

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

Show that each of these equations $f(x) = 0$ has a root in the given interval(s):

(a) $x^3 - x + 5 = 0 \quad -2 < x < -1.$

(b) $3 + x^2 - x^3 = 0 \quad 1 < x < 2.$

(c) $x^2 - \sqrt{x} - 10 = 0 \quad 3 < x < 4.$

(d) $x^3 - \frac{1}{x} - 2 = 0 \quad -0.5 < x < -0.2 \text{ and } 1 < x < 2.$

(e) $x^5 - 5x^3 - 10 = 0 \quad -2 < x < -1.8, -1.8 < x < -1 \text{ and } 2 < x < 3.$

(f) $\sin x - \ln x = 0 \quad 2.2 < x < 2.3$

(g) $e^x - \ln x - 5 = 0 \quad 1.65 < x < 1.75.$

(h) $\sqrt[3]{x} - \cos x = 0 \quad 0.5 < x < 0.6.$

Solution:

(a) Let $f(x) = x^3 - x + 5$

$$f(-2) = (-2)^3 - (-2) + 5 = -8 + 2 + 5 = -1$$

$$f(-1) = (-1)^3 - (-1) + 5 = -1 + 1 + 5 = 5$$

$f(-2) < 0$ and $f(-1) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -2$ and $x = -1$.

(b) Let $f(x) = 3 + x^2 - x^3$

$$f(1) = 3 + (1)^2 - (1)^3 = 3 + 1 - 1 = 3$$

$$f(2) = 3 + (2)^2 - (2)^3 = 3 + 4 - 8 = -1$$

$f(1) > 0$ and $f(2) < 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 1$ and $x = 2$.

(c) Let $f(x) = x^2 - \sqrt{x} - 10$

$$f(3) = 3^2 - \sqrt{3} - 10 = -2.73$$

$$f(4) = 4^2 - \sqrt{4} - 10 = 4$$

$f(3) < 0$ and $f(4) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 3$ and $x = 4$.

(d) Let $f(x) = x^3 - \frac{1}{x} - 2$

$$[1] f(-0.5) = (-0.5)^3 - \frac{1}{-0.5} - 2 = -0.125$$

$$f(-0.2) = (-0.2)^3 - \frac{1}{-0.2} - 2 = 2.992$$

$f(-0.5) < 0$ and $f(-0.2) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -0.5$ and $x = -0.2$.

$$[2] f(1) = (1)^3 - \frac{1}{1} - 2 = -2$$

$$f(2) = (2)^3 - \frac{1}{2} - 2 = 5\frac{1}{2}$$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 1$ and $x = 2$.

(e) Let $f(x) = x^5 - 5x^3 - 10$

$$[1] f(-2) = (-2)^5 - 5(-2)^3 - 10 = -2$$

$$f(-1.8) = (-1.8)^5 - 5(-1.8)^3 - 10 = 0.26432$$

$f(-2) < 0$ and $f(-1.8) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -2$ and $x = -1.8$.

$$[2] f(-1.8) = 0.26432$$

$$f(-1) = (-1)^5 - 5(-1)^3 - 10 = -6$$

$f(-1.8) > 0$ and $f(-1) < 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -1.8$ and $x = -1$.

$$[3] f(2) = (2)^5 - 5(2)^3 - 10 = -18$$

$$f(3) = (3)^5 - 5(3)^3 - 10 = 98$$

$f(2) < 0$ and $f(3) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 2$ and $x = 3$.

(f) Let $f(x) = \sin x - \ln x$

$$f(2.2) = \sin 2.2 - \ln 2.2 = 0.0200$$

$$f(2.3) = -0.0872$$

$f(2.2) > 0$ and $f(2.3) < 0$ so there is a change of sign.

⇒ There is a root between $x = 2.2$ and $x = 2.3$.

(g) Let $f(x) = e^x - \ln x - 5$

$$f(1.65) = e^{1.65} - \ln 1.65 - 5 = -0.294$$

$$f(1.75) = e^{1.75} - \ln 1.75 - 5 = 0.195$$

$f(1.65) < 0$ and $f(1.75) > 0$ so there is a change of sign.

⇒ There is a root between $x = 1.65$ and $x = 1.75$.

(h) Let $f(x) = \sqrt[3]{x} - \cos x$

$$f(0.5) = \sqrt[3]{0.5} - \cos 0.5 = -0.0839$$

$$f(0.6) = \sqrt[3]{0.6} - \cos 0.6 = 0.0181$$

$f(0.5) < 0$ and $f(0.6) > 0$ so there is a change of sign.

⇒ There is a root between $x = 0.5$ and $x = 0.6$.

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Exercise A, Question 2

Question:

Given that $f(x) = x^3 - 5x^2 + 2$, show that the equation $f(x) = 0$ has a root near to $x = 5$.

Solution:

$$\text{Let } f(x) = x^3 - 5x^2 + 2$$

$$f(4.9) = (4.9)^3 - 5(4.9)^2 + 2 = -0.401$$

$$f(5.0) = 2$$

$f(4.9) < 0$ and $f(5) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 4.9$ and $x = 5$.

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Exercise A, Question 3

Question:

Given that $f(x) \equiv 3 - 5x + x^3$, show that the equation $f(x) = 0$ has a root $x = a$, where a lies in the interval $1 < a < 2$.

Solution:

$$\text{Let } f(x) = 3 - 5x + x^3$$

$$f(1) = 3 - 5(1) + (1)^3 = -1$$

$$f(2) = 3 - 5(2) + (2)^3 = 1$$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 1$ and $x = 2$.

So if the root is $x = a$, then $1 < a < 2$.

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Exercise A, Question 4

Question:

Given that $f(x) \equiv e^x \sin x - 1$, show that the equation $f(x) = 0$ has a root $x = r$, where r lies in the interval $0.5 < r < 0.6$.

Solution:

$$f(x) = e^x \sin x - 1$$

$$f(0.5) = e^{0.5} \sin 0.5 - 1 = -0.210$$

$$f(0.6) = e^{0.6} \sin 0.6 - 1 = 0.0288$$

$f(0.5) < 0$ and $f(0.6) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 0.5$ and $x = 0.6$.

So if the root is $x = r$, then $0.5 < r < 0.6$.

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Exercise A, Question 5

Question:

It is given that $f(x) \equiv x^3 - 7x + 5$.

(a) Copy and complete the table below.

x	-3	-2	-1	0	1	2	3
$f(x)$							

(b) Given that the negative root of the equation $x^3 - 7x + 5 = 0$ lies between α and $\alpha + 1$, where α is an integer, write down the value of α .

Solution:

(a)

x	-3	-2	-1	0	1	2	3
$f(x)$	-1	11	11	5	-1	-1	11

(b) $f(-3) < 0$ and $f(-2) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -3$ and $x = -2$.

So $\alpha = -3$. (**Note.** $\alpha + 1 = -2$).

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Exercise A, Question 6

Question:

Given that $f(x) \equiv x - (\sin x + \cos x)^{\frac{1}{2}}$, $0 \leq x \leq \frac{3}{4}\pi$, show that the equation $f(x) = 0$ has a root lying between $\frac{\pi}{3}$ and $\frac{\pi}{2}$.

Solution:

$$f(x) = x - (\sin x + \cos x)^{\frac{1}{2}}$$

$$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - \left(\sin \frac{\pi}{3} + \cos \frac{\pi}{3}\right)^{\frac{1}{2}} = -0.122$$

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} - \left(\sin \frac{\pi}{2} + \cos \frac{\pi}{2}\right)^{\frac{1}{2}} = 0.571$$

$$f\left(\frac{\pi}{3}\right) < 0 \text{ and } f\left(\frac{\pi}{2}\right) > 0 \text{ so there is a change of sign.}$$

$$\Rightarrow \text{There is a root between } x = \frac{\pi}{3} \text{ and } x = \frac{\pi}{2}.$$

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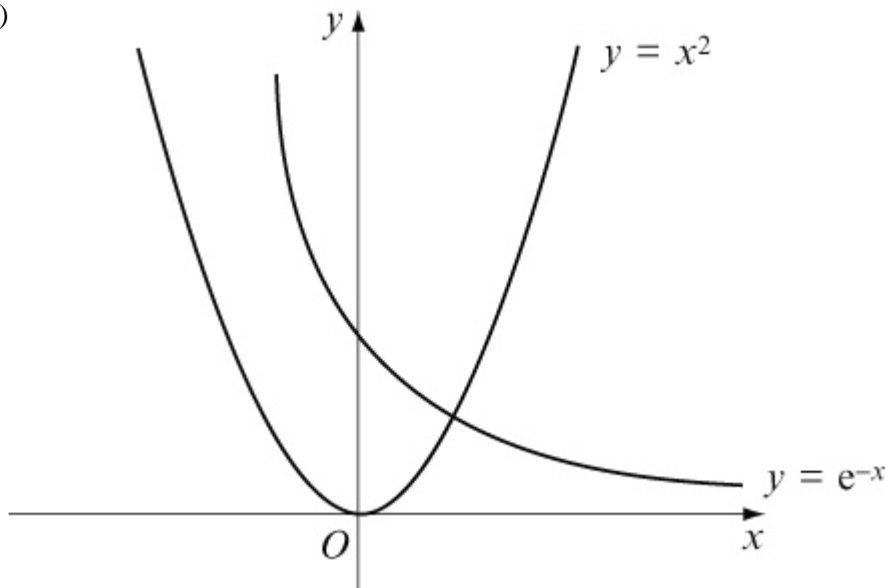
Exercise A, Question 7

Question:

- (a) Using the same axes, sketch the graphs of $y = e^{-x}$ and $y = x^2$.
- (b) Explain why the equation $e^{-x} = x^2$ has only one root.
- (c) Show that the equation $e^{-x} = x^2$ has a root between $x = 0.70$ and $x = 0.71$.

Solution:

(a)



- (b) The curves meet where $e^{-x} = x^2$
 The curves meet at one point, so there is one value of x that satisfies the equation $e^{-x} = x^2$.
 So $e^{-x} = x^2$ has one root.

- (c) Let $f(x) = e^{-x} - x^2$
 $f(0.70) = e^{-0.70} - 0.70^2 = 0.00659$
 $f(0.71) = e^{-0.71} - 0.71^2 = -0.0125$
 $f(0.70) > 0$ and $f(0.71) < 0$ so there is a change of sign.
 \Rightarrow There is a root between $x = 0.70$ and $x = 0.71$.

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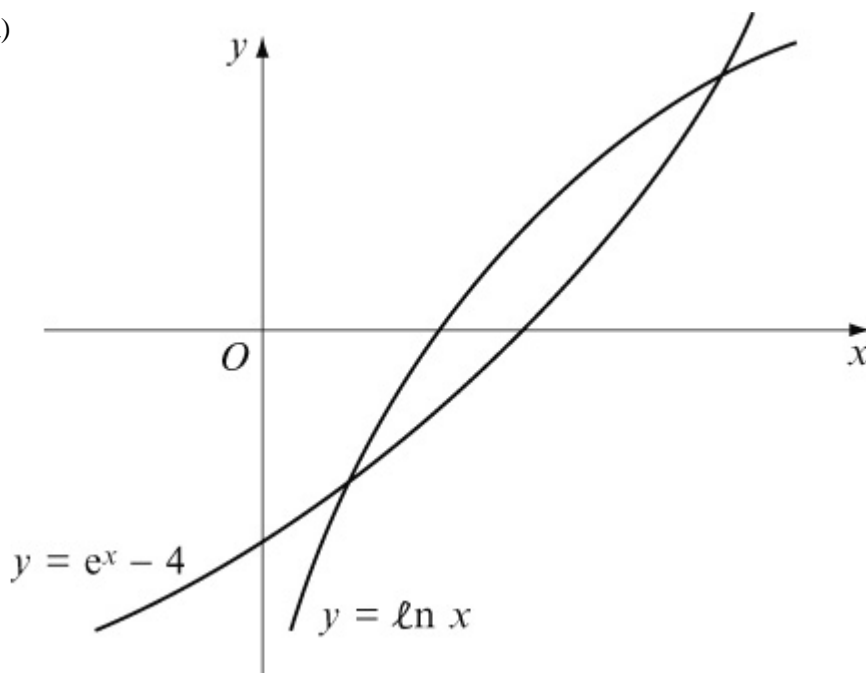
Exercise A, Question 8

Question:

- (a) On the same axes, sketch the graphs of $y = \ln x$ and $y = e^x - 4$.
- (b) Write down the number of roots of the equation $\ln x = e^x - 4$.
- (c) Show that the equation $\ln x = e^x - 4$ has a root in the interval $(1.4, 1.5)$.

Solution:

(a)



(b) The curves meet at two points, so there are two values of x that satisfy the equation $\ln x = e^x - 4$.

So $\ln x = e^x - 4$ has two roots.

(c) Let $f(x) = \ln x - e^x + 4$

$$f(1.4) = \ln 1.4 - e^{1.4} + 4 = 0.281$$

$$f(1.5) = \ln 1.5 - e^{1.5} + 4 = -0.0762$$

$f(1.4) > 0$ and $f(1.5) < 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 1.4$ and $x = 1.5$.

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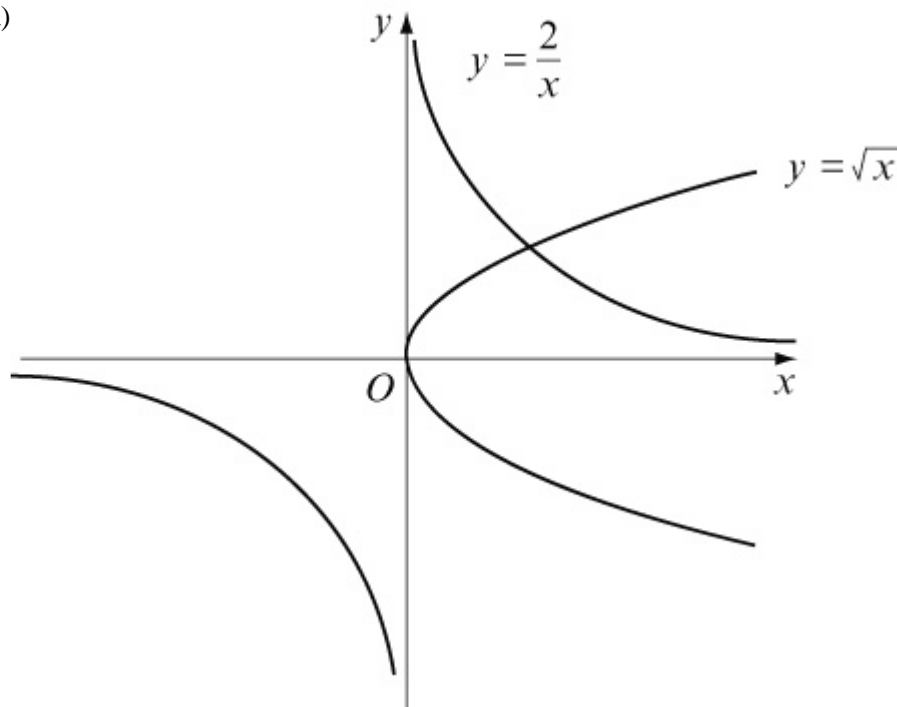
Exercise A, Question 9

Question:

- (a) On the same axes, sketch the graphs of $y = \sqrt{x}$ and $y = \frac{2}{x}$.
- (b) Using your sketch, write down the number of roots of the equation $\sqrt{x} = \frac{2}{x}$.
- (c) Given that $f(x) \equiv \sqrt{x} - \frac{2}{x}$, show that $f(x) = 0$ has a root r , where r lies between $x = 1$ and $x = 2$.
- (d) Show that the equation $\sqrt{x} = \frac{2}{x}$ may be written in the form $x^p = q$, where p and q are integers to be found.
- (e) Hence write down the exact value of the root of the equation $\sqrt{x} - \frac{2}{x} = 0$.

Solution:

(a)



- (b) The curves meet at one point, so there is one value of x that satisfies the

equation $\sqrt{x} = \frac{2}{x}$.

So $\sqrt{x} = \frac{2}{x}$ has **one** root.

(c) $f(x) = \sqrt{x} - \frac{2}{x}$

$f(1) = \sqrt{1} - \frac{2}{1} = -1$

$f(2) = \sqrt{2} - \frac{2}{2} = 0.414$

$f(1) < 0$ and $f(2) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 1$ and $x = 2$.

(d) $\sqrt{x} = \frac{2}{x}$

$x^{\frac{1}{2}} = \frac{2}{x}$

$x^{\frac{1}{2}} \times x = 2$

$x^{\frac{1}{2} + 1} = 2$

$x^{\frac{3}{2}} = 2$

$(x^{\frac{3}{2}})^2 = 2^2$

$x^3 = 4$

So $p = 3$ and $q = 4$

(e) $x^{\frac{3}{2}} = 2$

$\Rightarrow x = 2^{\frac{2}{3}} \quad [= (2^2)^{\frac{1}{3}} = 4^{\frac{1}{3}}]$

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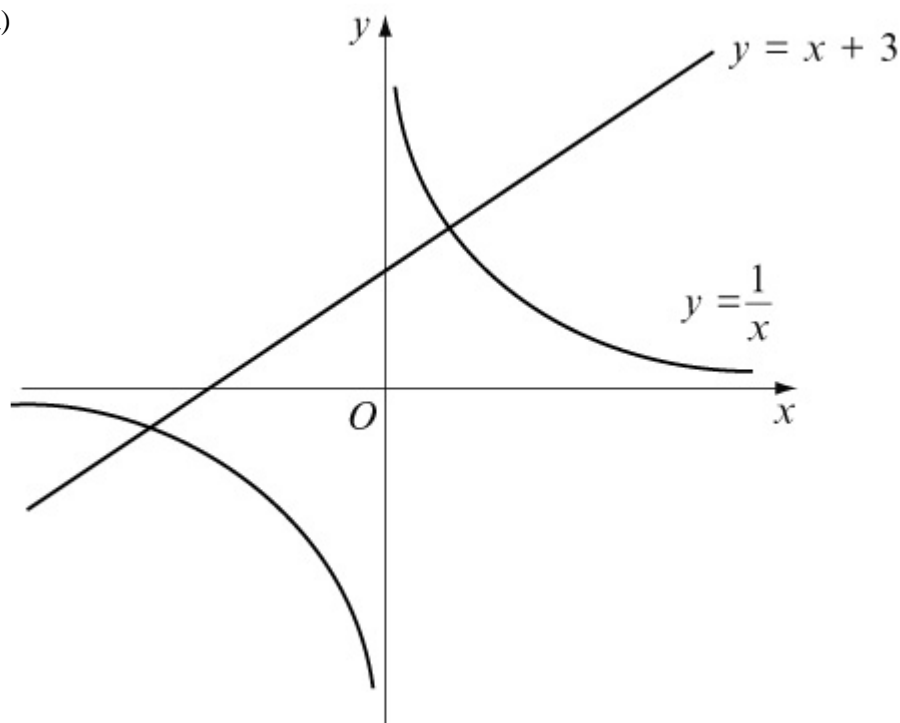
Exercise A, Question 10

Question:

- (a) On the same axes, sketch the graphs of $y = \frac{1}{x}$ and $y = x + 3$.
- (b) Write down the number of roots of the equation $\frac{1}{x} = x + 3$.
- (c) Show that the positive root of the equation $\frac{1}{x} = x + 3$ lies in the interval (0.30, 0.31).
- (d) Show that the equation $\frac{1}{x} = x + 3$ may be written in the form $x^2 + 3x - 1 = 0$.
- (e) Use the quadratic formula to find the positive root of the equation $x^2 + 3x - 1 = 0$ to 3 decimal places.

Solution:

(a)



- (b) The line meets the curve at two points, so there are two values of x that satisfy the equation $\frac{1}{x} = x + 3$.

So $\frac{1}{x} = x + 3$ has **two** roots.

(c) Let $f(x) = \frac{1}{x} - x - 3$

$$f(0.30) = \frac{1}{0.30} - (0.30) - 3 = 0.0333$$

$$f(0.31) = \frac{1}{0.31} - (0.31) - 3 = -0.0842$$

$f(0.30) > 0$ and $f(0.31) < 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 0.30$ and $x = 0.31$.

(d) $\frac{1}{x} = x + 3$

$$\frac{1}{x} \times x = x \times x + 3 \times x \quad (\times x)$$

$$1 = x^2 + 3x$$

$$\text{So } x^2 + 3x - 1 = 0$$

(e) Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with $a = 1$, $b = 3$, $c = -1$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)} = \frac{-3 \pm \sqrt{9+4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$$

$$\text{So } x = \frac{-3 + \sqrt{13}}{2} = 0.303$$

$$\text{and } x = \frac{-3 - \sqrt{13}}{2} = -3.303$$

The positive root is 0.303 to 3 decimal places.

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Exercise B, Question 1

Question:

Show that $x^2 - 6x + 2 = 0$ can be written in the form:

$$(a) x = \frac{x^2 + 2}{6}$$

$$(b) x = \sqrt{6x - 2}$$

$$(c) x = 6 - \frac{2}{x}$$

Solution:

$$(a) x^2 - 6x + 2 = 0$$

$$6x = x^2 + 2 \quad \text{Add } 6x \text{ to each side}$$

$$x = \frac{x^2 + 2}{6} \quad \text{Divide each side by } 6$$

$$(b) x^2 - 6x + 2 = 0$$

$$x^2 + 2 = 6x \quad \text{Add } 6x \text{ to each side}$$

$$x^2 = 6x - 2 \quad \text{Subtract } 2 \text{ from each side}$$

$$x = \sqrt{6x - 2} \quad \text{Take the square root of each side}$$

$$(c) x^2 - 6x + 2 = 0$$

$$x^2 + 2 = 6x \quad \text{Add } 6x \text{ to each side}$$

$$x^2 = 6x - 2 \quad \text{Subtract } 2 \text{ from each side}$$

$$\frac{x^2}{x} = \frac{6x}{x} - \frac{2}{x} \quad \text{Divide each term by } x$$

$$x = 6 - \frac{2}{x} \quad \text{Simplify}$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 2

Question:

Show that $x^3 + 5x^2 - 2 = 0$ can be written in the form:

$$(a) x = \sqrt[3]{2 - 5x^2}$$

$$(b) x = \frac{2}{x^2} - 5$$

$$(c) x = \sqrt{\frac{2 - x^3}{5}}$$

Solution:

$$(a) x^3 + 5x^2 - 2 = 0$$

$$x^3 + 5x^2 = 2 \quad \text{Add 2 to each side}$$

$$x^3 = 2 - 5x^2 \quad \text{Subtract } 5x^2 \text{ from each side}$$

$$x = \sqrt[3]{2 - 5x^2} \quad \text{Take the cube root of each side}$$

$$(b) x^3 + 5x^2 - 2 = 0$$

$$x^3 + 5x^2 = 2 \quad \text{Add 2 to each side}$$

$$x^3 = 2 - 5x^2 \quad \text{Subtract } 5x^2 \text{ from each side}$$

$$\frac{x^3}{x^2} = \frac{2}{x^2} - \frac{5x^2}{x^2} \quad \text{Divide each term by } x^2$$

$$x = \frac{2}{x^2} - 5 \quad \text{Simplify}$$

$$(c) x^3 + 5x^2 - 2 = 0$$

$$x^3 + 5x^2 = 2 \quad \text{Add 2 to each side}$$

$$5x^2 = 2 - x^3 \quad \text{Subtract } x^3 \text{ from each side}$$

$$x^2 = \frac{2 - x^3}{5} \quad \text{Divide each side by 5}$$

$$x = \sqrt{\frac{2 - x^3}{5}} \quad \text{Take the square root of each side}$$

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Exercise B, Question 3

Question:

Rearrange $x^3 - 3x + 4 = 0$ into the form $x = \frac{x^3}{3} + a$, where the value of a is to be found.

Solution:

$$x^3 - 3x + 4 = 0$$

$$3x = x^3 + 4 \quad \text{Add } 3x \text{ to each side}$$

$$\frac{3x}{3} = \frac{x^3}{3} + \frac{4}{3} \quad \text{Divide each term by 3}$$

$$x = \frac{x^3}{3} + \frac{4}{3} \quad \text{Simplify}$$

$$\text{So } a = \frac{4}{3}$$

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Exercise B, Question 4

Question:

Rearrange $x^4 - 3x^3 - 6 = 0$ into the form $x = \sqrt[3]{px^4 - 2}$, where the value of p is to be found.

Solution:

$$x^4 - 3x^3 - 6 = 0$$

$$3x^3 = x^4 - 6 \quad \text{Add } 3x^3 \text{ to each side}$$

$$\frac{3x^3}{3} = \frac{x^4}{3} - \frac{6}{3} \quad \text{Divide each term by 3}$$

$$x^3 = \frac{x^4}{3} - 2 \quad \text{Simplify}$$

$$x = \sqrt[3]{\frac{x^4}{3} - 2} \quad \text{Take the cube root of each side}$$

$$\text{So } p = \frac{1}{3}$$

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Exercise B, Question 5

Question:

(a) Show that the equation $x^3 - x^2 + 7 = 0$ can be written in the form $x = \sqrt[3]{x^2 - 7}$.

(b) Use the iteration formula $x_{n+1} = x_n^2 - 7$, starting with $x_0 = 1$, to find x_2 to 1 decimal place.

Solution:

$$(a) x^3 - x^2 + 7 = 0$$

$$x^3 + 7 = x^2 \quad \text{Add } x^2 \text{ to each side}$$

$$x^3 = x^2 - 7 \quad \text{Subtract 7 from each side}$$

$$x = \sqrt[3]{x^2 - 7} \quad \text{Take the cube root of each side}$$

$$(b) x_0 = 1$$

$$x_1 = \sqrt[3]{(1)^2 - 7} = -1.817\dots$$

$$x_2 = \sqrt[3]{(-1.817\dots)^2 - 7} = -1.546\dots$$

$$\text{So } x_2 = -1.5 \text{ (1 d.p.)}$$

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Exercise B, Question 6

Question:

(a) Show that the equation $x^3 + 3x^2 - 5 = 0$ can be written in the form $x = \sqrt{\frac{5}{x+3}}$.

(b) Use the iteration formula $x_{n+1} = \sqrt{\frac{5}{x_n+3}}$, starting with $x_0 = 1$, to find x_4 to 3 decimal places.

Solution:

$$(a) x^3 + 3x^2 - 5 = 0$$

$$x^2(x+3) - 5 = 0 \quad \text{Factorise } x^2$$

$$x^2(x+3) = 5 \quad \text{Add 5 to each side}$$

$$x^2 = \frac{5}{x+3} \quad \text{Divide each side by } (x+3)$$

$$x = \sqrt{\frac{5}{x+3}} \quad \text{Take the square root of each side}$$

$$(b) x_0 = 1$$

$$x_1 = \sqrt{\frac{5}{(1)+3}} = 1.118\dots$$

$$x_2 = \sqrt{\frac{5}{(1.118\dots)+3}} = 1.101\dots$$

$$x_3 = \sqrt{\frac{5}{(1.101\dots)+3}} = 1.104\dots$$

$$x_4 = \sqrt{\frac{5}{(1.104\dots)+3}} = 1.103768\dots$$

$$\text{So } x_4 = 1.104 \text{ (3 d.p.)}$$

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Edexcel AS and A Level Modular Mathematics

Exercise B, Question 7

Question:

(a) Show that the equation $x^6 - 5x + 3 = 0$ has a root between $x = 1$ and $x = 1.5$.

(b) Use the iteration formula $x_{n+1} = \sqrt[5]{5 - \frac{3}{x_n}}$ to find an approximation for the root of the equation $x^6 - 5x + 3 = 0$, giving your answer to 2 decimal places.

Solution:

(a) Let $f(x) = x^6 - 5x + 3$

$$f(1) = (1)^6 - 5(1) + 3 = -1$$

$$f(1.5) = (1.5)^6 - 5(1.5) + 3 = 6.89$$

$f(1) < 0$ and $f(1.5) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 1$ and $x = 1.5$.

(b) $x_0 = 1$

$$x_1 = \sqrt[5]{5 - \frac{3}{1}} = 1.148\dots$$

Similarly,

$$x_2 = 1.190\dots$$

$$x_3 = 1.199\dots$$

$$x_4 = 1.200\dots$$

$$x_5 = 1.201\dots$$

So the root is 1.20 (2 d.p.)

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Exercise B, Question 8

Question:

(a) Rearrange the equation $x^2 - 6x + 1 = 0$ into the form $x = p - \frac{1}{x}$, where p is a constant to be found.

(b) Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = p - \frac{1}{x_n}$ with your value of p , to find x_3 to 2 decimal places.

Solution:

$$(a) x^2 - 6x + 1 = 0$$

$$x^2 + 1 = 6x \quad \text{Add } 6x \text{ to each side}$$

$$x^2 = 6x - 1 \quad \text{Subtract 1 from each side}$$

$$\frac{x^2}{x} = \frac{6x}{x} - \frac{1}{x} \quad \text{Divide each term by } x$$

$$x = 6 - \frac{1}{x} \quad \text{Simplify}$$

$$\text{So } p = 6$$

$$(b) x_0 = 3$$

$$x_1 = 6 - \frac{1}{3} = 5.666\dots$$

$$x_2 = 6 - \frac{1}{5.666\dots} = 5.823\dots$$

$$x_3 = 6 - \frac{1}{5.823\dots} = 5.828\dots$$

$$\text{So } x_3 = 5.83 \text{ (2 d.p.)}$$

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Exercise B, Question 9

Question:

- (a) Show that the equation $x^3 - x^2 + 8 = 0$ has a root in the interval $(-2, -1)$.
- (b) Use a suitable iteration formula to find an approximation to 2 decimal places for the negative root of the equation $x^3 - x^2 + 8 = 0$.

Solution:

(a) Let $f(x) = x^3 - x^2 + 8$

$$f(-2) = (-2)^3 - (-2)^2 + 8 = -8 - 4 + 8 = -4$$

$$f(-1) = (-1)^3 - (-1)^2 + 8 = -1 - 1 + 8 = 6$$

$f(-2) < 0$ and $f(-1) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -2$ and $x = -1$.

(b) $x^3 - x^2 + 8 = 0$

$$x^3 + 8 = x^2 \quad \text{Add } x^2 \text{ to each side}$$

$$x^3 = x^2 - 8 \quad \text{Subtract 8 from each side}$$

$$x = \sqrt[3]{x^2 - 8} \quad \text{Take the cube root of each side}$$

Using $x_{n+1} = \sqrt[3]{x_n^2 - 8}$ and any value for x_0 , the root is -1.72 (2 d.p.).

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise B, Question 10

Question:

- (a) Show that $x^7 - 5x^2 - 20 = 0$ has a root in the interval (1.6, 1.7).
- (b) Use a suitable iteration formula to find an approximation to 3 decimal places for the root of $x^7 - 5x^2 - 20 = 0$ in the interval (1.6, 1.7).

Solution:

- (a) Let $f(x) = x^7 - 5x^2 - 20$
 $f(1.6) = (1.6)^7 - 5(1.6)^2 - 20 = -5.96$
 $f(1.7) = (1.7)^7 - 5(1.7)^2 - 20 = 6.58$
 $f(1.6) < 0$ and $f(1.7) > 0$ so there is a change of sign.
 \Rightarrow There is a root between $x = 1.6$ and $x = 1.7$.

- (b) $x^7 - 5x^2 - 20 = 0$
 $x^7 - 20 = 5x^2$ Add $5x^2$ to each side
 $x^7 = 5x^2 + 20$ Add 20 to each side
 $x = \sqrt[7]{5x^2 + 20}$ Take the seventh root of each side
 So let $x_{n+1} = \sqrt[7]{5x_n^2 + 20}$ and $x_0 = 1.6$, then
 $x_1 = \sqrt[7]{5(1.6)^2 + 20} = 1.6464\dots$

Similarly,

$$x_2 = 1.6518\dots$$

$$x_3 = 1.6524\dots$$

$$x_4 = 1.6525\dots$$

$$x_5 = 1.6525\dots$$

So the root is 1.653 (3 d.p.)

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 1

Question:

(a) Rearrange the cubic equation $x^3 - 6x - 2 = 0$ into the form $x = \pm \sqrt{a + \frac{b}{x}}$. State the values of the constants a and b .

(b) Use the iterative formula $x_{n+1} = \sqrt{a + \frac{b}{x_n}}$ with $x_0 = 2$ and your values of a and b to find the approximate positive solution x_4 of the equation, to an appropriate degree of accuracy. Show all your intermediate answers.

[E]

Solution:

$$(a) x^3 - 6x - 2 = 0$$

$$x^3 - 2 = 6x \quad \text{Add } 6x \text{ to each side}$$

$$x^3 = 6x + 2 \quad \text{Add } 2 \text{ to each side}$$

$$\frac{x^3}{x} = \frac{6x}{x} + \frac{2}{x} \quad \text{Divide each term by } x$$

$$x^2 = 6 + \frac{2}{x} \quad \text{Simplify}$$

$$x = \sqrt{6 + \frac{2}{x}} \quad \text{Take the square root of each side}$$

So $a = 6$ and $b = 2$

$$(b) x_0 = 2$$

$$x_1 = \sqrt{6 + \frac{2}{2}} = \sqrt{7} = 2.64575\dots$$

$$x_2 = \sqrt{6 + \frac{2}{2.64575\dots}} = 2.59921\dots$$

$$x_3 = \sqrt{6 + \frac{2}{2.59921\dots}} = 2.60181\dots$$

$$x_4 = \sqrt{6 + \frac{2}{2.60181\dots}} = 2.60167\dots$$

So $x_4 = 2.602$ (3 d.p.)

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 2

Question:

(a) By sketching the curves with equations $y = 4 - x^2$ and $y = e^x$, show that the equation $x^2 + e^x - 4 = 0$ has one negative root and one positive root.

(b) Use the iteration formula $x_{n+1} = - (4 - e^{x_n})^{\frac{1}{2}}$ with $x_0 = -2$ to find in turn x_1, x_2, x_3 and x_4 and hence write down an approximation to the negative root of the equation, giving your answer to 4 decimal places.

An attempt to evaluate the positive root of the equation is made using the iteration formula

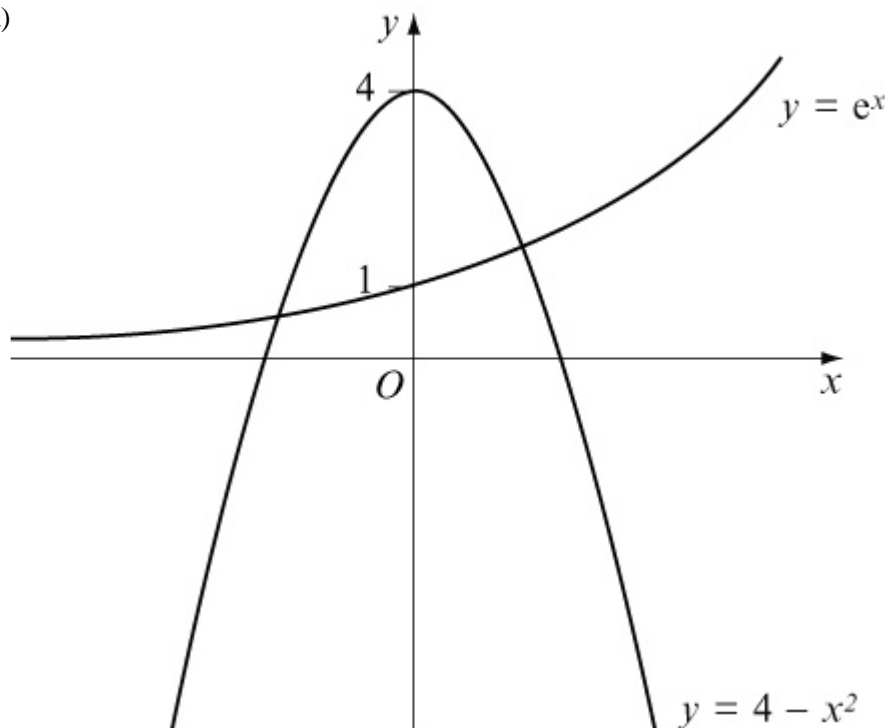
$$x_{n+1} = (4 - e^{x_n})^{\frac{1}{2}} \text{ with } x_0 = 1.3.$$

(c) Describe the result of such an attempt.

[E]

Solution:

(a)



The curves meet when $x < 0$ and $x > 0$, so the equation $e^x = 4 - x^2$ has one negative and one positive root.

(Note that $e^x = 4 - x^2$ is the same as $x^2 + e^x - 4 = 0$).

$$(b) x_0 = -2$$

$$x_1 = - (4 - e^{-2})^{\frac{1}{2}} = -1.965875051$$

$$x_2 = - (4 - e^{-1.965875051})^{\frac{1}{2}} = -1.964679797$$

$$x_3 = - (4 - e^{-1.964679797})^{\frac{1}{2}} = -1.964637175$$

$$x_4 = - (4 - e^{-1.964637175})^{\frac{1}{2}} = -1.964635654$$

$$\text{So } x_4 = -1.9646 \text{ (4 d.p.)}$$

$$(c) x_0 = 1.3$$

$$x_1 = (4 - e^{1.3})^{\frac{1}{2}} = 0.575\dots$$

$$x_2 = (4 - e^{0.575\dots})^{\frac{1}{2}} = 1.490\dots$$

$$x_3 = (4 - e^{1.490\dots})^{\frac{1}{2}} \quad \text{No solution}$$

The value of $4 - e^{1.490\dots}$ is **negative**.

You can not take the square root of a negative number.

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 3

Question:

- (a) Show that the equation $x^5 - 5x - 6 = 0$ has a root in the interval (1, 2).
- (b) Stating the values of the constants p , q and r , use an iteration of the form $x_{n+1} = (px_n + q)^{\frac{1}{r}}$ an appropriate number of times to calculate this root of the equation $x^5 - 5x - 6 = 0$ correct to 3 decimal places. Show sufficient working to justify your final answer.

[E]

Solution:

- (a) Let $f(x) = x^5 - 5x - 6$
 $f(1) = (1)^5 - 5(1) - 6 = 1 - 5 - 6 = -10$
 $f(2) = (2)^5 - 5(2) - 6 = 32 - 10 - 6 = 16$
 $f(1) < 0$ and $f(2) > 0$ so there is a change of sign.
 \Rightarrow There is a root between $x = 1$ and $x = 2$.

- (b) $x^5 - 5x - 6 = 0$
 $x^5 - 6 = 5x$ Add $5x$ to each side
 $x^5 = 5x + 6$ Add 6 to each side
 $x = (5x + 6)^{\frac{1}{5}}$ Take the fifth root of each side
 So $p = 5$, $q = 6$ and $r = 5$
 Let $x_0 = 1$ then

$$x_1 = [5(1) + 6]^{\frac{1}{5}} = 1.6153\dots$$

$$x_2 = [5(1.6153\dots) + 6]^{\frac{1}{5}} = 1.6970\dots$$

$$x_3 = 1.7068\dots$$

$$x_4 = 1.7079\dots$$

$$x_5 = 1.7080\dots$$

$$x_6 = 1.7081\dots$$

$$x_7 = 1.7081\dots$$

So the root is 1.708 (3 d.p.)

Solutionbank

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Exercise C, Question 4

Question:

$f(x) \equiv 5x - 4 \sin x - 2$, where x is in radians.

- (a) Evaluate, to 2 significant figures, $f(1.1)$ and $f(1.15)$.
- (b) State why the equation $f(x) = 0$ has a root in the interval $(1.1, 1.15)$.
An iteration formula of the form $x_{n+1} = p \sin x_n + q$ is applied to find an approximation to the root of the equation $f(x) = 0$ in the interval $(1.1, 1.15)$.
- (c) Stating the values of p and q , use this iteration formula with $x_0 = 1.1$ to find x_4 to 3 decimal places. Show the intermediate results in your working.

[E]

Solution:

$$\begin{aligned} \text{(a) } f(1.1) &= 5(1.1) - 4 \sin(1.1) - 2 = -0.0648\dots \\ f(1.15) &= 5(1.15) - 4 \sin(1.15) - 2 = 0.0989\dots \end{aligned}$$

- (b) $f(1.1) < 0$ and $f(1.15) > 0$ so there is a change of sign.
 \Rightarrow There is a root between $x = 1.1$ and $x = 1.15$.

$$\begin{aligned} \text{(c) } 5x - 4 \sin x - 2 &= 0 \\ 5x - 2 &= 4 \sin x && \text{Add } 4 \sin x \text{ to each side} \\ 5x &= 4 \sin x + 2 && \text{Add } 2 \text{ to each side} \\ \frac{5x}{5} &= \frac{4 \sin x}{5} + \frac{2}{5} && \text{Divide each term by } 5 \end{aligned}$$

$$x = 0.8 \sin x + 0.4 \quad \text{Simplify}$$

$$\text{So } p = 0.8 \text{ and } q = 0.4$$

$$x_0 = 1.1$$

$$x_1 = 0.8 \sin(1.1) + 0.4 = 1.112965888$$

$$x_2 = 0.8 \sin(1.112965888) + 0.4 = 1.117610848$$

$$x_3 = 0.8 \sin(1.117610848) + 0.4 = 1.11924557$$

$$x_4 = 0.8 \sin(1.11924557) + 0.4 = 1.119817195$$

$$\text{So } x_4 = 1.120 \text{ (3 d.p.)}$$

Solutionbank

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Exercise C, Question 5

Question:

$f(x) \equiv 2 \sec x + 2x - 3$, where x is in radians.

(a) Evaluate $f(0.4)$ and $f(0.5)$ and deduce the equation $f(x) = 0$ has a solution in the interval $0.4 < x < 0.5$.

(b) Show that the equation $f(x) = 0$ can be arranged in the form $x = p + \frac{q}{\cos x}$, where p and q are constants, and state the value of p and the value of q .

(c) Using the iteration formula $x_{n+1} = p + \frac{q}{\cos x_n}$, $x_0 = 0.4$, with the values of p and q found in part (b), calculate x_1, x_2, x_3 and x_4 , giving your final answer to 4 decimal places.

[E]

Solution:

$$(a) f(0.4) = 2 \sec(0.4) + 2(0.4) - 3 = -0.0286$$

$$f(0.5) = 2 \sec(0.5) + 2(0.5) - 3 = 0.279$$

$f(0.4) < 0$ and $f(0.5) > 0$ so there is a change of sign.

\Rightarrow There is a root between $x = 0.4$ and $x = 0.5$.

$$(b) 2 \sec x + 2x - 3 = 0$$

$$2 \sec x + 2x = 3 \quad \text{Add 3 to each side}$$

$$2x = 3 - 2 \sec x \quad \text{Subtract } 2 \sec x \text{ from each side}$$

$$\frac{2x}{2} = \frac{3}{2} - \frac{2 \sec x}{2} \quad \text{Divide each term by 2}$$

$$x = 1.5 - \sec x \quad \text{Simplify}$$

$$x = 1.5 - \frac{1}{\cos x} \quad \text{Use } \sec x = \frac{1}{\cos x}$$

So $p = 1.5$ and $q = -1$

$$(c) x_0 = 0.4$$

$$x_1 = 1.5 - \frac{1}{\cos(0.4)} = 0.4142955716$$

$$x_2 = 1.5 - \frac{1}{\cos(0.4142955716)} = 0.4075815187$$

$$x_3 = 1.5 - \frac{1}{\cos(0.4075815187)} = 0.4107728765$$

$$x_4 = 1.5 - \frac{1}{\cos(0.4107728765)} = 0.4092644032$$

So $x_4 = 0.4093$ (4 d.p.)

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Solutionbank

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Exercise C, Question 6

Question:

$$f(x) \equiv e^{0.8x} - \frac{1}{3-2x}, x \neq \frac{3}{2}$$

- (a) Show that the equation $f(x) = 0$ can be written as $x = 1.5 - 0.5e^{-0.8x}$.
- (b) Use the iteration formula $x_{n+1} = 1.5 - 0.5e^{-0.8x_n}$ with $x_0 = 1.3$ to obtain x_1, x_2 and x_3 . Give the value of x_3 , an approximation to a root of $f(x) = 0$, to 3 decimal places.
- (c) Show that the equation $f(x) = 0$ can be written in the form $x = p \ln(3 - 2x)$, stating the value of p .
- (d) Use the iteration formula $x_{n+1} = p \ln(3 - 2x_n)$ with $x_0 = -2.6$ and the value of p found in part (c) to obtain x_1, x_2 and x_3 . Give the value of x_3 , an approximation to the second root of $f(x) = 0$, to 3 decimal places.

[E]

Solution:

$$(a) e^{0.8x} - \frac{1}{3-2x} = 0$$

$$e^{0.8x} = \frac{1}{3-2x} \quad \text{Add } \frac{1}{3-2x} \text{ to each side}$$

$$\left(\frac{1}{3-2x} \right) e^{0.8x} = \frac{1}{3-2x} \times \left(\frac{1}{3-2x} \right) \quad \text{Multiply each side by}$$

$$\frac{e^{0.8x}}{3-2x} = \frac{1}{(3-2x)^2} \quad \text{Simplify}$$

$$\frac{(3-2x)e^{0.8x}}{e^{0.8x}} = \frac{1}{e^{0.8x}} \quad \text{Divide each side by } e^{0.8x}$$

$$3-2x = e^{-0.8x} \quad \text{Simplify (remember } \frac{1}{e^a} = e^{-a} \text{)}$$

$$3 = e^{-0.8x} + 2x \quad \text{Add } 2x \text{ to each side}$$

$$2x = 3 - e^{-0.8x} \quad \text{Subtract } e^{-0.8x} \text{ from each side}$$

$$\frac{2x}{2} = \frac{3}{2} - \frac{e^{-0.8x}}{2} \quad \text{Divide each term by 2}$$

$$x = 1.5 - 0.5e^{-0.8x} \quad \text{Simplify}$$

$$(b) x_0 = 1.3$$

$$x_1 = 1.5 - 0.5e^{-0.8(1.3)} = 1.323272659$$

$$x_2 = 1.5 - 0.5e^{-0.8(1.323272659)} = 1.32653255$$

$$x_3 = 1.5 - 0.5e^{-0.8(1.32653255)} = 1.326984349$$

$$\text{So } x_3 = 1.327 \text{ (3 d.p.)}$$

$$(c) e^{0.8x} - \frac{1}{3-2x} = 0$$

$$e^{0.8x} = \frac{1}{3-2x} \quad \text{Add } \frac{1}{3-2x} \text{ to each side}$$

$$0.8x = \ln \left(\frac{1}{3-2x} \right) \quad \text{Taking logs}$$

$$0.8x = -\ln(3-2x) \quad \text{Simplify using } \ln \left(\frac{1}{c} \right) = -\ln c$$

$$\frac{0.8x}{0.8} = -\frac{\ln(3-2x)}{0.8} \quad \text{Divide each side by 0.8}$$

$$x = -1.25 \ln(3-2x) \quad \text{Simplify } \left(\frac{1}{0.8} = 1.25 \right)$$

$$\text{So } p = -1.25$$

$$(d) x_0 = -2.6$$

$$x_1 = -1.25 \ln [3 - 2(-2.6)] = -2.630167693$$

$$x_2 = -1.25 \ln [3 - 2(-2.630167693)] = -2.639331488$$

$$x_3 = -1.25 \ln [3 - 2(-2.639331488)] = -2.642101849$$

$$\text{So } x_3 = -2.642 \text{ (3 d.p.)}$$

Solutionbank

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Exercise C, Question 7

Question:

(a) Use the iteration $x_{n+1} = (3x_n + 3)^{\frac{1}{3}}$ with $x_0 = 2$ to find, to 3 significant figures, x_4 .

The only real root of the equation $x^3 - 3x - 3 = 0$ is α . It is given that, to 3 significant figures, $\alpha = x_4$.

(b) Use the substitution $y = 3^x$ to express $27^x - 3^{x+1} - 3 = 0$ as a cubic equation.

(c) Hence, or otherwise, find an approximate solution to the equation $27^x - 3^{x+1} - 3 = 0$, giving your answer to 2 significant figures.

[E]

Solution:

(a) $x_0 = 2$

$$x_1 = [3(2) + 3]^{\frac{1}{3}} = 2.080083823$$

$$x_2 = [3(2.080083823) + 3]^{\frac{1}{3}} = 2.098430533$$

$$x_3 = [3(2.098430533) + 3]^{\frac{1}{3}} = 2.102588765$$

$$x_4 = [3(2.102588765) + 3]^{\frac{1}{3}} = 2.103528934$$

So $x_4 = 2.10$ (3 s.f.)

(b) $27^x - 3^{x+1} - 3 = 0$

$$(3^3)^x - 3(3^x) - 3 = 0$$

$$3^{3x} - 3(3^x) - 3 = 0$$

$$(3^x)^3 - 3(3^x) - 3 = 0$$

Let $y = 3^x$

then $y^3 - 3y - 3 = 0$

(c) The root of the equation $y^3 - 3y - 3 = 0$ is x_4

so $y = 2.10$ (3 s.f.)

but $y = 3^x$

so $3^x = 2.10$

$\ln 3^x = \ln 2.10$ Take logs of each side

$x \ln 3 = \ln 2.10$ Simplify using $\ln a^b = b \ln a$

$\frac{x \ln 3}{\ln 3} = \frac{\ln 2.10}{\ln 3}$ Divide each side by $\ln 3$

$x = \frac{\ln 2.10}{\ln 3}$ Simplify

$x = 0.6753\dots$

So $x = 0.68$ (2 s.f.)

Solutionbank

Edexcel AS and A Level Modular Mathematics

Exercise C, Question 8

Question:

The equation $x^x = 2$ has a solution near $x = 1.5$.

(a) Use the iteration formula $x_{n+1} = 2 \frac{1}{x_n}$ with $x_0 = 1.5$ to find the approximate solution x_5 of the equation. Show the intermediate iterations and give your final answer to 4 decimal places.

(b) Use the iteration formula $x_{n+1} = 2x_n^{(1-x_n)}$ with $x_0 = 1.5$ to find x_1, x_2, x_3, x_4 . Comment briefly on this sequence.

[E]

Solution:

$$(a) x_0 = 1.5$$

$$x_1 = 2 \frac{1}{1.5} = 1.587401052$$

$$x_2 = 2 \frac{1}{1.587401052} = 1.54752265$$

$$x_3 = 2 \frac{1}{1.54752265} = 1.565034105$$

$$x_4 = 2 \frac{1}{1.565034105} = 1.557210213$$

$$x_5 = 2 \frac{1}{1.557210213} = 1.560679241$$

So $x_5 = 1.5607$ (4 d.p.)

$$(b) x_0 = 1.5$$

$$x_1 = 2 \times (1.5)^{1-(1.5)} = 1.632993162$$

$$x_2 = 2 \times (1.632993162)^{1-(1.632993162)} = 1.466264596$$

$$x_3 = 2 \times (1.466264596)^{1-(1.466264596)} = 1.673135301$$

$$x_4 = 2 \times (1.673135301)^{1-(1.673135301)} = 1.414371012$$

The sequence x_0, x_1, x_2, x_3, x_4 gets further from the root. It is a divergent sequence.

Solutionbank

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Exercise C, Question 9

Question:

(a) Show that the equation $2^{1-x} = 4x + 1$ can be arranged in the form $x = \frac{1}{2} \left(2^{-x} \right) + q$, stating the value of the constant q .

(b) Using the iteration formula $x_{n+1} = \frac{1}{2} \left(2^{-x_n} \right) + q$ with $x_0 = 0.2$ and the value of q found in part (a), find x_1 , x_2 , x_3 and x_4 . Give the value of x_4 , to 4 decimal places.

[E]

Solution:

$$(a) 2^{1-x} = 4x + 1$$

$$4x = 2^{1-x} - 1 \quad \text{Subtract 1 from each side}$$

$$4x = 2 \left(2^{-x} \right) - 1 \quad \text{Use } 2^{a+b} = 2^a \times 2^b \text{ and } 2^1 = 2$$

$$\frac{4x}{4} = \frac{2}{4} \left(2^{-x} \right) - \frac{1}{4} \quad \text{Divide each term by 4}$$

$$x = \frac{1}{2} \left(2^{-x} \right) - \frac{1}{4} \quad \text{Simplify}$$

$$\text{So } q = -\frac{1}{4}$$

$$(b) x_0 = 0.2$$

$$x_1 = \frac{1}{2} \left(2^{-0.2} \right) - \frac{1}{4} = 0.1852752816$$

$$x_2 = \frac{1}{2} \left(2^{-0.1852752816} \right) - \frac{1}{4} = 0.1897406227$$

$$x_3 = \frac{1}{2} \left(2^{-0.1897406227} \right) - \frac{1}{4} = 0.1883816687$$

$$x_4 = \frac{1}{2} \left(2^{-0.1883816687} \right) - \frac{1}{4} = 0.1887947991$$

$$\text{So } x_4 = 0.1888 \text{ (4 d.p.)}$$

Solutionbank

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Exercise C, Question 10

Question:

The curve with equation $y = \ln(3x)$ crosses the x -axis at the point P $(p, 0)$.

(a) Sketch the graph of $y = \ln(3x)$, showing the exact value of p .

The normal to the curve at the point Q, with x -coordinate q , passes through the origin.

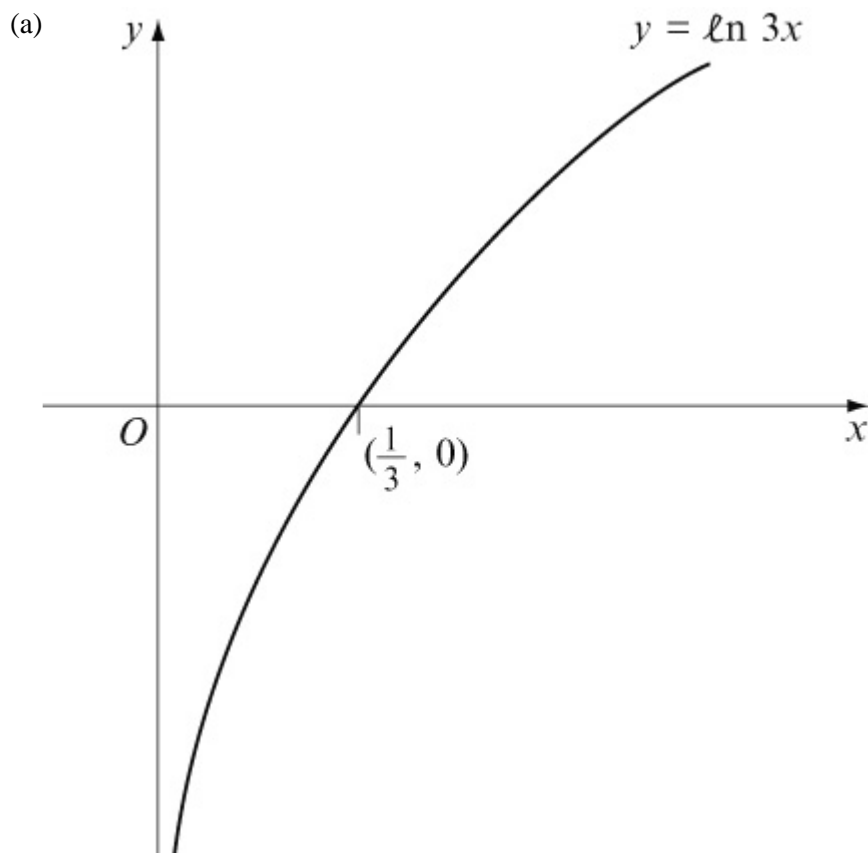
(b) Show that $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$.

(c) Show that the equation in part (b) can be rearranged in the form $x = \frac{1}{3}e^{-x^2}$.

(d) Use the iteration formula $x_{n+1} = \frac{1}{3}e^{-x_n^2}$, with $x_0 = \frac{1}{3}$, to find x_1 , x_2 , x_3 and x_4 . Hence write down, to 3 decimal places, an approximation for q .

[E]

Solution:



So $p = \frac{1}{3}$

(b)① $\frac{d}{dx} \ln 3x = \frac{1}{x}$

So the gradient of the tangent at Q is $\frac{1}{q}$.

The gradient of the normal is $-q$ (because the product of the gradients of perpendicular lines is -1).

The equation of the line with gradient $-q$ that passes through $(0, 0)$ is

$$y - y_1 = m (x - x_1)$$

$$y - 0 = -q (x - 0)$$

$$y = -qx$$

② The line $y = -qx$ meets the curve $y = \ln 3x$ when

$$\ln 3x = -qx$$

We know they meet at Q.

So, substitute $x = q$ into $\ln 3x = -qx$:

$$\ln 3q = -q(q)$$

$$\ln 3q = -q^2$$

$$q^2 + \ln 3q = 0 \quad \text{Add } q^2 \text{ to each side}$$

This is $x^2 + \ln 3x = 0$ with $x = q$

So $x = q$ is a solution of the equation $x^2 + \ln 3x = 0$

$$(c) x^2 + \ln 3x = 0$$

$$\ln 3x = -x^2 \quad \text{Subtract } x^2 \text{ from each side}$$

$$3x = e^{-x^2} \quad \text{Use } \ln a = b \Rightarrow a = e^b$$

$$x = \frac{1}{3}e^{-x^2} \quad \text{Divide each term by 3}$$

$$(d) x_0 = \frac{1}{3}$$

$$x_1 = \frac{1}{3}e^{-\left(\frac{1}{3}\right)^2} = 0.2982797723$$

$$x_2 = \frac{1}{3}e^{-\left(0.2982797723\right)^2} = 0.3049574223$$

$$x_3 = \frac{1}{3}e^{-\left(0.3049574223\right)^2} = 0.3037314616$$

$$x_4 = \frac{1}{3}e^{-\left(0.3037314616\right)^2} = 0.3039581993$$

$$\text{So } x_4 = 0.304 \text{ (3 d.p.)}$$

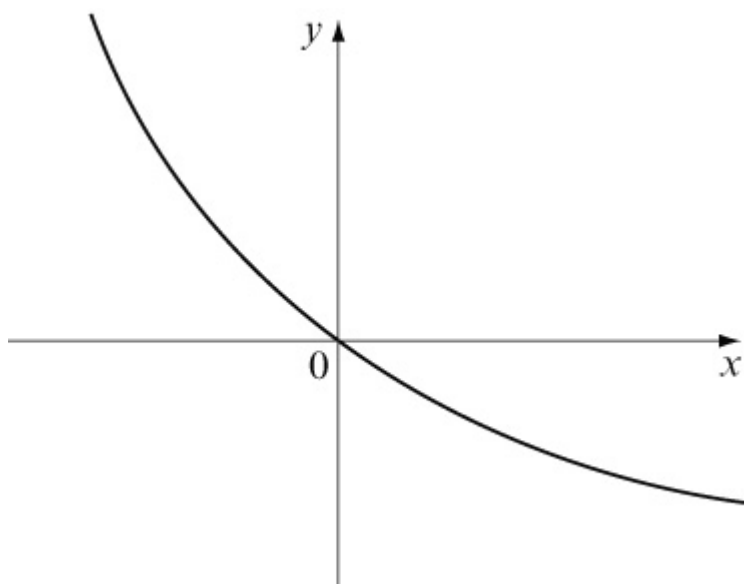
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Exercise C, Question 11

Question:

(a) Copy this sketch of the curve with equation $y = e^{-x} - 1$. On the same axes sketch the graph of $y = \frac{1}{2} \left(x - 1 \right)$, for $x \geq 1$, and $y = -\frac{1}{2} \left(x - 1 \right)$, for $x < 1$. Show the coordinates of the points where the graph meets the axes.



The x -coordinate of the point of intersection of the graphs is α .

(b) Show that $x = \alpha$ is a root of the equation $x + 2e^{-x} - 3 = 0$.

(c) Show that $-1 < \alpha < 0$.

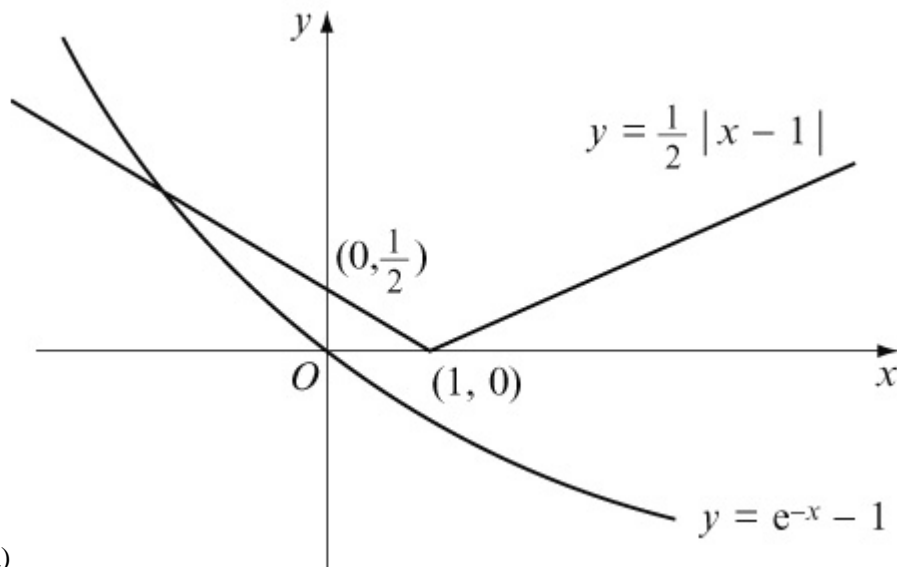
The iterative formula $x_{n+1} = -\ln \left[\frac{1}{2} \left(3 - x_n \right) \right]$ is used to solve the equation $x + 2e^{-x} - 3 = 0$.

(d) Starting with $x_0 = -1$, find the values of x_1 and x_2 .

(e) Show that, to 2 decimal places, $\alpha = -0.58$.

[E]

Solution:



① Substitute $x = 0$ into $y = \frac{1}{2} |x - 1|$:

$$y = \frac{1}{2} |-1| = \frac{1}{2}$$

So $y = \frac{1}{2} |x - 1|$ meets the y -axis at $(0, \frac{1}{2})$

② Substitute $y = 0$ into $y = \frac{1}{2} |x - 1|$:

$$\frac{1}{2} |x - 1| = 0$$

$$x = 1$$

So $y = \frac{1}{2} |x - 1|$ meets the x -axis at $(1, 0)$

(b) The equation of the branch of the curve for $x < 1$ is $y = \frac{1}{2} (1 - x)$.

This line meets the curve $y = e^{-x} - 1$ when

$$\frac{1}{2} (1 - x) = e^{-x} - 1$$

$$(1 - x) = 2(e^{-x} - 1) \quad \text{Multiply each side by 2}$$

$$1 - x = 2e^{-x} - 2 \quad \text{Simplify}$$

$$-x = 2e^{-x} - 3 \quad \text{Subtract 1 from each side}$$

$$0 = x + 2e^{-x} - 3 \quad \text{Add } x \text{ to each side}$$

$$\text{or } x + 2e^{-x} - 3 = 0$$

The line meets the curve when $x = \alpha$, so $x = \alpha$ is a root of the equation

$$x + 2e^{-x} - 3 = 0$$

$$(c) \text{ Let } f(x) = x + 2e^{-x} - 3$$

$$f(-1) = (-1) + 2e^{-(-1)} - 3 = 1.44$$

$$f(0) = (0) + 2e^{-(0)} - 3 = -1$$

$f(-1) > 0$ and $f(0) < 0$ so there is a change of sign.

\Rightarrow There is a root between $x = -1$ and $x = 0$,

i.e. $-1 < \alpha < 0$

$$(d) x_0 = -1$$

$$x_1 = -\ln \left\{ \frac{1}{2} \left[3 - \left(-1 \right) \right] \right\} = -0.6931471806$$

$$x_2 = -\ln \left\{ \frac{1}{2} \left[3 - \left(-0.6931471806 \right) \right] \right\} = -0.6133318084$$

$$(e) x_3 = -\ln \left\{ \frac{1}{2} \left[3 - \left(-0.6133318084 \right) \right] \right\} = -0.5914831048$$

$$x_4 = -\ln \left\{ \frac{1}{2} \left[3 - \left(-0.5914831048 \right) \right] \right\} = -0.5854180577$$

$$x_5 = -\ln \left\{ \frac{1}{2} \left[3 - \left(-0.5854180577 \right) \right] \right\} = -0.5837278997$$

$$x_6 = -\ln \left\{ \frac{1}{2} \left[3 - \left(-0.5837278997 \right) \right] \right\} = -0.5832563908$$

So $\alpha = -0.58$ (2 d.p.)