acute, write down the values of:

(i) $\cos \frac{\theta}{2}$

(ii) $\sin \frac{\theta}{2}$

(iii) $\tan \frac{\theta}{2}$

(c) Show that $\cos^4 \frac{A}{2} \equiv \frac{1}{8}(3 + 4\cos A + \cos 2A)$

Solution:

(a) (i) Using $\cos 2A \equiv 2\cos^2 A - 1$ with $A = \frac{x}{2}$

$\Rightarrow \quad \cos x \equiv 2\cos^2 \frac{x}{2} - 1$

$\Rightarrow \quad 2\cos^2 \frac{x}{2} \equiv 1 + \cos x$

$\Rightarrow \quad \cos^2 \frac{x}{2} \equiv \frac{1 + \cos x}{2}$

(ii) Using $\cos 2A \equiv 1 - 2\sin^2 A$

$\Rightarrow \quad \cos x \equiv 1 - 2\sin^2 \frac{x}{2}$

$\Rightarrow \quad 2\sin^2 \frac{x}{2} \equiv 1 - \cos x$

$\Rightarrow \quad \sin^2 \frac{x}{2} \equiv \frac{1 - \cos x}{2}$

(b) Given that $\cos\theta = 0.6$ and θ acute

(i) using (a) (i) $\cos^2 \frac{\theta}{2} = \frac{1.6}{2} = 0.8 = \frac{4}{5}$

$\Rightarrow \quad \cos \frac{\theta}{2} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \quad$ (as $\frac{\theta}{2}$ acute)

(ii) using (a) (ii) $\sin^2 \frac{\theta}{2} = \frac{0.4}{2} = 0.2 = \frac{1}{5}$

$\Rightarrow \quad \sin \frac{\theta}{2} = \sqrt{\frac{1}{5}} = \frac{\sqrt{5}}{5}$

(iii) $\tan \frac{\theta}{2} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} = \frac{\sqrt{5}}{5} \times \frac{5}{2\sqrt{5}} = \frac{1}{2}$

(c) Using (a) (i) and squaring

$\cos^4 \frac{A}{2} = \left(\frac{1 + \cos A}{2} \right)^2 = \frac{1 + 2\cos A + \cos^2 A}{4}$

but using (a) (i) again

$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

So $\cos^4 \frac{A}{2} = \frac{1 + 2\cos A + \frac{1}{2}(1 + \cos 2A)}{4} = \frac{2 + 4\cos A + 1 + \cos 2A}{8} =$

$\frac{3 + 4\cos A + \cos 2A}{8}$

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Exercise C, Question 9

Question:

(a) Show that $3\cos^2 x - \sin^2 x \equiv 1 + 2\cos 2x$.

(b) Hence sketch, for $-\pi \leq x \leq \pi$, the graph of $y = 3\cos^2 x - \sin^2 x$, showing the coordinates of points where the curve meets the axes.

Solution:

(a) R.H.S. $\equiv 1 + 2\cos 2x$

$\equiv 1 + 2(\cos^2 x - \sin^2 x)$

$\equiv 1 + 2\cos^2 x - 2\sin^2 x$

$\equiv \cos^2 x + \sin^2 x + 2\cos^2 x - 2\sin^2 x$ (using $\sin^2 x + \cos^2 x \equiv 1$)

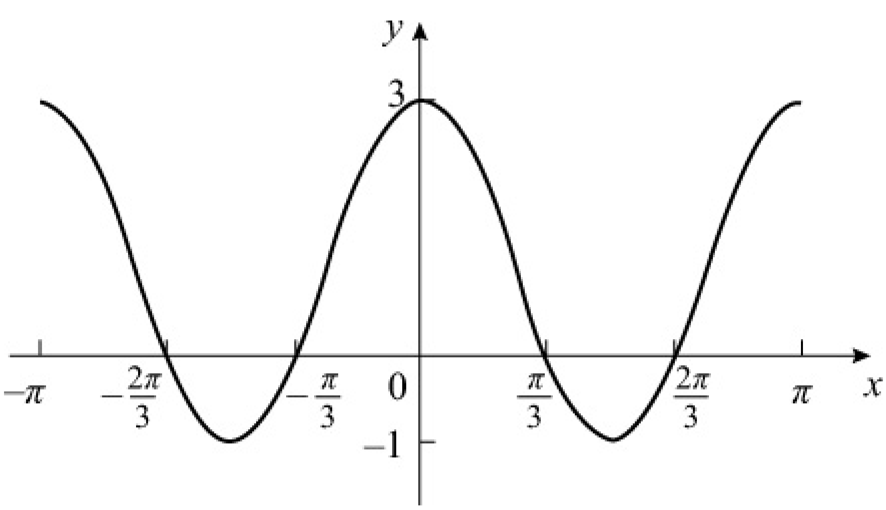
$\equiv 3\cos^2 x - \sin^2 x$

$\equiv$ L.H.S.

(b) $y = 3\cos^2 x - \sin^2 x$
is the same as $y = 1 + 2\cos 2x$
Using your work on transformations this curve is the result of

(i) stretching $y = \cos x$ by scale factor $\frac{1}{2}$ in the x direction, then

(ii) stretching the result by scale factor 2 in the y direction, then
(iii) translating by 1 in the +ve y direction.

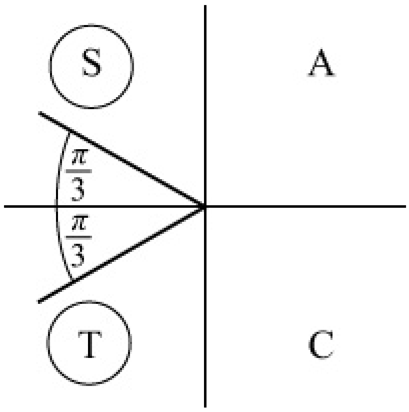


The curve crosses y-axis at (0, 3).

It crosses x-axis where $y = 0$

i.e. where $1 + 2\cos 2x = 0 \quad -\pi \leq x \leq \pi$

$\Rightarrow \quad \cos 2x = -\frac{1}{2} \quad -2\pi \leq 2x \leq 2\pi$



So $2x = -\frac{4\pi}{3}, \frac{-2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

$\Rightarrow \quad x = \frac{-2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

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Exercise C, Question 10

Question:

(a) Express $2\cos^2\frac{\theta}{2} - 4\sin^2\frac{\theta}{2}$ in the form $a\cos\theta + b$, where a and b are constants.

(b) Hence solve $2\cos^2\frac{\theta}{2} - 4\sin^2\frac{\theta}{2} = -3$, in the interval $0 \leq \theta < 360°$, giving answers to 1 decimal place.

Solution:

(a) $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$, $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

So $2\cos^2\frac{\theta}{2} - 4\sin^2\frac{\theta}{2} = (1+\cos\theta) - 2(1-\cos\theta) = 3\cos\theta - 1$

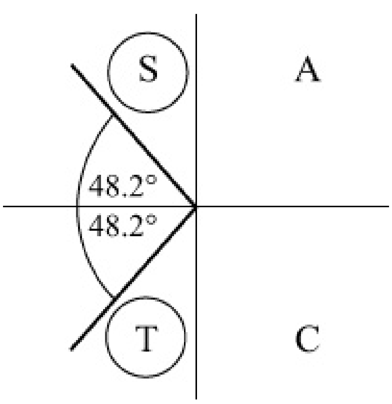
(b) Hence solve $3\cos\theta - 1 = -3$, $0 \leq \theta < 360°$

$\Rightarrow \quad 3\cos\theta = -2$

$\Rightarrow \quad \cos\theta = -\frac{2}{3}$

As $\cos\theta$ is $-$ve, θ is in 2nd and 3rd quadrants.

Calculator value is $\cos^{-1}\left(-\frac{2}{3}\right) = 131.8°$



Solutions are 131.8°, $360° - 131.8° = 131.8°$, 228.2°

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Exercise C, Question 11

Question:

(a) Use the identity $\sin^2 A + \cos^2 A \equiv 1$ to show that $\sin^4 A + \cos^4 A \equiv \frac{1}{2}(2 - \sin^2 2A)$.

(b) Deduce that $\sin^4 A + \cos^4 A \equiv \frac{1}{4}(3 + \cos 4A)$.

(c) Hence solve $8\sin^4\theta + 8\cos^4\theta = 7$, for $0 < \theta < \pi$.

Solution:

(a) As $\sin^2 A + \cos^2 A \equiv 1$

so $(\sin^2 A + \cos^2 A)^2 \equiv 1$

$\Rightarrow \quad \sin^4 A + \cos^4 A + 2\sin^2 A\cos^2 A \equiv 1$

$\Rightarrow \quad \sin^4 A + \cos^4 A \equiv 1 - 2\sin^2 A\cos^2 A$

$\equiv 1 - \frac{1}{2}(4\sin^2 A\cos^2 A)$

$\equiv 1 - \frac{1}{2}\left[(2\sin A\cos A)^2\right]$

$\equiv 1 - \frac{1}{2}\sin^2 2A$

$\equiv \frac{1}{2}(2 - \sin^2 2A)$

(b) As $\cos 2A \equiv 1 - 2\sin^2 A$

so $\cos 4A \equiv 1 - 2\sin^2 2A$

so $\sin^2 2A \equiv \frac{1 - \cos 4A}{2}$

$\Rightarrow \quad$ from (a) $\sin^4 A + \cos^4 A \equiv \frac{1}{2}\left(2 - \frac{1 - \cos 4A}{2}\right) \equiv \frac{1}{2}\left(\frac{4 - 1 + \cos 4A}{2}\right) \equiv \frac{1}{4}\left(3 + \cos 4A\right)$

(c) Using part (b)

$8\sin^4\theta + 8\cos^4\theta = 7$

$\Rightarrow \quad 8 \times \frac{1}{4}(3 + \cos 4\theta) = 7$

$\Rightarrow \quad 3 + \cos 4\theta = \frac{7}{2}$

$\Rightarrow \quad \cos 4\theta = \frac{1}{2}$

Solve $\cos 4\theta = \frac{1}{2}$ in $0 < 4\theta < 4\pi$

$\Rightarrow \quad 4\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$

$\Rightarrow \quad \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}$

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Exercise C, Question 12

Question:

(a) By expanding $\cos(2A + A)$ show that $\cos 3A \equiv 4\cos^3 A - 3\cos A$.

(b) Hence solve $8\cos^3\theta - 6\cos\theta - 1 = 0$, for $\{0 \leq \theta \leq 360°\}$.

Solution:

(a) $\cos(2A + A) \equiv \cos 2A \cos A - \sin 2A \sin A$

$\equiv (2\cos^2 A - 1)\cos A - (2\sin A \cos A)\sin A$

$\equiv 2\cos^3 A - \cos A - 2\sin^2 A \cos A$

$\equiv 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$\equiv 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$\equiv 4\cos^3 A - 3\cos A$

(b) $8\cos^3\theta - 6\cos\theta - 1 = 0 \quad 0 \leq \theta \leq 360°$

$\Rightarrow \quad 2(4\cos^3\theta - 3\cos\theta) - 1 = 0$

$\Rightarrow \quad 2\cos 3\theta - 1 = 0$ [using part (a)]

$\Rightarrow \quad \cos 3\theta = \frac{1}{2}$

Solve $\cos 3\theta = \frac{1}{2}$ in $0 \leq 3\theta \leq 1080°$

$\Rightarrow \quad 3\theta = 60°, 300°, 420°, 660°, 780°, 1020°$

$\Rightarrow \quad \theta = 20°, 100°, 140°, 220°, 260°, 340°$

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Exercise C, Question 13

Question:

(a) Show that $\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$.

(b) Given that θ is acute such that $\cos\theta = \frac{1}{3}$, show that $\tan 3\theta = \frac{10\sqrt{2}}{23}$.

Solution:

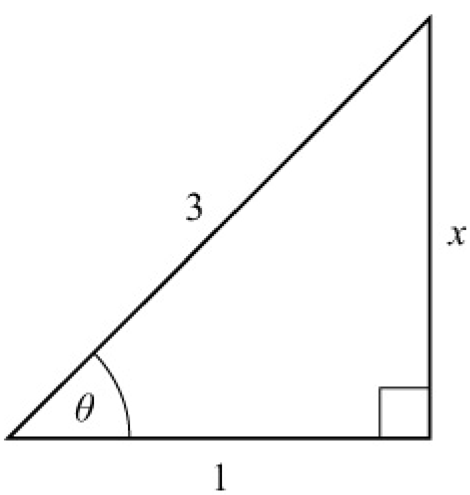
(a) $\tan 3\theta \equiv \tan\left(2\theta + \theta\right) \equiv \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$

Numerator $= \frac{2\tan\theta}{1 - \tan^2\theta} + \tan\theta \equiv \frac{2\tan\theta + \tan\theta - \tan^3\theta}{1 - \tan^2\theta} \equiv \frac{3\tan\theta - \tan^3\theta}{1 - \tan^2\theta}$

Denominator $= 1 - \frac{2\tan\theta}{1 - \tan^2\theta}\tan\theta \equiv \frac{1 - \tan^2\theta - 2\tan^2\theta}{1 - \tan^2\theta} \equiv \frac{1 - 3\tan^2\theta}{1 - \tan^2\theta}$

So $\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - \tan^2\theta} \times \frac{1 - \tan^2\theta}{1 - 3\tan^2\theta} \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

(b) Draw a right-angled triangle.



Using Pythagoras' theorem
$x^2 = 9 - 1 = 8$
So $x = 2\sqrt{2}$
So $\tan\theta = 2\sqrt{2}$
Using part (a)

$$\tan 3\theta = \frac{3(2\sqrt{2}) - (2\sqrt{2})^3}{1 - 3(2\sqrt{2})^2} = \frac{6\sqrt{2} - 16\sqrt{2}}{1 - 24} = \frac{-10\sqrt{2}}{-23} = \frac{10\sqrt{2}}{23}$$

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Exercise D, Question 1

Question:

Given that $5\sin\theta + 12\cos\theta \equiv R\sin(\theta + \alpha)$, find the value of R, $R > 0$, and the value of $\tan\alpha$.

Solution:

$5\sin\theta + 12\cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

Comparing $\sin\theta$: $R\cos\alpha = 5$

Comparing $\cos\theta$ $R\sin\alpha = 12$

Divide the equations:

$\frac{\sin\alpha}{\cos\alpha} = \frac{12}{5} \Rightarrow \tan\alpha = 2\frac{2}{5}$

Square and add the equations:

$R^2\cos^2\alpha + R^2\sin^2\alpha = 5^2 + 12^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 13^2$

$R = 13$

since $\cos^2\alpha + \sin^2\alpha \equiv 1$

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Exercise D, Question 2

Question:

Given that $\sqrt{3}\sin\theta + \sqrt{6}\cos\theta \equiv 3\cos(\theta - \alpha)$, where $0 < \alpha < 90°$, find the value of α to the nearest 0.1°.

Solution:

$\sqrt{3}\sin\theta + \sqrt{6}\cos\theta \equiv 3\cos\theta\cos\alpha + 3\sin\theta\sin\alpha$

Comparing $\sin\theta$: $\sqrt{3} = 3\sin\alpha$ ①

Comparing $\cos\theta$: $\sqrt{6} = 3\cos\alpha$ ②

Divide ① by ②:

$\tan\alpha = \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}}$

So $\alpha = 35.3°$

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Exercise D, Question 3

Question:

Given that $2\sin\theta - \sqrt{5}\cos\theta \equiv -3\cos(\theta + \alpha)$, where $0 < \alpha < 90°$, find the value of α to the nearest 0.1°.

Solution:

$2\sin\theta - \sqrt{5}\cos\theta \equiv -3\cos\theta\cos\alpha + 3\sin\theta\sin\alpha$

Comparing $\sin\theta$: $2 = 3\sin\alpha$ ①

Comparing $\cos\theta$: $+\sqrt{5} = +3\cos\alpha$ ②

Divide ① by ②:

$\tan\alpha = \frac{2}{\sqrt{5}}$

So $\alpha = 41.8°$

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Exercise D, Question 4

Question:

Show that:

(a) $\cos\theta + \sin\theta \equiv \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

(b) $\sqrt{3}\sin 2\theta - \cos 2\theta \equiv 2\sin\left(2\theta - \frac{\pi}{6}\right)$

Solution:

(a) R.H.S. $\equiv \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

$$\equiv \sqrt{2}\left(\sin\theta\cos\frac{\pi}{4} + \cos\theta\sin\frac{\pi}{4}\right)$$

$$\equiv \sqrt{2}\left(\sin\theta \times \frac{1}{\sqrt{2}} + \cos\theta \times \frac{1}{\sqrt{2}}\right)$$

$$\equiv \sin\theta + \cos\theta$$

$\equiv$ L.H.S.

(b) R.H.S. $\equiv 2\sin\left(2\theta - \frac{\pi}{6}\right)$

$$\equiv 2\left(\sin 2\theta\cos\frac{\pi}{6} - \cos 2\theta\sin\frac{\pi}{6}\right)$$

$$\equiv 2\left(\sin 2\theta \times \frac{\sqrt{3}}{2} - \cos 2\theta \times \frac{1}{2}\right)$$

$$\equiv \sqrt{3}\sin 2\theta - \cos 2\theta$$

$\equiv$ L.H.S.

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Exercise D, Question 5

Question:

Prove that $\cos 2\theta - \sqrt{3}\sin 2\theta \equiv 2\cos\left(2\theta + \frac{\pi}{3}\right) \equiv -2\sin\left(2\theta - \frac{\pi}{6}\right)$.

Solution:

Let $\cos 2\theta - \sqrt{3}\sin 2\theta \equiv R\cos(2\theta + \alpha) \equiv R\cos 2\theta\cos\alpha - R\sin 2\theta\sin\alpha$

Compare $\cos 2\theta$: $R\cos\alpha = 1$ ①

Compare $\sin 2\theta$: $R\sin\alpha = \sqrt{3}$ ②

Divide ② by ①:

$\tan\alpha = \sqrt{3} \quad \Rightarrow \quad \alpha = \frac{\pi}{3}$

Square and add equations:

$R^2 = 1 + 3 = 4$

$\Rightarrow \quad R = 2$

So $\cos 2\theta - \sqrt{3}\sin 2\theta \equiv 2\cos\left(2\theta + \frac{\pi}{3}\right)$

$$\cos\left(2\theta + \frac{\pi}{3}\right) \equiv \cos 2\theta\cos\frac{\pi}{3} - \sin 2\theta\sin\frac{\pi}{3}$$
$$\equiv \cos 2\theta \times \frac{1}{2} - \sin 2\theta \times \frac{\sqrt{3}}{2}$$
$$\equiv \cos 2\theta\sin\frac{\pi}{6} - \sin 2\theta\cos\frac{\pi}{6}$$
$$\equiv -\left(\sin 2\theta\cos\frac{\pi}{6} - \cos 2\theta\sin\frac{\pi}{6}\right)$$
$$\equiv -\sin\left(2\theta - \frac{\pi}{6}\right)$$

So $\cos 2\theta - \sqrt{3}\sin 2\theta \equiv 2\cos\left(2\theta + \frac{\pi}{3}\right) \equiv -2\sin\left(2\theta - \frac{\pi}{6}\right)$

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Exercise D, Question 6

Question:

Give all angles to the nearest 0.1° and non-exact values of R in surd form.
Find the value of R, where $R > 0$, and the value of α, where $0 < \alpha < 90°$, in each of the following cases:

(a) $\sin\theta + 3\cos\theta \equiv R\sin(\theta + \alpha)$

(b) $3\sin\theta - 4\cos\theta \equiv R\sin(\theta - \alpha)$

(c) $2\cos\theta + 7\sin\theta \equiv R\cos(\theta - \alpha)$

(d) $\cos 2\theta - 2\sin 2\theta \equiv R\cos(2\theta + \alpha)$

Solution:

(a) $\sin\theta + 3\cos\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$
Compare $\sin\theta$: $R\cos\alpha = 1$ ①
Compare $\cos\theta$: $R\sin\alpha = 3$ ②
Dividing ② by ①:
$\tan\alpha = 3 \Rightarrow \alpha = 71.6°$
Square and add equations:
$R^2 = 3^2 + 1^2 \Rightarrow R = \sqrt{10}$

(b) $3\sin\theta - 4\cos\theta \equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$
Compare $\sin\theta$: $R\cos\alpha = 3$ ①
Compare $\cos\theta$: $R\sin\alpha = 4$ ②
Divide ② by ①:
$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.1°$
Square and add equations:
$R^2 = 3^2 + 4^2 \Rightarrow R = 5$

(c) $2\cos\theta + 7\sin\theta \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$
Compare $\cos\theta$: $R\cos\alpha = 2$ ①
Compare $\sin\theta$: $R\sin\alpha = 7$ ②
Divide ② by ①:
$\tan\alpha = \frac{7}{2} \Rightarrow \alpha = 74.1°$

Square and add equations:

$R^2 = 2^2 + 7^2 = 53 \quad \Rightarrow \quad R = \sqrt{53}$

(d) $\cos 2\theta - 2\sin 2\theta \equiv R\cos 2\theta \cos\alpha - R\sin 2\theta \sin\alpha$

Compare $\cos 2\theta$: $\quad R\cos\alpha = 1 \quad$ ①

Compare $\sin 2\theta$: $\quad R\sin\alpha = 2 \quad$ ②

Divide ② by ①:

$\tan\alpha = 2 \quad \Rightarrow \quad \alpha = 63.4°$

Square and add equations:

$R^2 = 1^2 + 2^2 = 5 \quad \Rightarrow \quad R = \sqrt{5}$

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Exercise D, Question 7

Question:

(a) Show that $\cos\theta - \sqrt{3}\sin\theta$ can be written in the form $R\cos(\theta + \alpha)$, with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Hence sketch the graph of $y = \cos\theta - \sqrt{3}\sin\theta$, $0 < \alpha < 2\pi$, giving the coordinates of points of intersection with the axes.

Solution:

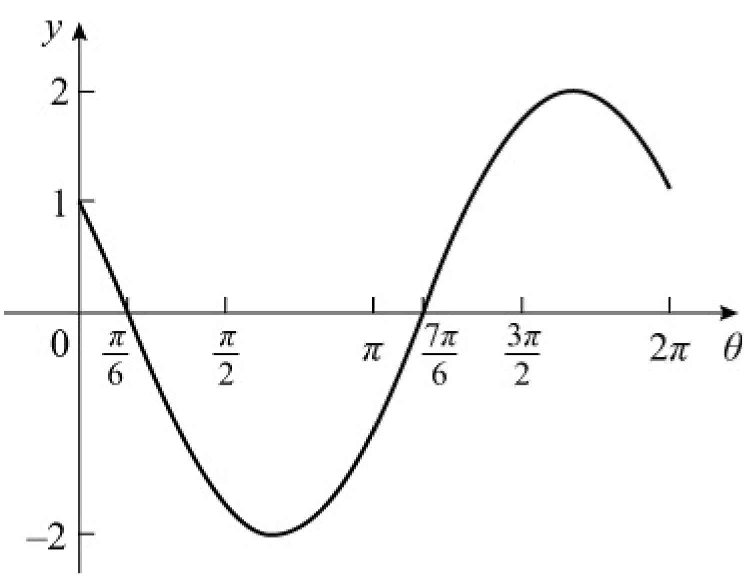
(a) Let $\cos\theta - \sqrt{3}\sin\theta \equiv R\cos(\theta + \alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$
Compare $\cos\theta$: $R\cos\alpha = 1$ ①
Compare $\sin\theta$: $R\sin\alpha = \sqrt{3}$ ②

Divide ② by ①: $\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

Square and add: $R^2 = 1 + 3 = 4 \Rightarrow R = 2$

So $\cos\theta - \sqrt{3}\sin\theta \equiv 2\cos\left(\theta + \frac{\pi}{3}\right)$

(b) This is the graph of $y = \cos\theta$, translated by $\frac{\pi}{3}$ to the left and then stretched in the y direction by scale factor 2.



Meets y-axis at (0, 1)

Meets x-axis at $\left(\frac{\pi}{6}, 0\right)$, $\left(\frac{7\pi}{6}, 0\right)$

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Exercise D, Question 8

Question:

(a) Show that $3\sin 3\theta - 4\cos 3\theta$ can be written in the form $R\sin(3\theta - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.

(b) Deduce the minimum value of $3\sin 3\theta - 4\cos 3\theta$ and work out the smallest positive value of θ (to the nearest 0.1°) at which it occurs.

Solution:

(a) Let $3\sin 3\theta - 4\cos 3\theta \equiv R\sin(3\theta - \alpha) \equiv R\sin 3\theta\cos\alpha - R\cos 3\theta\sin\alpha$
Compare $\sin 3\theta$: $R\cos\alpha = 3$ ①
Compare $\cos 3\theta$: $R\sin\alpha = 4$ ②
Divide ② by ①: $\tan\alpha = \frac{4}{3}$ $\Rightarrow$ $\alpha = 53.1°$
Square and add: $R^2 = 3^2 + 4^2$ $\Rightarrow$ $R = 5$
So $3\sin 3\theta - 4\cos 3\theta \equiv 5\sin(3\theta - 53.1°)$

(b) Minimum value occurs when $\sin(3\theta - 53.1°) = -1$
So minimum value is -5
To find smallest +ve value of θ solve $\sin(3\theta - 53.1°) = -1$
So $3\theta - 53.1° = 270°$
$\Rightarrow$ $3\theta = 323.1°$
$\Rightarrow$ $\theta = 107.7°$

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Exercise D, Question 9

Question:

(a) Show that $\cos 2\theta + \sin 2\theta$ can be written in the form $R\sin(2\theta + \alpha)$, with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(b) Hence solve, in the interval $0 \le \theta < 2\pi$, the equation $\cos 2\theta + \sin 2\theta = 1$, giving your answers as rational multiples of π.

Solution:

(a) Let $\cos 2\theta + \sin 2\theta \equiv R\sin(2\theta + \alpha) \equiv R\sin 2\theta\cos\alpha + R\cos 2\theta\sin\alpha$
Compare $\cos 2\theta$: $R\sin\alpha = 1$ ①
Compare $\sin 2\theta$: $R\cos\alpha = 1$ ②

Divide ① by ②: $\tan\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$

Square and add: $R^2 = 1^2 + 1^2 = 2 \Rightarrow R = \sqrt{2}$

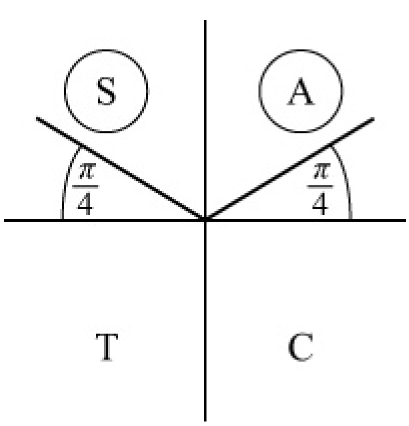
So $\cos 2\theta + \sin 2\theta \equiv \sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right)$

(b) Solve $\sqrt{2}\sin\left(2\theta + \frac{\pi}{4}\right) = 1$, $0 \le \theta < 2\pi$

so $\sin\left(2\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\frac{\pi}{4} \le 2\theta + \frac{\pi}{4} < \frac{17\pi}{4}$

As $\sin\left(2\theta + \frac{\pi}{4}\right)$ is +ve, $\left(2\theta + \frac{\pi}{4}\right)$ is in 1st and 2nd quadrants.

Calculator value is $\sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$



So $2\theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

$\Rightarrow \quad 2\theta = 0, \frac{\pi}{2}, 2\pi, \frac{5\pi}{2}$

$\Rightarrow \quad \theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$

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Exercise D, Question 10

Question:

(a) Express $7\cos\theta - 24\sin\theta$ in the form $R\cos(\theta + \alpha)$, with $R > 0$ and $0 < \alpha < 90°$. Give α to the nearest 0.1°.

(b) The graph of $y = 7\cos\theta - 24\sin\theta$ meets the y-axis at P. State the coordinates of P.

(c) Write down the maximum and minimum values of $7\cos\theta - 24\sin\theta$.

(d) Deduce the number of solutions, in the interval $0 < \theta < 360°$, of the following equations:
(i) $7\cos\theta - 24\sin\theta = 15$
(ii) $7\cos\theta - 24\sin\theta = 26$
(iii) $7\cos\theta - 24\sin\theta = -25$

Solution:

(a) Let $7\cos\theta - 24\sin\theta \equiv R\cos(\theta + \alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$
Compare $\cos\theta$: $R\cos\alpha = 7$ ①
Compare $\sin\theta$: $R\sin\alpha = 24$ ②
Divide ② by ①: $\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 73.7°$
Square and add: $R^2 = 24^2 + 7^2 \Rightarrow R = 25$
So $7\cos\theta - 24\sin\theta \equiv 25\cos(\theta + 73.7°)$

(b) Graph meets y-axis where $\theta = 0$,
i.e. $y = 7\cos 0° - 24\sin 0° = 7$
so coordinates are (0, 7)

(c) Maximum value of $25\cos(\theta + 73.7°)$ is when $\cos(\theta + 73.7°) = 1$
So maximum is 25
Minimum value is $25(-1) = -25$

(d) (i) The line $y = 15$ will meet the graph twice in $0 < \theta < 360°$, so there are 2 solutions.
(ii) As the maximum value is 25 it can never be 26, so there are 0 solutions.
(iii) As -25 is a minimum, line $y = -25$ only meets curve once, so only 1 solution.

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Exercise D, Question 11

Question:

(a) Express $5\sin^2\theta - 3\cos^2\theta + 6\sin\theta\cos\theta$ in the form $a\sin 2\theta + b\cos 2\theta + c$, where a, b and c are constants.

(b) Hence find the maximum and minimum values of $5\sin^2\theta - 3\cos^2\theta + 6\sin\theta\cos\theta$.

Solution:

(a) As $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$ and $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

so $5\sin^2\theta - 3\cos^2\theta + 6\sin\theta\cos\theta$

$\equiv 5\frac{1 - \cos 2\theta}{2} - 3\frac{1 + \cos 2\theta}{2} + 3(2\sin\theta\cos\theta)$

$\equiv \frac{5}{2} - \frac{5}{2}\cos 2\theta - \frac{3}{2} - \frac{3}{2}\cos 2\theta + 3\sin 2\theta$

$\equiv 1 - 4\cos 2\theta + 3\sin 2\theta$

(b) Write $3\sin 2\theta - 4\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$
The maximum value of $R\sin(2\theta - \alpha)$ is R
The minimum value of $R\sin(2\theta - \alpha)$ is $-R$
You know that $R^2 = 3^2 + 4^2$ so $R = 5$
So maximum value of $1 - 4\cos 2\theta + 3\sin 2\theta$ is $1 + 5 = 6$
and minimum value of $1 - 4\cos 2\theta + 3\sin 2\theta$ is $1 - 5 = -4$

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Exercise D, Question 12

Question:

Solve the following equations, in the interval given in brackets. Give all angles to the nearest 0.1°.

(a) $6\sin x + 8\cos x = 5\sqrt{3}$ [0, 360°]

(b) $2\cos 3\theta - 3\sin 3\theta = -1$ [0, 90°]

(c) $8\cos\theta + 15\sin\theta = 10$ [0, 360°]

(d) $5\sin\frac{x}{2} - 12\cos\frac{x}{2} = -6.5$ [$-360°$, 360°]

Solution:

(a) Write $6\sin x + 8\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$, $0 < \alpha < 90°$
so $6\sin x + 8\cos x \equiv R\sin x\cos\alpha + R\cos x\sin\alpha$
Compare $\sin x$: $R\cos\alpha = 6$ ①
Compare $\cos x$: $R\sin\alpha = 8$ ②
Divide ② by ①: $\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.13°$

$R^2 = 6^2 + 8^2 \Rightarrow R = 10$
So $6\sin x + 8\cos x \equiv 10\sin(x + 53.13°)$
Solve $10\sin(x + 53.13°) = 5\sqrt{3}$, $0 \le x \le 360°$
so $\sin(x + 53.13°) = \frac{\sqrt{3}}{2}$

$\Rightarrow x + 53.13° = 60°, 120°$

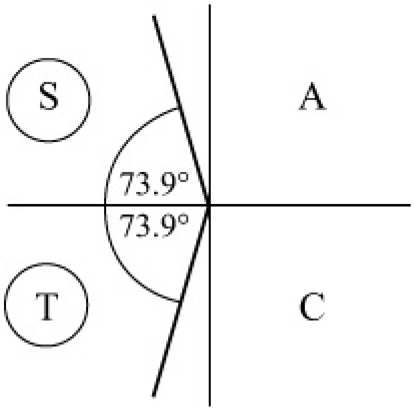
$\Rightarrow x = 6.9°, 66.9°$

(b) Let $2\cos 3\theta - 3\sin 3\theta \equiv R\cos(3\theta + \alpha) \equiv R\cos 3\theta\cos\alpha - R\sin 3\theta\sin\alpha$
Compare $\cos 3\theta$: $R\cos\alpha = 2$ ①
Compare $\sin 3\theta$: $R\sin\alpha = 3$ ②
Divide ② by ①: $\tan\alpha = \frac{3}{2} \Rightarrow \alpha = 56.31°$

$R^2 = 2^2 + 3^2 \Rightarrow R = \sqrt{13}$
Solve $\sqrt{13}\cos(3\theta + 56.31°) = -1$, $0 \le \theta \le 90°$

so $\cos(3\theta + 56.31°) = -\frac{1}{\sqrt{13}}$ for

$56.31° \leq 3\theta + 56.31° \leq 326.31°$



$\Rightarrow \quad 3\theta + 56.31° = 106.1°, 253.9°$

$\Rightarrow \quad 3\theta = 49.8°, 197.6°$

$\Rightarrow \quad \theta = 16.6°, 65.9°$

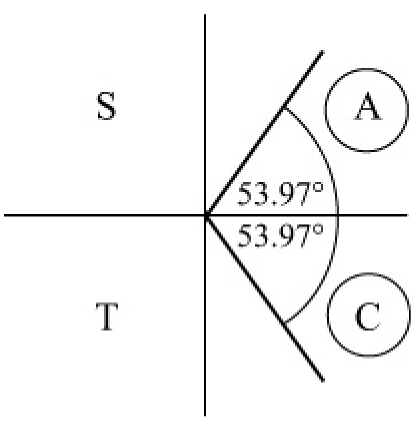
(c) Let $8\cos\theta + 15\sin\theta \equiv R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$
Compare $\cos\theta$: $\quad R\cos\alpha = 8 \quad$ ①
Compare $\sin\theta$: $\quad R\sin\alpha = 15 \quad$ ②

Divide ② by ①: $\quad \tan\alpha = \frac{15}{8} \quad \Rightarrow \quad \alpha = 61.93°$

$R^2 = 8^2 + 15^2 \quad \Rightarrow \quad R = 17$
Solve $17\cos(\theta - 61.93°) = 10, 0 \leq \theta \leq 360°$

so $\cos\left(\theta - 61.93°\right) = \frac{10}{17}, \; -61.93° \leq \theta - 61.93° \leq 298.1°$

$\cos^{-1}\left(\frac{10}{17}\right) = 53.97°$



So $\theta - 61.93° = -53.97°, +53.97°$

$\Rightarrow \quad \theta = 8.0°, 115.9°$

(d) Let $5\sin\frac{x}{2} - 12\cos\frac{x}{2} \equiv R\sin\left(\frac{x}{2} - \alpha\right) \equiv R\sin\frac{x}{2}\cos\alpha - R\cos\frac{x}{2}\sin\alpha$

Compare $\sin\frac{x}{2}$: $\quad R\cos\alpha = 5 \quad$ ①

Compare $\cos\frac{x}{2}$: $\quad R\sin\alpha = 12 \quad$ ②

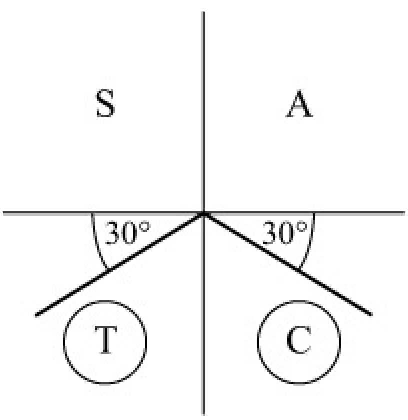
Divide ② by ①: $\quad \tan\alpha = \frac{12}{5} \quad \Rightarrow \quad \alpha = 67.38°$

$R = 13$

Solve $13\sin\left(\frac{x}{2} - 67.38°\right) = -6.5, \ -360° \le x \le 360°$

so $\sin\left(\frac{x}{2} - 67.38°\right) = -\frac{1}{2}, \quad -247.4° \le$

$\frac{x}{2} - 67.4° \le 112.6°$



From quadrant diagram:

$\frac{x}{2} - 67.4° = -150°, \ -30°$

$\Rightarrow \quad \frac{x}{2} = -82.6°, 37.4°$

$\Rightarrow \quad x = -165.2°, 74.8°$

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Exercise D, Question 13

Question:

Solve the following equations, in the interval given in brackets. Give all angles to the nearest 0.1°.

(a) $\sin x \cos x = 1 - 2.5 \cos 2x \quad [0, 360°]$

(b) $\cot \theta + 2 = \operatorname{cosec} \theta \quad [0 < \theta < 360, \theta \neq 180]$

(c) $\sin \theta = 2 \cos \theta - \sec \theta \quad [0, 180°]$

(d) $\sqrt{2} \cos \left(\theta - \frac{\pi}{4} \right) + \left(\sqrt{3} - 1 \right) \sin \theta = 2 \quad [0, 2\pi]$

Solution:

(a) $\sin x \cos x = 1 - 2.5 \cos 2x, 0 \leq x \leq 360°$

$\Rightarrow \quad \frac{1}{2} \sin 2x = 1 - 2.5 \cos 2x$

$\Rightarrow \quad \sin 2x + 5 \cos 2x = 2$

Let $\sin 2x + 5 \cos 2x \equiv R \sin (2x + \alpha) \equiv R \sin 2x \cos \alpha + R \cos 2x \sin \alpha$

Compare $\sin 2x$: $\quad R \cos \alpha = 1 \quad$ ①

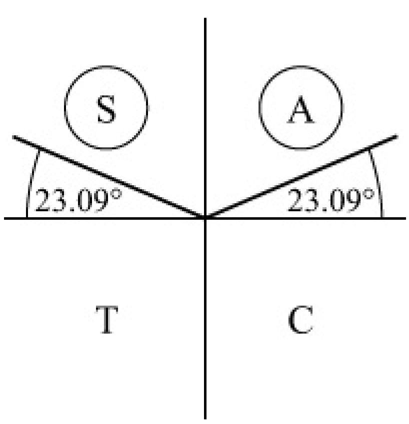
Compare $\cos 2x$: $\quad R \sin \alpha = 5 \quad$ ②

Divide ② by ①: $\quad \tan \alpha = 5 \quad \Rightarrow \quad \alpha = \tan^{-1} 5 = 78.7°$

$R^2 = 5^2 + 1^2 \quad \Rightarrow \quad R = \sqrt{26}$

Solve $\sqrt{26} \sin (2x + 78.7°) = 2, 0 \leq x \leq 360°$

$\Rightarrow \quad \sin \left(2x + 78.7° \right) = \frac{2}{\sqrt{26}}, 78.7° \leq 2x + 78.7° \leq 798.7°$



$\Rightarrow \quad 2x + 78.7° = 156.9°, 383.1°, 516.9°, 743.1°$

$\Rightarrow \quad 2x = 78.2°, 304.4°, 438.2°, 664.4°$

$\Rightarrow \quad x = 39.1°, 152.2°, 219.1°, 332.2°$

(b) $\cot\theta + 2 = \operatorname{cosec}\theta,\ 0 < \theta < 360°,\ \theta \neq 180°$

$\Rightarrow \quad \frac{\cos\theta}{\sin\theta} + 2 = \frac{1}{\sin\theta} \quad$ (as $\sin\theta \neq 0$)

$\Rightarrow \quad \cos\theta + 2\sin\theta = 1$

Let $\cos\theta + 2\sin\theta \equiv R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

Compare $\cos\theta$: $\quad R\cos\alpha = 1 \quad$ ①

Compare $\sin\theta$: $\quad R\sin\alpha = 2 \quad$ ②

Divide ② by ①: $\quad \tan\alpha = 2 \quad \Rightarrow \quad \alpha = 63.43°$

$R^2 = 2^2 + 1^2 \quad \Rightarrow \quad R = \sqrt{5}$

Solve $\sqrt{5}\cos(\theta - 63.43°) = 1,\ 0 < \theta < 360°$

$\Rightarrow \quad \cos(\theta - 63.43°) = \frac{1}{\sqrt{5}},\ -63.43° < \theta - 63.43° < 296.6°$

$\Rightarrow \quad \theta - 63.43° = 63.43°$

$\Rightarrow \quad \theta = 126.9°$

(c) $\sin\theta = 2\cos\theta - \sec\theta,\ 0 \leq \theta \leq 180°$

$\Rightarrow \quad \sin\theta\cos\theta = 2\cos^2\theta - 1 \quad (\times \cos\theta)$

$\Rightarrow \quad \frac{1}{2}\sin 2\theta = \cos 2\theta$

$\Rightarrow \quad \tan 2\theta = 2,\ 0 \leq 2\theta \leq 360°$

$\Rightarrow \quad 2\theta = \tan^{-1}2,\ 180° + \tan^{-1}2 = 63.43°, 243.43°$

$\Rightarrow \quad \theta = 31.7°, 121.7°$

(d) $\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) + (\sqrt{3} - 1)\sin\theta$

$\equiv \sqrt{2}\left(\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right) + (\sqrt{3} - 1)\sin\theta$

$\equiv \cos\theta + \sin\theta + \sqrt{3}\sin\theta - \sin\theta$

$\equiv \cos\theta + \sqrt{3}\sin\theta$

Let $\cos\theta + \sqrt{3}\sin\theta \equiv R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

Compare $\cos\theta$: $\quad R\cos\alpha = 1 \quad$ ①

Compare $\sin\theta$: $\quad R\sin\alpha = \sqrt{3} \quad$ ②

Divide ② by ①: $\quad \tan\alpha = \sqrt{3} \quad \Rightarrow \quad \alpha = \frac{\pi}{3}$

$R^2 = (\sqrt{3})^2 + 1^2 \Rightarrow R = 2$

Solve $2\cos\left(\theta - \frac{\pi}{3}\right) = 2,\ 0 \leq \theta \leq 2\pi$

$\Rightarrow \cos\left(\theta - \frac{\pi}{3}\right) = 1,\ -\frac{\pi}{3} \leq \theta - \frac{\pi}{3} \leq \frac{5\pi}{3}$

$\Rightarrow \theta - \frac{\pi}{3} = 0$

$\Rightarrow \theta = \frac{\pi}{3}$

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Exercise D, Question 14

Question:

Solve, if possible, in the interval $0 < \theta < 360°$, $\theta \neq 180°$, the equation $\frac{4 - 2\sqrt{2}\sin\theta}{1 + \cos\theta} = k$ in the case when k is equal to:

(a) 4

(b) 2

(c) 1

(d) 0

(e) −1

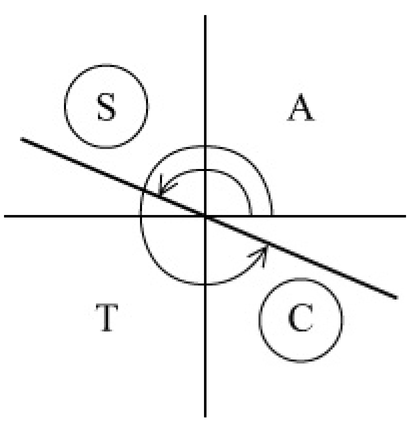
Give all angles to the nearest 0.1°.

Solution:

(a) When $k = 4$, $4 - 2\sqrt{2}\sin\theta = 4 + 4\cos\theta$

$\Rightarrow \quad -2\sqrt{2}\sin\theta = 4\cos\theta$

$\Rightarrow \quad \tan\theta = -\frac{4}{2\sqrt{2}} = -\sqrt{2}$



$\theta = 180° + \tan^{-1}(-\sqrt{2}), 360° + \tan^{-1}(-\sqrt{2}) = 125.3°, 305.3°$

(b) When $k = 2$, $4 - 2\sqrt{2}\sin\theta = 2 + 2\cos\theta$

$\Rightarrow \quad 2\cos\theta + 2\sqrt{2}\sin\theta = 2$

$\Rightarrow \quad \cos\theta + \sqrt{2}\sin\theta = 1$

Using the 'R formula' L.H.S. $\equiv \sqrt{3}\cos(\theta - 54.74°)$

Solve $\sqrt{3}\cos(\theta - 54.74°) = 1$

$\Rightarrow \quad \cos\left(\theta - 54.74°\right) = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta - 54.74° = 54.74°$

$\Rightarrow \quad \theta = 109.5°$

(c) When $k = 1$, $4 - 2\sqrt{2}\sin\theta = 1 + \cos\theta$

$\Rightarrow \quad \cos\theta + 2\sqrt{2}\sin\theta = 3$

Using the R formula, $\cos\theta + 2\sqrt{2}\sin\theta \equiv 3\cos(\theta - 70.53°)$

Solve $3\cos(\theta - 70.53°) = 3$

$\Rightarrow \quad \cos(\theta - 70.53°) = 1$

$\Rightarrow \quad \theta - 70.53° = 0°$

$\Rightarrow \quad \theta = 70.5°$

(d) When $k = 0$, $4 - 2\sqrt{2}\sin\theta = 0$

$\Rightarrow \quad \sin\theta = \sqrt{2}$

No solutions as $-1 \leq \sin\theta \leq 1$

(e) When $k = -1$, $4 - 2\sqrt{2}\sin\theta = -1 - \cos\theta$

$\Rightarrow \quad \cos\theta - 2\sqrt{2}\sin\theta = -5$

Using the R formula, $\cos\theta - 2\sqrt{2}\sin\theta \equiv 3\cos(\theta + 70.53°)$

This lies between −3 and +3, so there can be no solutions.

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Exercise D, Question 15

Question:

Give all angles to the nearest 0.1° and non-exact values of R in surd form.
A class were asked to solve $3\cos\theta = 2 - \sin\theta$ for $0 \leq \theta \leq 360°$. One student expressed the equation in the form $R\cos(\theta - \alpha) = 2$, with $R > 0$ and $0 < \alpha < 90°$, and correctly solved the equation.

(a) Find the values of R and α and hence find her solutions.
Another student decided to square both sides of the equation and then form a quadratic equation in $\sin\theta$.

(b) Show that the correct quadratic equation is $10\sin^2\theta - 4\sin\theta - 5 = 0$.

(c) Solve this equation, for $0 \leq \theta < 360°$.

(d) Explain why not all of the answers satisfy $3\cos\theta = 2 - \sin\theta$.

Solution:

(a) Let $3\cos\theta + \sin\theta \equiv R\cos(\theta - \alpha) \equiv R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$
Compare $\cos\theta$: $R\cos\alpha = 3$ ①
Compare $\sin\theta$: $R\sin\alpha = 1$ ②
Divide ② by ①: $\tan\alpha = \frac{1}{3} \Rightarrow \alpha = 18.43°$

$R^2 = 3^2 + 1^2 = 10 \Rightarrow R = \sqrt{10} = 3.16$
Solve $\sqrt{10}\cos(\theta - 18.43°) = 2,\ 0 \leq \theta \leq 360°$

$\Rightarrow \cos(\theta - 18.43°) = \frac{2}{\sqrt{10}}$

$\Rightarrow \theta - 18.43° = 50.77°, 309.23°$

$\Rightarrow \theta = 69.2°, 327.7°$

(b) Squaring $3\cos\theta = 2 - \sin\theta$
gives $9\cos^2\theta = 4 + \sin^2\theta - 4\sin\theta$
$\Rightarrow 9(1 - \sin^2\theta) = 4 + \sin^2\theta - 4\sin\theta$
$\Rightarrow 10\sin^2\theta - 4\sin\theta - 5 = 0$

(c) $10\sin^2\theta - 4\sin\theta - 5 = 0$

$\Rightarrow \quad \sin\theta = \frac{4 \pm \sqrt{216}}{20}$

For $\sin\theta = \frac{4 + \sqrt{216}}{20}$, $\sin\theta$ is +ve, so θ is in 1st and 2nd quadrants.

$\Rightarrow \quad \theta = 69.2°, 180° - 69.2° = 69.2°, 110.8°$

For $\sin\theta = \frac{4 - \sqrt{216}}{20}$, $\sin\theta$ is −ve, so θ is in 3rd and 4th quadrants.

$\Rightarrow \quad \theta = 180° - (-32.3°), 360° + (-32.3°) = 212.3°, 327.7°$

So solutions of quadratic in (b) are 69.2°, 110.8°, 212.3°, 327.7°

(d) In squaring the equation, you are also including the solutions to $3\cos\theta = -(2 - \sin\theta)$, which when squared produces the same quadratic.
The extra two solutions satisfying this equation.

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Exercise E, Question 1

Question:

(a) Show that $\sin(A+B)+\sin(A-B)\equiv 2\sin A\cos B$.

(b) Deduce that $\sin P+\sin Q\equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$.

(c) Use part (a) to express the following as the sum of two sines:
(i) $2\sin 7\theta\cos 2\theta$
(ii) $2\sin 12\theta\cos 5\theta$

(d) Use the result in (b) to solve, in the interval $0\leq\theta\leq 180°$, $\sin 3\theta+\sin\theta=0$.

(e) Prove that $\frac{\sin 7\theta+\sin\theta}{\sin 5\theta+\sin 3\theta}\equiv\frac{\cos 3\theta}{\cos\theta}$.

Solution:

(a) $\sin(A+B)+\sin(A-B)\equiv\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B$
$\equiv 2\sin A\cos B$

(b) Let $P=A+B$ and $Q=A-B$, so $A=\frac{P+Q}{2}$, $B=\frac{P-Q}{2}$

Substitute in (a): $\sin P+\sin Q\equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

(c) (i) $2\sin 7\theta\cos 2\theta\equiv\sin(7\theta+2\theta)+\sin(7\theta-2\theta)$ [from (a)]
$\equiv\sin 9\theta+\sin 5\theta$
(ii) $2\sin 12\theta\sin 5\theta\equiv\sin(12\theta+5\theta)+\sin(12\theta-5\theta)\equiv\sin 17\theta+\sin 7\theta$

(d) $\sin 3\theta+\sin\theta=0 \Rightarrow 2\sin\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)=0$
so $2\sin 2\theta\cos\theta=0$
$\Rightarrow \sin 2\theta=0$ or $\cos\theta=0$
$\sin 2\theta=0$ in $0\leq 2\theta\leq 360°$
$\Rightarrow 2\theta=0°, 180°, 360°$
$\Rightarrow \theta=0°, 90°, 180°$
$\cos\theta=0$ in $0\leq\theta\leq 180° \Rightarrow \theta=90°$
Solution set: 0°, 90°, 180°

(e)

$$\frac{\sin 7\theta + \sin\theta}{\sin 5\theta + \sin 3\theta} \equiv \frac{\cancel{2}\,\cancel{\sin 4\theta}\cos 3\theta}{\cancel{2}\,\cancel{\sin 4\theta}\cos\theta}\,[\text{using (b)}] \equiv \frac{\cos 3\theta}{\cos\theta}$$

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Exercise E, Question 2

Question:

(a) Show that $\sin(A+B) - \sin(A-B) \equiv 2\cos A \sin B$.

(b) Express the following as the difference of two sines:
(i) $2\cos 5x \sin 3x$
(ii) $\cos 2x \sin x$
(iii) $6\cos \frac{3}{2}x \sin \frac{1}{2}x$

(c) Using the result in (a) show that $\sin P - \sin Q \equiv 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$.

(d) Deduce that $\sin 56° - \sin 34° = \sqrt{2}\sin 11°$.

Solution:

(a) $\sin(A+B) - \sin(A-B) \equiv \sin A\cos B + \cos A\sin B - (\sin A\cos B - \cos A\sin B)$
$\equiv 2\cos A\sin B$

(b) (i) $2\cos 5x\sin 3x \equiv \sin(5x+3x) - \sin(5x-3x) \equiv \sin 8x - \sin 2x$

(ii) $\cos 2x\sin x \equiv \frac{1}{2}[\sin(2x+x) - \sin(2x-x)] \equiv \frac{1}{2}(\sin 3x - \sin x)$

(iii) $6\cos\frac{3}{2}x\sin\frac{1}{2}x \equiv 3\left[\sin\left(\frac{3}{2}x+\frac{1}{2}x\right) - \sin\left(\frac{3}{2}x-\frac{1}{2}x\right)\right] \equiv 3(\sin 2x - \sin x)$

(c) In (a) let $P = A + B$ and $Q = A - B$, so $A = \frac{P+Q}{2}$, $B = \frac{P-Q}{2}$

So $\sin P - \sin Q \equiv 2\cos\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

(d) $\sin 56° - \sin 34° = 2\cos\left(\frac{56° + 34°}{2}\right)\sin\left(\frac{56° - 34°}{2}\right)$

$= 2\cos 45°\sin 11° = 2 \times \frac{1}{\sqrt{2}}\sin 11° = \sqrt{2}\sin 11°$

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Exercise E, Question 3

Question:

(a) Show that $\cos(A+B)+\cos(A-B)\equiv 2\cos A\cos B$.

(b) Express as a sum of cosines (i) $2\cos\frac{5\theta}{2}\cos\frac{\theta}{2}$

(ii) $5\cos 2x\cos 3x$

(c) Show that $\cos P+\cos Q\equiv 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$.

(d) Prove that $\frac{\sin 3\theta-\sin\theta}{\cos 3\theta+\cos\theta}\equiv\tan\theta$.

Solution:

(a) $\cos(A+B)+\cos(A-B)\equiv\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B$

$\equiv 2\cos A\cos B$

(b) Hence, using (a),

(i) $2\cos\frac{5\theta}{2}\cos\frac{\theta}{2}\equiv\cos\left(\frac{5\theta}{2}+\frac{\theta}{2}\right)+\cos\left(\frac{5\theta}{2}-\frac{\theta}{2}\right)\equiv\cos 3\theta+\cos 2\theta$

(ii) $5\cos 2x\cos 3x\equiv\frac{5}{2}(2\cos 3x\cos 2x)$

$\equiv\frac{5}{2}[\cos(3x+2x)+\cos(3x-2x)]\equiv\frac{5}{2}$

$(\cos 5x+\cos x)$

(c) In (a) let $P=A+B$, $Q=A-B$, so $A=\frac{P+Q}{2}$, $B=\frac{P-Q}{2}$

So $\cos P+\cos Q\equiv 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$

(d)

$$\text{L.H.S.}\equiv\frac{\sin 3\theta-\sin\theta}{\cos 3\theta+\cos\theta}\equiv\frac{2\cos\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)}{2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)}$$

$$\equiv\tan\theta$$

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Exercise E, Question 4

Question:

(a) Show that $\cos(A+B) - \cos(A-B) \equiv -2\sin A \sin B$.

(b) Hence show that $\cos P - \cos Q \equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$.

(c) Deduce that $\cos 2\theta - 1 \equiv -2\sin^2\theta$.

(d) Solve, in the interval $0 \leq \theta \leq 180°$, $\cos 3\theta + \sin 2\theta - \cos\theta = 0$.

Solution:

(a) $\cos(A+B) - \cos(A-B) \equiv \cos A\cos B - \sin A\sin B - (\cos A\cos B + \sin A\sin B)$
$\equiv \cos A\cos B - \sin A\sin B - \cos A\cos B - \sin A\sin B \equiv -2\sin A\sin B$

(b) Let $P = A+B$, $Q = A-B$, so $A = \frac{P+Q}{2}$, $B = \frac{P-Q}{2}$

then $\cos P - \cos Q \equiv -2\sin\left(\frac{P+Q}{2}\right)\sin\left(\frac{P-Q}{2}\right)$

(c) Let $P = 2\theta$, $Q = 0$
then $\cos 2\theta - \cos 0 \equiv -2\sin\theta\sin\theta$
$\Rightarrow \quad \cos 2\theta - 1 \equiv -2\sin^2\theta$

(d) As $\cos 3\theta - \cos\theta \equiv -2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)$

$\cos 3\theta - \cos\theta \equiv -2\sin 2\theta\sin\theta$
So $\cos 3\theta + \sin 2\theta - \cos\theta = 0$

$\Rightarrow \quad \sin 2\theta - 2\sin 2\theta\sin\theta = 0$

$\Rightarrow \quad \sin 2\theta(1 - 2\sin\theta) = 0$

$\Rightarrow \quad \sin 2\theta = 0$ or $\sin\theta = \frac{1}{2}$

For $\sin\theta = \frac{1}{2}$, $\theta = 30°, 150°$

For $\sin 2\theta = 0$, $2\theta = 0°, 180°, 360°$
So $\theta = 0°, 90°, 180°$
Solution set: $0°, 30°, 90°, 150°, 180°$

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Exercise E, Question 5

Question:

Express the following as a sum or difference of sines or cosines:

(a) $2\sin 8x\cos 2x$

(b) $\cos 5x\cos x$

(c) $3\sin x\sin 7x$

(d) $\cos 100^\circ \cos 40^\circ$

(e) $10\cos\frac{3x}{2}\sin\frac{x}{2}$

(f) $2\sin 30^\circ \cos 10^\circ$

Solution:

(a) $2\sin 8x\cos 2x \equiv \sin(8x+2x)+\sin(8x-2x)$
$\equiv \sin 10x+\sin 6x$ [question 1(a)]

(b) $\cos 5x\cos x \equiv \frac{1}{2}(2\cos 5x\cos x)$ [question 3(a)]

$\equiv \frac{1}{2}[\cos(5x+x)+\cos(5x-x)] \equiv \frac{1}{2}(\cos 6x+\cos 4x)$

(c) $3\sin x\sin 7x \equiv -\frac{3}{2}(-2\sin 7x\sin x)$ [question 4(a)]

$\equiv -\frac{3}{2}[\cos(7x+x)-\cos(7x-x)] \equiv -\frac{3}{2}(\cos 8x-\cos 6x)$

(d) $\cos 100^\circ \cos 40^\circ \equiv \frac{1}{2}(2\cos 100^\circ \cos 40^\circ)$

$\equiv \frac{1}{2}[\cos(100+40)^\circ+\cos(100-40)^\circ] \equiv \frac{1}{2}(\cos 140^\circ+\cos 60^\circ)$

(e) $10\cos\frac{3x}{2}\sin\frac{x}{2} \equiv 5\left(2\cos\frac{3x}{2}\sin\frac{x}{2}\right)$ [question 2(a)]

$$\equiv 5\left[\sin\left(\frac{3x}{2}+\frac{x}{2}\right)-\sin\left(\frac{3x}{2}-\frac{x}{2}\right)\right] \equiv 5\left(\sin 2x - \sin x\right)$$

(f) $2\sin 30° \cos 10° \equiv \sin(30° + 10°) + \sin(30° - 10°)$

$$\equiv \sin 40° + \sin 20°$$

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Exercise E, Question 6

Question:

Show, without using a calculator, that $2\sin 82\frac{1}{2}° \cos 37\frac{1}{2}° = \frac{1}{2}\left(\sqrt{3} + \sqrt{2}\right)$.

Solution:

$$2\sin 82\frac{1}{2}° \cos 37\frac{1}{2}° = \sin\left(82\frac{1}{2}° + 37\frac{1}{2}°\right) + \sin\left(82\frac{1}{2}° - 37\frac{1}{2}°\right)$$

$$= \sin 120° + \sin 45°$$

$$= \sin 60° + \sin 45°$$

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2}\left(\sqrt{3} + \sqrt{2}\right)$$

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Exercise E, Question 7

Question:

Express, in their simplest form, as a product of sines and/or cosines:

(a) $\sin 12x + \sin 8x$

(b) $\cos(x + 2y) - \cos(2y - x)$

(c) $(\cos 4x + \cos 2x)\sin x$

(d) $\sin 95° - \sin 5°$

(e) $\cos\frac{\pi}{15} + \cos\frac{\pi}{12}$

(f) $\sin 150° + \sin 20°$

Solution:

(a) $\sin 12x + \sin 8x \equiv 2\sin\left(\frac{12x + 8x}{2}\right)\cos\left(\frac{12x - 8x}{2}\right)$

$\equiv 2\sin 10x\cos 2x$

(b) $\cos(x + 2y) - \cos(2y - x) \equiv -2\sin\left[\frac{(x + 2y) + (2y - x)}{2}\right]\sin\left[\frac{(x + 2y) - (2y - x)}{2}\right]$

$\equiv -2\sin 2y\sin x$

(c) $\cos 4x + \cos 2x \equiv 2\cos\left(\frac{4x + 2x}{2}\right)\cos\left(\frac{4x - 2x}{2}\right)$

$\equiv 2\cos 3x\cos x$

So $(\cos 4x + \cos 2x)\sin x \equiv 2\cos 3x\cos x\sin x$

$\equiv \cos 3x(2\sin x\cos x) \equiv \sin 2x\cos 3x$

(d) $\sin 95° - \sin 5° \equiv 2\cos\left(\frac{95° + 5°}{2}\right)\sin\left(\frac{95° - 5°}{2}\right)$

$\equiv 2\cos 50°\sin 45° \equiv \sqrt{2}\cos 50°$

(e) $\cos\frac{\pi}{15} + \cos\frac{\pi}{12} \equiv 2\cos\left(\frac{\frac{\pi}{15}+\frac{\pi}{12}}{2}\right)\cos\left(\frac{\frac{\pi}{15}-\frac{\pi}{12}}{2}\right)$

$$\equiv 2\cos\frac{9\pi}{120}\cos\left(-\frac{\pi}{120}\right) \equiv 2\cos\frac{9\pi}{120}\cos\frac{\pi}{120}$$

(f) $\sin 150° + \sin 20° \equiv 2\sin\left(\frac{150° + 20°}{2}\right)\cos\left(\frac{150° - 20°}{2}\right)$

$$\equiv 2\sin 85°\cos 65°$$

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Exercise E, Question 8

Question:

Using the identity $\cos P + \cos Q \equiv 2\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right)$, show that $\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) = 0$.

Solution:

$$\cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right)$$

$$\equiv \left[\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\theta\right] + \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$\equiv 2\cos\left[\frac{\left(\theta + \frac{4\pi}{3}\right) + \theta}{2}\right]\cos\left[\frac{\left(\theta + \frac{4\pi}{3}\right) - \theta}{2}\right] + \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$\equiv 2\cos\left(\theta + \frac{2\pi}{3}\right)\cos\frac{2\pi}{3} + \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$\equiv 2\cos\left(\theta + \frac{2\pi}{3}\right)\left(-\frac{1}{2}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$\equiv -\cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$\equiv 0$$

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Exercise E, Question 9

Question:

Prove that $\frac{\sin 75° + \sin 15°}{\cos 15° - \cos 75°} = \sqrt{3}$.

Solution:

$\sin 75° + \sin 15° = 2 \sin \left(\frac{75 + 15}{2} \right)° \cos \left(\frac{75 - 15}{2} \right)° = 2 \sin 45° \cos 30°$

$\cos 15° - \cos 75° = - (\cos 75° - \cos 15°)$

$= - \left[- 2 \sin \left(\frac{75 + 15}{2} \right)° \sin \left(\frac{75 - 15}{2} \right)° \right]$

$= 2 \sin 45° \sin 30°$

So $\frac{\sin 75° + \sin 15°}{\cos 15° - \cos 75°} = \frac{2 \sin 45° \cos 30°}{2 \sin 45° \sin 30°} = \cot 30° = \frac{1}{\tan 30°} = \sqrt{3}$

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Exercise E, Question 10

Question:

Solve the following equations:

(a) $\cos 4x = \cos 2x$, for $0 \leq x \leq 180°$

(b) $\sin 3\theta - \sin \theta = 0$, for $0 \leq \theta \leq 2\pi$

(c) $\sin(x + 20°) + \sin(x - 10°) = \cos 15°$, for $0 \leq x \leq 360°$

(d) $\sin 3\theta - \sin \theta = \cos 2\theta$, for $0 \leq \theta \leq 2\pi$

Solution:

(a) $\cos 4x - \cos 2x = 0$

$\Rightarrow \quad -2\sin\left(\frac{4x + 2x}{2}\right)\sin\left(\frac{4x - 2x}{2}\right) = 0$

$\Rightarrow \quad \sin 3x \sin x = 0$

$\Rightarrow \quad \sin 3x = 0$ or $\sin x = 0$, $0 \leq x \leq 180°$

$\sin x = 0$, $0 \leq x \leq 180°$

$\Rightarrow \quad x = 0°, 180°$

$\sin 3x = 0$, $0 \leq 3x \leq 540°$

$\Rightarrow \quad 3x = 0°, 180°, 360°, 540°$

$\Rightarrow \quad x = 0°, 60°, 120°, 180°$

Solution set: 0°, 60°, 120°, 180°

(b) $\sin 3\theta - \sin \theta = 0$

$\Rightarrow \quad 2\cos\left(\frac{3\theta + \theta}{2}\right)\sin\left(\frac{3\theta - \theta}{2}\right) = 0$

$\Rightarrow \quad 2\cos 2\theta \sin \theta = 0$, $0 \leq \theta \leq 2\pi$

$\sin \theta = 0$, $0 \leq \theta \leq 2\pi \quad \Rightarrow \quad \theta = 0, \pi, 2\pi$

$\cos 2\theta = 0$, $0 \leq 2\theta \leq 4\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solution set: $0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

(c) $\sin\left(x + 20°\right) + \sin\left(x - 10°\right) \equiv 2\sin\left(\frac{x + 20° + x - 10°}{2}\right)\cos\left[\frac{x + 20° - (x - 10°)}{2}\right]$

$\equiv 2\sin(x + 5°)\cos 15°$

So $\sin(x + 20°) + \sin(x - 10°) = \cos 15°, 0 \le x \le 360°$

$\Rightarrow \quad 2\sin(x + 5°) = 1$

So $\sin\left(x + 5°\right) = \frac{1}{2}, 5° \le (x + 5°) \le 365°$

$\Rightarrow \quad x + 5° = 30°, 150°$

$\Rightarrow \quad x = 25°, 145°$

(d) $\sin 3\theta - \sin\theta \equiv 2\cos\left(\frac{3\theta + \theta}{2}\right)\sin\left(\frac{3\theta - \theta}{2}\right)$

$\equiv 2\cos 2\theta\sin\theta$

So $\sin 3\theta - \sin\theta = \cos 2\theta$

$\Rightarrow \quad 2\cos 2\theta\sin\theta = \cos 2\theta$

$\Rightarrow \quad \cos 2\theta(2\sin\theta - 1) = 0$

$\Rightarrow \quad \cos 2\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\sin\theta = \frac{1}{2}, 0 \le \theta \le 2\pi$

$\Rightarrow \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\cos 2\theta = 0, 0 \le 2\theta \le 4\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Solution set: $\frac{\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{6}, \frac{5\pi}{4}, \frac{7\pi}{4}$

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Exercise E, Question 11

Question:

Prove the identities

(a) $\frac{\sin 7\theta - \sin 3\theta}{\sin \theta \cos \theta} \equiv 4 \cos 5\theta$

(b) $\frac{\cos 2\theta + \cos 4\theta}{\sin 2\theta - \sin 4\theta} \equiv - \cot \theta$

(c) $\sin^2 (x + y) - \sin^2 (x - y) \equiv \sin 2x \sin 2y$

(d) $\cos x + 2 \cos 3x + \cos 5x \equiv 4 \cos^2 x \cos 3x$

Solution:

(a) L.H.S. $\equiv \frac{\sin 7\theta - \sin 3\theta}{\sin \theta \cos \theta}$

$$\equiv \frac{2 \cos \frac{1}{2} (7\theta + 3\theta) \sin \frac{1}{2} (7\theta - 3\theta)}{\frac{1}{2} (2 \sin \theta \cos \theta)}$$

$$\equiv \frac{2 \cos 5\theta \sin 2\theta}{\frac{1}{2} \sin 2\theta}$$

$\equiv 4 \cos 5\theta$

$\equiv$ R.H.S.

(b) L.H.S. $\equiv \frac{\cos 2\theta + \cos 4\theta}{\sin 2\theta - \sin 4\theta}$

$$\equiv \frac{2 \cos \frac{1}{2} (4\theta + 2\theta) \cos \frac{1}{2} (4\theta - 2\theta)}{2 \cos \frac{1}{2} (2\theta + 4\theta) \sin \frac{1}{2} (2\theta - 4\theta)}$$

$$\equiv \frac{2 \cos 3\theta \cos \theta}{2 \cos 3\theta \sin (- \theta)}$$

$\equiv \dfrac{\cos\theta}{-\sin\theta}$ [as $\sin(-\theta) \equiv -\sin\theta$]

$\equiv -\cot\theta$

$\equiv$ R.H.S.

(c) L.H.S. $\equiv \sin^2(x+y) - \sin^2(x-y)$

$\equiv [\sin(x+y) + \sin(x-y)][\sin(x+y) - \sin(x-y)]$

$$\equiv \left[2\sin\left(\frac{x+y+x-y}{2}\right)\cos\left(\frac{x+y-x+y}{2}\right)\right]\left[2\cos\left(\frac{x+y+x-y}{2}\right)\sin\left(\frac{x+y-x+y}{2}\right)\right]$$

$\equiv (2\sin x\cos y)(2\cos x\sin y)$

$\equiv (2\sin x\cos x)(2\sin y\cos y)$

$\equiv \sin 2x\sin 2y$

$\equiv$ R.H.S.

(d) L.H.S. $\equiv \cos x + 2\cos 3x + \cos 5x$

$\equiv \cos 5x + \cos x + 2\cos 3x$

$\equiv 2\cos\left(\dfrac{5x+x}{2}\right)\cos\left(\dfrac{5x-x}{2}\right) + 2\cos 3x$

$\equiv 2\cos 3x\cos 2x + 2\cos 3x$

$\equiv 2\cos 3x(\cos 2x + 1)$

$\equiv 2\cos 3x(2\cos^2 x - 1 + 1)$ $(\cos 2x \equiv 2\cos^2 x - 1)$

$\equiv 2\cos 3x \times 2\cos^2 x$

$\equiv 4\cos^2 x\cos 3x$

$\equiv$ R.H.S.

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Exercise E, Question 12

Question:

(a) Prove that $\cos\theta + \sin 2\theta - \cos 3\theta \equiv \sin 2\theta\ (1 + 2\sin\theta)$.

(b) Hence solve, for $0 \le \theta \le 2\pi$, $\cos\theta + \sin 2\theta = \cos 3\theta$.

Solution:

(a) L.H.S. $\equiv \cos\theta + \sin 2\theta - \cos 3\theta$

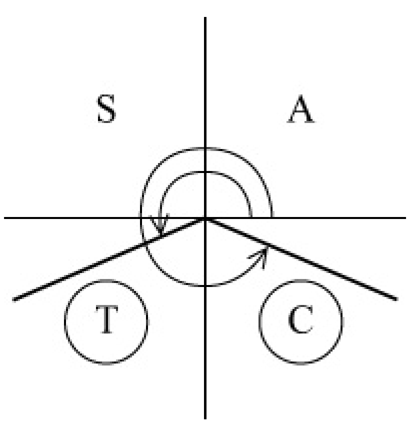
$$\equiv -(\cos 3\theta - \cos\theta) + \sin 2\theta$$
$$\equiv -\left[-2\sin\left(\frac{3\theta+\theta}{2}\right)\sin\left(\frac{3\theta-\theta}{2}\right)\right] + \sin 2\theta$$
$$\equiv 2\sin 2\theta \sin\theta + \sin 2\theta$$
$$\equiv \sin 2\theta\ (2\sin\theta + 1)$$
$$\equiv \text{R.H.S.}$$

(b) So to solve $\cos\theta + \sin 2\theta = \cos 3\theta$
or $\cos\theta + \sin 2\theta - \cos 3\theta = 0$
solve $\sin 2\theta\ (1 + 2\sin\theta) = 0$ [using (a)]
Either $\sin 2\theta = 0$

$\Rightarrow\ 2\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

$\Rightarrow\ \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

or $\sin\theta = -\frac{1}{2}$



$\Rightarrow\ \theta = \pi - \sin^{-1}\left(-\frac{1}{2}\right), 2\pi + \sin^{-1}\left(-\frac{1}{2}\right) = \pi + \frac{\pi}{6}, 2\pi -$

$\frac{\pi}{6} = \frac{7\pi}{6}, \frac{11\pi}{6}$

Solution set: $0, \frac{\pi}{2}, \pi, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}, 2\pi$

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Exercise F, Question 1

Question:

The lines l_1 and l_2, with equations $y = 2x$ and $3y = x - 1$ respectively, are drawn on the same set of axes. Given that the scales are the same on both axes and that the angles that l_1 and l_2 make with the positive x-axis are A and B respectively,

(a) write down the value of tan A and the value of tan B;

(b) without using your calculator, work out the acute angle between l_1 and l_2.

Solution:

(a) $\tan A = 2, \tan B = \frac{1}{3}$ since $y = \frac{1}{3}x - \frac{1}{3}$

(b) The angle required is $(A - B)$.

Using $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{2 - \frac{1}{3}}{1 + 2 \times \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1$

$\Rightarrow \quad A - B = 45°$

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Exercise F, Question 2

Question:

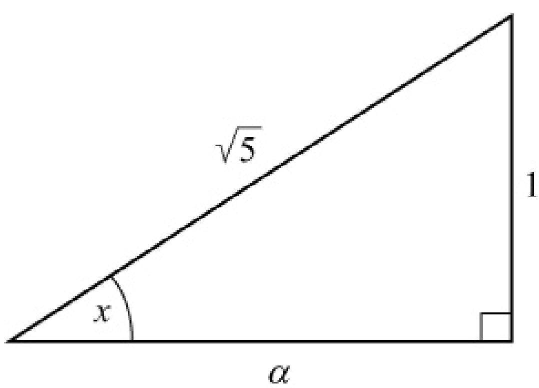
Given that $\sin x = \frac{1}{\sqrt{5}}$ where x is acute, and that $\cos(x - y) = \sin y$, show that $\tan y = \frac{\sqrt{5} + 1}{2}$.

Solution:

As $\cos(x - y) = \sin y$

$\cos x \cos y + \sin x \sin y = \sin y$ ①

Draw a right-angled triangle where $\sin x = \frac{1}{\sqrt{5}}$



Using Pythagoras' theorem,

$a^2 = (\sqrt{5})^2 - 1 = 4 \Rightarrow a = 2$

So $\cos x = \frac{2}{\sqrt{5}}$

Substitute into ①:

$\frac{2}{\sqrt{5}} \cos y + \frac{1}{\sqrt{5}} \sin y = \sin y$

$\Rightarrow 2 \cos y + \sin y = \sqrt{5} \sin y$

$\Rightarrow 2 \cos y = \sin y (\sqrt{5} - 1)$

$\Rightarrow \frac{2}{\sqrt{5} - 1} = \tan y \quad \left(\tan y = \frac{\sin y}{\cos y} \right)$

$\Rightarrow \tan y = \frac{2(\sqrt{5} + 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)} = \frac{2(\sqrt{5} + 1)}{4} = \frac{\sqrt{5} + 1}{2}$

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Exercise F, Question 3

Question:

Using $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$ with an appropriate value of θ,

(a) show that $\tan\frac{\pi}{8} = \sqrt{2} - 1$.

(b) Use the result in (a) to find the exact value of $\tan\frac{3\pi}{8}$.

Solution:

(a) Using $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$ with $\theta = \frac{\pi}{8}$

$$\Rightarrow \quad \tan\frac{\pi}{4} = \frac{2\tan\frac{\pi}{8}}{1 - \tan^2\frac{\pi}{8}}$$

Let $t = \tan\frac{\pi}{8}$

So $1 = \frac{2t}{1 - t^2}$

$$\Rightarrow \quad 1 - t^2 = 2t$$

$$\Rightarrow \quad t^2 + 2t - 1 = 0$$

$$\Rightarrow \quad t = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

As $\frac{\pi}{8}$ is acute, $\tan\frac{\pi}{8}$ is +ve, so $\tan\frac{\pi}{8} = \sqrt{2} - 1$

(b) $\tan\frac{3\pi}{8} = \tan\left(\frac{\pi}{4} + \frac{\pi}{8}\right) = \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{8}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{8}}$

$$= \frac{1 + (\sqrt{2} - 1)}{1 - (\sqrt{2} - 1)} = \frac{\sqrt{2}}{2 - \sqrt{2}} = \frac{\sqrt{2}(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})} = \frac{\sqrt{2}}{2}$$

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$$(2 + \sqrt{2}) = \sqrt{2} + 1$$

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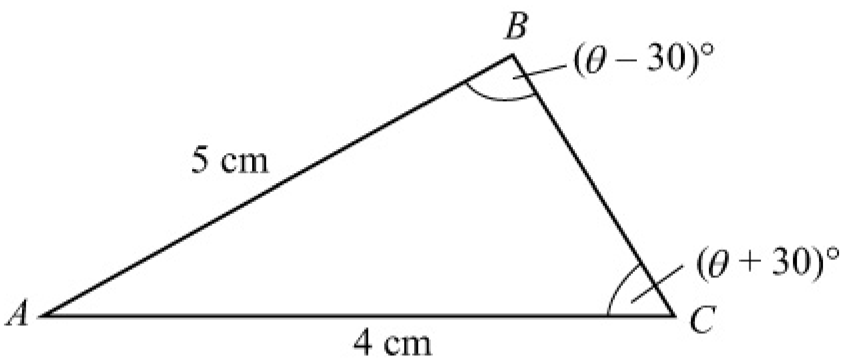
Exercise F, Question 4

Question:

In △ABC, AB = 5 cm and AC = 4 cm, ∠ ABC = $(\theta - 30)°$ and ∠ ACB = $(\theta + 30)°$. Using the sine rule, show that $\tan\theta = 3\sqrt{3}$.

Solution:



Using $\frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin(\theta - 30)°}{4} = \frac{\sin(\theta + 30)°}{5}$

$\Rightarrow \quad 5\sin(\theta - 30)° = 4\sin(\theta + 30)°$

$\Rightarrow \quad 5(\sin\theta\cos 30° - \cos\theta\sin 30°) = 4(\sin\theta\cos 30° + \cos\theta\sin 30°)$

$\Rightarrow \quad \sin\theta\cos 30° = 9\cos\theta\sin 30°$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 9\frac{\sin 30°}{\cos 30°} = 9\tan 30°$

$\Rightarrow \quad \tan\theta = 9 \times \frac{\sqrt{3}}{3} = 3\sqrt{3}$

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Exercise F, Question 5

Question:

Two of the angles, A and B, in $\triangle ABC$ are such that $\tan A = \frac{3}{4}$, $\tan B = \frac{5}{12}$.

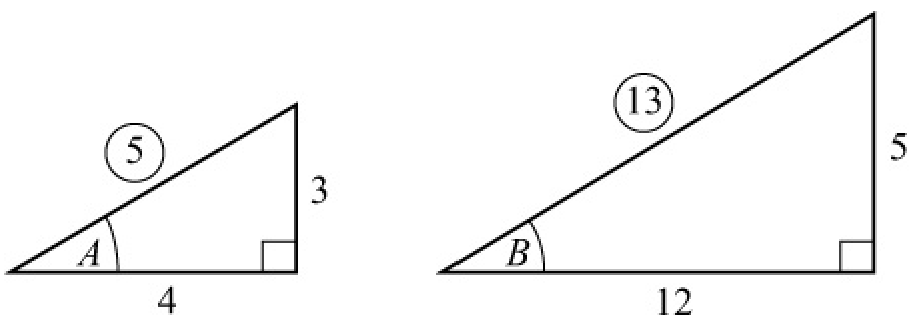
(a) Find the exact value of
(i) $\sin(A+B)$

(ii) $\tan 2B$

(b) By writing C as $180° - (A+B)$, show that $\cos C = -\frac{33}{65}$.

Solution:

(a) Draw right-angled triangles.



$\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$ $\quad$ $\sin B = \frac{5}{13}$, $\cos B = \frac{12}{13}$

(i) $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65}$$

(ii) $$\tan 2B = \frac{2\tan B}{1-\tan^2 B} = \frac{2 \times \frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} = \frac{\frac{5}{6}}{\frac{119}{144}} = \frac{5}{6} \times \frac{144}{119} = \frac{120}{119}$$

(b) $\cos C = \cos[180° - (A+B)] = -\cos(A+B)$

$$= -(\cos A \cos B - \sin A \sin B) = -\left(\frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}\right)$$

$$= -$$

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Exercise F, Question 6

Question:

Show that

(a) $\sec\theta \operatorname{cosec}\theta \equiv 2\operatorname{cosec} 2\theta$

(b) $\frac{1 - \cos 2x}{1 + \cos 2x} \equiv \sec^2 x - 1$

(c) $\cot \theta - 2\cot 2\theta \equiv \tan\theta$

(d) $\cos^4 2\theta - \sin^4 2\theta \equiv \cos 4\theta$

(e) $\tan\left(\frac{\pi}{4} + x\right) - \tan\left(\frac{\pi}{4} - x\right) \equiv 2\tan 2x$

(f) $\sin(x + y)\sin(x - y) \equiv \cos^2 y - \cos^2 x$

(g) $1 + 2\cos 2\theta + \cos 4\theta \equiv 4\cos^2\theta\cos 2\theta$

Solution:

(a) L.H.S. $\equiv \sec\theta\operatorname{cosec}\theta$

$$\equiv \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$

$$\equiv \frac{2}{2\sin\theta\cos\theta}$$

$$\equiv \frac{2}{\sin 2\theta}$$

$$\equiv 2\operatorname{cosec} 2\theta$$

$\equiv$ R.H.S.

(b) L.H.S. $\equiv \frac{1 - \cos 2x}{1 + \cos 2x}$

$$\equiv \frac{1 - (1 - 2\sin^2 x)}{1 + (2\cos^2 x - 1)}$$

$$\equiv \frac{2\sin^2 x}{2\cos^2 x}$$

$\equiv \tan^2 x$
$\equiv \sec^2 x - 1 \qquad (1 + \tan^2 x \equiv \sec^2 x)$
$\equiv$ R.H.S.

(c) L.H.S. $\equiv \cot \theta - 2\cot 2\theta$

$$\equiv \frac{1}{\tan\theta} - \frac{2}{\tan 2\theta}$$

$$\equiv \frac{1}{\tan\theta} - \frac{2(1-\tan^2\theta)}{2\tan\theta}$$

$$\equiv \frac{1 - 1 + \tan^2\theta}{\tan\theta}$$

$$\equiv \frac{\tan^2\theta}{\tan\theta}$$

$\equiv \tan\theta$
$\equiv$ R.H.S.

(d) L.H.S. $\equiv \cos^4 2\theta - \sin^4 2\theta$
$\equiv (\cos^2 2\theta + \sin^2 2\theta)(\cos^2 2\theta - \sin^2 2\theta)$
$\equiv (1)(\cos 4\theta) \qquad (\cos^2 A + \sin^2 A \equiv 1,$
$\cos^2 A - \sin^2 A \equiv \cos 2A)$
$\equiv \cos 4\theta$
$\equiv$ R.H.S.

(e) L.H.S. $\equiv \tan\left(\frac{\pi}{4} + x\right) - \tan\left(\frac{\pi}{4} - x\right)$

$$\equiv \frac{1+\tan x}{1-\tan x} - \frac{1-\tan x}{1+\tan x}$$

$$\equiv \frac{(1+\tan x)^2 - (1-\tan x)^2}{(1-\tan x)(1+\tan x)}$$

$$\equiv \frac{1 + 2\tan x + \tan^2 x - (1 - 2\tan x + \tan^2 x)}{1 - \tan^2 x}$$

$$\equiv \frac{4\tan x}{1-\tan^2 x}$$

$$\equiv 2\left(\frac{2\tan x}{1-\tan^2 x}\right)$$

$\equiv 2\tan 2x$
$\equiv$ R.H.S.

(f) R.H.S. $\equiv \cos^2 y - \cos^2 x$

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$$\equiv (\cos y + \cos x)(\cos y - \cos x)$$

$$\equiv \left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]\left[-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{y-x}{2}\right)\right]$$

$$\equiv \left[2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]\left[2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)\right] \quad [\text{as } \sin(-\theta) = \sin\theta]$$

$$\equiv \left[2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right)\right]\left[2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$\equiv \sin 2\left(\frac{x+y}{2}\right)\sin 2\left(\frac{x-y}{2}\right)$$

$$\equiv \sin(x+y)\sin(x-y)$$

$$\equiv \text{L.H.S.}$$

(g) L.H.S. $\equiv 1 + 2\cos 2\theta + \cos 4\theta$

$$\equiv 1 + 2\cos 2\theta + (2\cos^2 2\theta - 1)$$

$$\equiv 2\cos 2\theta + 2\cos^2 2\theta$$

$$\equiv 2\cos 2\theta(1 + \cos 2\theta)$$

$$\equiv 2\cos 2\theta[1 + (2\cos^2\theta - 1)]$$

$$\equiv 4\cos^2\theta\cos 2\theta$$

$$\equiv \text{R.H.S.}$$

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Exercise F, Question 7

Question:

The angles x and y are acute angles such that $\sin x = \frac{2}{\sqrt{5}}$ and $\cos y = \frac{3}{\sqrt{10}}$

(a) Show that $\cos 2x = -\frac{3}{5}$.

(b) Find the value of $\cos 2y$.

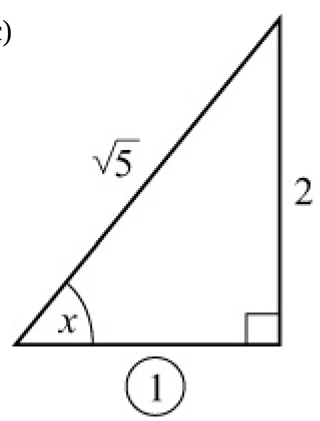
(c) Show without using your calculator, that
(i) $\tan(x + y) = 7$
(ii) $x - y = \frac{\pi}{4}$

Solution:

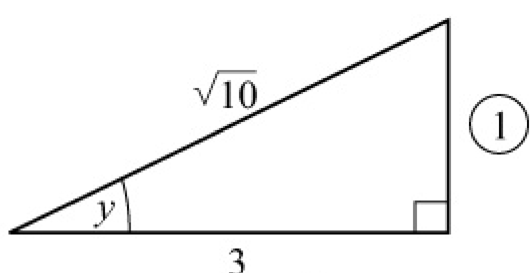
(a) $\cos 2x \equiv 1 - 2\sin^2 x = 1 - 2\left(\frac{2}{\sqrt{5}}\right)^2 = 1 - \frac{8}{5} = -\frac{3}{5}$

(b) $\cos 2y \equiv 2\cos^2 y - 1 = 2\left(\frac{3}{\sqrt{10}}\right)^2 - 1 = 2\left(\frac{9}{10}\right) - 1 = \frac{4}{5}$

(c)



$\tan x = 2$



$\tan y = \frac{1}{3}$

(i) $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{2\frac{1}{3}}{1 - \frac{2}{3}} = \frac{\frac{7}{3}}{\frac{1}{3}} = 7$

(ii) $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1$

As x and y are acute, $x - y = \frac{\pi}{4}$ (it cannot be $\frac{5\pi}{4}$)

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Exercise F, Question 8

Question:

Given that $\sin x \cos y = \frac{1}{2}$ and $\cos x \sin y = \frac{1}{3}$,

(a) show that $\sin(x + y) = 5 \sin(x - y)$.
Given also that $\tan y = k$, express in terms of k:

(b) $\tan x$

(c) $\tan 2x$

Solution:

(a) $\sin(x + y) \equiv \sin x \cos y + \cos x \sin y = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$5 \sin(x - y) \equiv 5(\sin x \cos y - \cos x \sin y) = 5\left(\frac{1}{2} - \frac{1}{3}\right) = 5 \times \frac{1}{6} = \frac{5}{6}$

(b) $\frac{\sin x \cos y}{\cos x \sin y} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$

$\Rightarrow \quad \frac{\tan x}{\tan y} = \frac{3}{2}$

so $\tan x = \frac{3}{2} \tan y = \frac{3}{2} k$

(c) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{3k}{1 - \frac{9}{4}k^2} \quad \left(= \frac{12k}{4 - 9k^2} \right)$

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Exercise F, Question 9

Question:

Solve the following equations in the interval given in brackets:

(a) $\sqrt{3}\sin 2\theta + 2\sin^2\theta = 1 \quad \{0 \le \theta \le \pi\{$

(b) $\sin 3\theta\cos 2\theta = \sin 2\theta\cos 3\theta \quad \{0 \le \theta \le 2\pi\{$

(c) $\sin(\theta + 40°) + \sin(\theta + 50°) = 0 \quad \{0 \le \theta \le 360°\{$

(d) $\sin^2\frac{\theta}{2} = 2\sin\theta \quad \{0 \le \theta \le 360°\{$

(e) $2\sin\theta = 1 + 3\cos\theta \quad \{0 \le \theta \le 360°\{$

(f) $\cos 5\theta = \cos 3\theta \quad \{0 \le \theta \le \pi\{$

(g) $\cos 2\theta = 5\sin\theta \quad \{-\pi \le \theta \le \pi\{$.

Solution:

(a) $\sqrt{3}\sin 2\theta = 1 - 2\sin^2\theta,\ 0 \le \theta \le \pi$

$\Rightarrow \quad \sqrt{3}\sin 2\theta = \cos 2\theta$

$\Rightarrow \quad \tan 2\theta = \frac{1}{\sqrt{3}},\ 0 \le 2\theta \le 2\pi$

$\Rightarrow \quad 2\theta = \frac{\pi}{6}, \pi + \frac{\pi}{6} = \frac{\pi}{6}, \frac{7\pi}{6}$

$\Rightarrow \quad \theta = \frac{\pi}{12}, \frac{7\pi}{12}$

(b) $\sin 3\theta\cos 2\theta - \cos 3\theta\sin 2\theta = 0$

$\Rightarrow \quad \sin(3\theta - 2\theta) = 0$

$\Rightarrow \quad \sin\theta = 0,\ 0 \le \theta \le 2\pi$

$\Rightarrow \quad \theta = 0, \pi, 2\pi$

(c) $\sin(\theta + 40°) + \sin(\theta + 50°) = 0,\ 0 \le \theta \le 360°$

$\Rightarrow \quad 2\sin\left[\frac{(\theta + 40°) + (\theta + 50°)}{2}\right]\cos\left[\frac{(\theta + 40°) - (\theta + 50°)}{2}\right]$

$= 0$

$\Rightarrow \quad 2\sin(\theta + 45°)\cos(-5°) = 0$

$\Rightarrow \quad \sin(\theta + 45°) = 0, 45° \leq \theta + 45° \leq 405°$

$\Rightarrow \quad \theta + 45° = 180°, 360°$

$\Rightarrow \quad \theta = 135°, 315°$

(d) $\sin^2 \frac{\theta}{2} = 2\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) \qquad \left(\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$

$\Rightarrow \quad \sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - 4\cos\frac{\theta}{2}\right) = 0$

$\Rightarrow \quad \sin\frac{\theta}{2} = 0$ or $\sin\frac{\theta}{2} = 4\cos\frac{\theta}{2}$, i.e. $\tan\frac{\theta}{2} = 4$

For $\sin\frac{\theta}{2} = 0 \quad \Rightarrow \quad \frac{\theta}{2} = 0°, 180° \quad \Rightarrow \quad \theta = 0°, 360°$

For $\tan\frac{\theta}{2} = 4 \quad \Rightarrow \quad \frac{\theta}{2} = \tan^{-1} 4 = 75.96° \quad \Rightarrow \quad \theta = 151.9°$

Solution set: 0°, 151.9°, 360°

(e) $2\sin\theta - 3\cos\theta = 1$

Let $2\sin\theta - 3\cos\theta \equiv R\sin(\theta - \alpha) \equiv R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$\Rightarrow \quad R\cos\alpha = 2$ and $R\sin\alpha = 3$

$\Rightarrow \quad \tan\alpha = \frac{3}{2} \quad (\Rightarrow \quad \alpha = 56.3°), R = \sqrt{13}$

$\Rightarrow \quad \sqrt{13}\sin(\theta - 56.3°) = 1$

$\Rightarrow \quad \sin(\theta - 56.3°) = \frac{1}{\sqrt{13}}$

$\Rightarrow \quad \theta - 56.3° = \sin^{-1}\frac{1}{\sqrt{13}}, 180° - \sin^{-1}\frac{1}{\sqrt{13}} = 16.1°, 163.9°$

$\Rightarrow \quad \theta = 72.4°, 220.2°$

(f) $\cos 5\theta - \cos 3\theta = 0$

$\Rightarrow \quad -2\sin\left(\frac{5\theta + 3\theta}{2}\right)\sin\left(\frac{5\theta - 3\theta}{2}\right) = 0$

$\Rightarrow \quad \sin 4\theta \sin\theta = 0, 0 \leq \theta \leq \pi$

$\Rightarrow \quad \sin\theta = 0 \quad \Rightarrow \quad \theta = 0, \pi$

or $\sin 4\theta = 0 \quad \Rightarrow \quad 4\theta = 0, \pi, 2\pi, 3\pi, 4\pi$

$\Rightarrow \quad \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

Solution set: $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

(g) $\cos 2\theta = 5\sin\theta$

$\Rightarrow \quad 1 - 2\sin^2\theta = 5\sin\theta$

$\Rightarrow \quad 2\sin^2\theta + 5\sin\theta - 1 = 0$

$\Rightarrow \quad \sin\theta = \frac{-5 \pm \sqrt{33}}{4}$

As $-1 \le \sin\theta \le 1$, $\sin\theta = \frac{-5 + \sqrt{33}}{4}$

In radian mode: $\theta = 0.187, \pi - 0.187 = 0.187, 2.95$

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Exercise F, Question 10

Question:

The first three terms of an arithmetic series are $\sqrt{3}\cos\theta$, $\sin(\theta - 30°)$ and $\sin\theta$, where θ is acute. Find the value of θ.

Solution:

As the three values are consecutive terms of an arithmetic progression,
$\sin(\theta - 30°) - \sqrt{3}\cos\theta = \sin\theta - \sin(\theta - 30°)$

$\Rightarrow \quad 2\sin(\theta - 30°) = \sin\theta + \sqrt{3}\cos\theta$

$\Rightarrow \quad 2(\sin\theta\cos 30° - \cos\theta\sin 30°) = \sin\theta + \sqrt{3}\cos\theta$

$\Rightarrow \quad \sqrt{3}\sin\theta - \cos\theta = \sin\theta + \sqrt{3}\cos\theta$

$\Rightarrow \quad \sin\theta(\sqrt{3} - 1) = \cos\theta(\sqrt{3} + 1)$

$\Rightarrow \quad \tan\theta = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$

Calculator value is $\theta = \tan^{-1}\frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 75°$

No other values as θ is acute.

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Exercise F, Question 11

Question:

Solve, for $0 \le \theta \le 360°$, $\cos(\theta + 40°)\cos(\theta - 10°) = 0.5$.

Solution:

$2\cos(\theta + 40°)\cos(\theta - 10°) = 1$

$\Rightarrow \quad \cos\left[\left(\theta + 40°\right) + \left(\theta - 10°\right)\right] + \cos\left[\left(\theta + 40°\right) - \left(\theta - 10°\right)\right] = 1$

$\Rightarrow \quad \cos(2\theta + 30°) + \cos 50° = 1$

$\Rightarrow \quad \cos(2\theta + 30°) = 1 - \cos 50° = 0.3572$

$\Rightarrow \quad 2\theta + 30° = 69.07°, 290.9°, 429.07°, 650.9°$

$\Rightarrow \quad 2\theta = 39.07°, 260.9°, 399.07°, 620.9°$

$\Rightarrow \quad \theta = 19.5°, 130.5°, 199.5°, 310.5°$

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Exercise F, Question 12

Question:

Without using calculus, find the maximum and minimum value of the following expressions. In each case give the smallest positive value of θ at which each occurs.

(a) $\sin\theta\cos 10° - \cos\theta\sin 10°$

(b) $\cos 30° \cos\theta - \sin 30° \sin\theta$

(c) $\sin\theta + \cos\theta$

Solution:

(a) $\sin\theta\cos 10° - \cos\theta\sin 10° = \sin(\theta - 10°)$ $[\sin(A - B)]$

Maximum value $= +1$ when $\theta - 10° = 90°$ $\Rightarrow$ $\theta = 100°$

Minimum value $= -1$ when $\theta - 10° = 270°$ $\Rightarrow$ $\theta = 280°$

(b) $\cos 30° \cos\theta - \sin 30° \sin\theta = \cos(\theta + 30°)$

Maximum value $= +1$ when $\theta + 30° = 360°$ $\Rightarrow$ $\theta = 330°$

Minimum value $= -1$ when $\theta + 30° = 180°$ $\Rightarrow$ $\theta = 150°$

(c) $\sin\theta + \cos\theta$

$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right)$

$= \sqrt{2}(\sin\theta\cos 45° + \cos\theta\sin 45°)$

$= \sqrt{2}\sin(\theta + 45°)$

Maximum value $= \sqrt{2}$ when $\theta + 45° = 90°$ $\Rightarrow$ $\theta = 45°$

Minimum value $= -\sqrt{2}$ when $\theta + 45° = 270°$ $\Rightarrow$ $\theta = 225°$

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Exercise F, Question 13

Question:

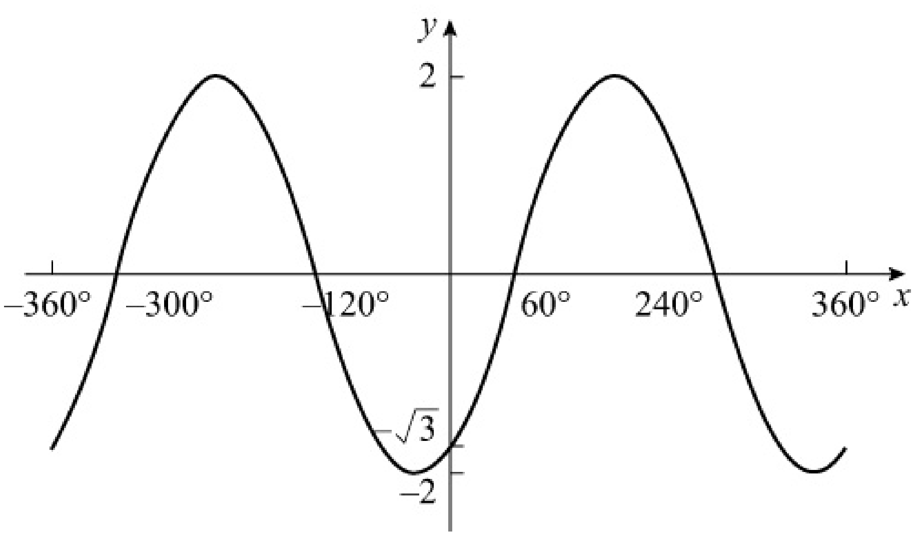
(a) Express $\sin x - \sqrt{3}\cos x$ in the form $R\sin(x - \alpha)$, with $R > 0$ and $0 < \alpha < 90°$.

(b) Hence sketch the graph of $y = \sin x - \sqrt{3}\cos x$ $\{-360° \le x \le 360°\}$, giving the coordinates of all points of intersection with the axes.

Solution:

(a) Let $\sin x - \sqrt{3}\cos x \equiv R\sin(x - \alpha) \equiv R\sin x\cos\alpha - R\cos x\sin\alpha$
$R > 0$, $0 < \alpha < 90°$
Compare $\sin x$: $R\cos\alpha = 1$ ①
Compare $\cos x$: $R\sin\alpha = \sqrt{3}$ ②
Divide ② by ①: $\tan\alpha = \sqrt{3} \Rightarrow \alpha = 60°$
$R^2 = (\sqrt{3})^2 + 1^2 = 4 \Rightarrow R = 2$
So $\sin x - \sqrt{3}\cos x \equiv 2\sin(x - 60°)$

(b) Sketch $y = 2\sin(x - 60°)$ by first translating $y = \sin x$ by 60° to the right and then stretching the result in the y direction by scale factor 2.



Graph meets y-axis when $x = 0$, i.e. $y = 2\sin(-60°) = -\sqrt{3}$
Graph meets x-axis when $y = 0$, i.e. $(-300°, 0)$, $(-120°, 0)$, $(60°, 0)$, $240°, 0)$

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Exercise F, Question 14

Question:

Given that $7\cos 2\theta + 24\sin 2\theta \equiv R\cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, find:

(a) the value of R and the value of α, to 2 decimal places

(b) the maximum value of $14\cos^2\theta + 48\sin\theta\cos\theta$

Solution:

(a) Let $7\cos 2\theta + 24\sin 2\theta \equiv R\cos(2\theta - \alpha) \equiv R\cos 2\theta\cos\alpha + R\sin 2\theta\sin\alpha$

$R > 0, 0 < \alpha < \frac{\pi}{2}$

Compare $\cos 2\theta$: $R\cos\alpha = 7$ ①
Compare $\sin 2\theta$: $R\sin\alpha = 24$ ②

Divide ② by ①: $\tan\alpha = \frac{24}{7} \Rightarrow \alpha = 1.29$ (1.287)

$R^2 = 24^2 + 7^2 \Rightarrow R = 25$
So $7\cos 2\theta + 24\sin 2\theta \equiv 25\cos(2\theta - 1.29)$

(b) $14\cos^2\theta + 48\sin\theta\cos\theta$
$\equiv 14\left(\frac{1 + \cos 2\theta}{2}\right) + 24(2\sin\theta\cos\theta)$
$\equiv 7(1 + \cos 2\theta) + 24\sin 2\theta$
$\equiv 7 + 7\cos 2\theta + 24\sin 2\theta$
The maximum value of $7\cos 2\theta + 24\sin 2\theta$ is 25 [using (a) with $\cos(2\theta - 1.29) = 1$].
So maximum value of $7 + 7\cos 2\theta + 24\sin 2\theta = 7 + 25 = 32$.

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Exercise F, Question 15

Question:

(a) Given that α is acute and $\tan\alpha = \frac{3}{4}$, prove that

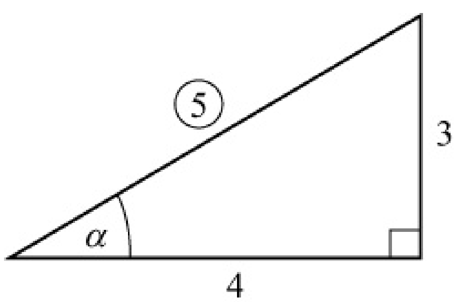
$3\sin(\theta + \alpha) + 4\cos(\theta + \alpha) \equiv 5\cos\theta$

(b) Given that $\sin x = 0.6$ and $\cos x = -0.8$, evaluate $\cos(x + 270)^\circ$ and $\cos(x + 540)^\circ$.

[E]

Solution:

(a) Draw a right-angled triangle and find sin α and cos α.



$\Rightarrow \quad \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$

So $3\sin(\theta + \alpha) + 4\cos(\theta + \alpha)$

$\equiv 3(\sin\theta\cos\alpha + \cos\theta\sin\alpha) + 4(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

$\equiv 3\left(\frac{4}{5}\sin\theta + \frac{3}{5}\cos\theta\right) + 4\left(\frac{4}{5}\cos\theta - \frac{3}{5}\sin\theta\right)$

$\equiv \frac{12}{5}\sin\theta + \frac{9}{5}\cos\theta + \frac{16}{5}\cos\theta - \frac{12}{5}\sin\theta$

$\equiv \frac{25}{5}\cos\theta$

$\equiv 5\cos\theta$

(b) $\cos(x + 270)^\circ \equiv \cos x^\circ \cos 270^\circ - \sin x^\circ \sin 270^\circ$

$= (-0.8)(0) - (0.6)(-1) = 0 + 0.6 = 0.6$

$\cos(x + 540)^\circ \equiv \cos x^\circ \cos 540^\circ - \sin x^\circ \sin 540^\circ$

$= (-0.8)(-1) - (0.6)(0) = 0.8 - 0 = 0.8$

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Exercise F, Question 16

Question:

(a) Without using a calculator, find the values of:

(i) $\sin 40° \cos 10° - \cos 40° \sin 10°$

(ii) $\frac{1}{\sqrt{2}} \cos 15° - \frac{1}{\sqrt{2}} \sin 15°$

(iii) $\frac{1 - \tan 15°}{1 + \tan 15°}$

(b) Find, to 1 decimal place, the values of x, $0 \le x \le 360°$, which satisfy the equation $2 \sin x = \cos (x - 60)$

[E]

Solution:

(a) (i) $\sin 40° \cos 10° - \cos 40° \sin 10° = \sin (40° - 10°) = \sin 30° = \frac{1}{2}$

(ii) $\frac{1}{\sqrt{2}} \cos 15° - \frac{1}{\sqrt{2}} \sin 15°$

$= \cos 45° \cos 15° - \sin 45° \sin 15° = \cos (45° + 15°) = \cos 60° = \frac{1}{2}$

(iii) $\frac{1 - \tan 15°}{1 + \tan 15°} = \frac{\tan 45° - \tan 15°}{1 + \tan 45° \tan 15°}$

$= \tan (45° - 15°) = \tan 30° = \frac{\sqrt{3}}{3}$

(b) $2 \sin x = \cos (x - 60°)$

$\Rightarrow 2 \sin x = \cos x \cos 60° + \sin x \sin 60°$

$\Rightarrow 2 \sin x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$

$\Rightarrow (4 - \sqrt{3}) \sin x = \cos x$

$\Rightarrow \frac{\sin x}{\cos x} = \frac{1}{4 - \sqrt{3}}$

$\Rightarrow \tan x = \frac{1}{4 - \sqrt{3}}$

So $x = \tan^{-1}\left(\frac{1}{4-\sqrt{3}}\right), 180° + \tan^{-1}\left(\frac{1}{4-\sqrt{3}}\right)$

$\Rightarrow \quad x = 23.8°, 203.8°$

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Exercise F, Question 17

Question:

(a) Prove, by counter example, that the statement
'$\sec(A+B) \equiv \sec A + \sec B$, for all A and B'
is false.

(b) Prove that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$, $\theta \neq \frac{n\pi}{2}$, $n \in \mathbb{Z}$.

[E]

Solution:

(a) One example is sufficient to disprove a statement.
E.g. $A = 60°$, $B = 0°$

$\sec(A+B) = \sec(60° + 0°) = \sec 60° = \frac{1}{\cos 60°} = 2$

$\sec A = \sec 60° = \frac{1}{\cos 60°} = 2$

$\sec B = \sec 0° = \frac{1}{\cos 0°} = 1$

So $\sec(60° + 0°) \neq \sec 60° + \sec 0°$
$\Rightarrow \quad \sin(A+B) \equiv \sec A + \sec B$ not true for all values of A, B.

(b) L.H.S. $\equiv \tan\theta + \cot\theta$

$$\equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$\equiv \frac{1}{\frac{1}{2}\sin 2\theta} \qquad (\sin^2\theta + \cos^2\theta \equiv 1,\ \sin 2\theta \equiv 2\sin\theta\cos\theta)$$

$$\equiv \frac{2}{\sin 2\theta}$$

$\equiv 2\operatorname{cosec} 2\theta$
$\equiv$ R.H.S.

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Exercise F, Question 18

Question:

Using the formula $\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$:

(a) Show that $\cos(A-B) - \cos(A+B) \equiv 2\sin A\sin B$.

(b) Hence show that $\cos 2x - \cos 4x \equiv 2\sin 3x\sin x$.

(c) Find all solutions in the range $0 \le x \le \pi$ of the equation
$\cos 2x - \cos 4x = \sin x$
giving all your solutions in multiples of π radians.

[E]

Solution:

(a) $\cos(A+B) \equiv \cos A\cos B - \sin A\sin B$

$\Rightarrow \quad \cos(A-B) \equiv \cos A\cos(-B) - \sin A\sin(-B)$

$\equiv \cos A\cos B + \sin A\sin B$

so $\cos(A+B) - \cos(A-B) \equiv (\cos A\cos B - \sin A\sin B) - (\cos A\cos B + \sin A\sin B)$

$\equiv -2\sin A\sin B$

(b) Let $A+B = 2x$, $A-B = 4x$

Add: $2A = 6x \Rightarrow A = 3x$

Subtract: $2B = -2x \Rightarrow B = -x$

Using (a) $\cos 2x - \cos 4x \equiv -2\sin 3x\sin(-x) \equiv 2\sin 3x\sin x$
as $\sin(-x) = -\sin x$

(c) Solve $2\sin 3x\sin x = \sin x$

$\Rightarrow \quad \sin x(2\sin 3x - 1) = 0$

$\Rightarrow \quad \sin x = 0$ or $\sin 3x = \frac{1}{2}$

$\sin x = 0 \Rightarrow x = 0, \pi$

$\sin 3x = \frac{1}{2}, 0 \le 3x \le 3\pi \Rightarrow 3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$\Rightarrow \quad x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$

Solution set: $0, \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \pi$

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Exercise F, Question 19

Question:

(a) Given that $\cos(x + 30°) = 3\cos(x - 30°)$, prove that $\tan x = -\frac{\sqrt{3}}{2}$.

(b) (i) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan\theta$.

(ii) Verify that $\theta = 180°$ is a solution of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

(iii) Using the result in part (i), or otherwise, find the two other solutions, $0 < \theta < 360°$, of the equation $\sin 2\theta = 2 - 2\cos 2\theta$.

[E]

Solution:

(a) $\cos(x + 30°) = 3\cos(x - 30°)$

$\Rightarrow \quad \cos x\cos 30° - \sin x\sin 30° = 3(\cos x\cos 30° + \sin x\sin 30°)$

$\Rightarrow \quad -2\cos x\cos 30° = 4\sin x\sin 30°$

$\Rightarrow \quad -2\cos x \times \frac{\sqrt{3}}{2} = 4\sin x \times \frac{1}{2}$

$\Rightarrow \quad -\frac{\sqrt{3}}{2} = \frac{\sin x}{\cos x}$

$\Rightarrow \quad \tan x = -\frac{\sqrt{3}}{2}$

(b) (i) L.H.S. $\equiv \frac{1 - \cos 2\theta}{\sin 2\theta}$

$\equiv \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta}$

$\equiv \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$

$\equiv \frac{\sin\theta}{\cos\theta}$

$\equiv \tan\theta$

(ii) L.H.S. $= \sin 360° = 0$

R.H.S. $= 2 - 2\cos 360° = 2 - 2(1) = 0$ ✓

(iii) Using (i) this is equivalent to solving $\tan\theta = \frac{1}{2}$.

From (i) $1 - \cos 2\theta = \sin 2\theta \tan\theta$

So $\sin 2\theta = 2 - 2\cos 2\theta \quad \Rightarrow \quad \sin 2\theta = 2\sin 2\theta \tan\theta$

$\sin 2\theta = 0$ gives $\theta = 180\ °$, so $\tan\theta = \frac{1}{2} \quad \Rightarrow \quad \theta =$ 26.6°, 206.6°

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Exercise F, Question 20

Question:

(a) Express $1.5\sin 2x + 2\cos 2x$ in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, giving your values of R and α to 3 decimal places where appropriate.

(b) Express $3\sin x\cos x + 4\cos^2 x$ in the form $a\sin 2x + b\cos 2x + c$, where a, b and c are constants to be found.

(c) Hence, using your answer to part (a), deduce the maximum value of $3\sin x\cos x + 4\cos^2 x$.

[E]

Solution:

(a) Let $1.5\sin 2x + 2\cos 2x \equiv R\sin(2x + \alpha) \equiv R\sin 2x\cos\alpha + R\cos 2x\sin\alpha$

$R > 0,\ 0 < \alpha < \frac{\pi}{2}$

Compare $\sin 2x$: $R\cos\alpha = 1.5$ ①
Compare $\cos 2x$: $R\sin\alpha = 2$ ②

Divide ② by ①: $\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 0.927$

$R^2 = 2^2 + 1.5^2 \Rightarrow R = 2.5$

(b) $3\sin x\cos x + 4\cos^2 x \equiv \frac{3}{2}(2\sin x\cos x) + 4\left(\frac{1 + \cos 2x}{2}\right)$

$\equiv \frac{3}{2}\sin 2x + 2 + 2\cos 2x \equiv \frac{3}{2}\sin 2x + 2\cos 2x + 2$

(c) From part (a) $\frac{3}{2}\sin 2x + 2\cos 2x \equiv 2.5\sin(2x + 0.927)$

So maximum value of $\frac{3}{2}\sin 2x + 2\cos 2x = 2.5 \times 1 = 2.5$

So maximum value of $3\sin x\cos x + 4\cos^2 x = 2.5 + 2 = 4.5$

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