## Chapter review

**1 a** $f(x) = x^3 - 6x - 2$

$f(x) = 0 \Rightarrow x^3 = 6x + 2$

$x^2 = 6 + \dfrac{2}{x}$

$x = \pm\sqrt{6 + \dfrac{2}{x}}$

$a = 6,\ b = 2$

**b** $x_{n+1} = \sqrt{6 + \dfrac{2}{x_n}}$

$x_0 = 2 \Rightarrow x_1 = \sqrt{6 + \dfrac{2}{2}} = \sqrt{7} = 2.64575...$

$x_2 = \sqrt{6 + \dfrac{2}{2.64575...}} = 2.59921...$

$x_3 = \sqrt{6 + \dfrac{2}{2.59921...}} = 2.60181...$

$x_4 = \sqrt{6 + \dfrac{2}{2.60181...}} = 2.60167...$

To 4 d.p., the values are $x_1 = 2.6458$, $x_2 = 2.5992$, $x_3 = 2.6018$, $x_4 = 2.6017$.

**c** $f(2.6015) = 2.6015^3 - 6 \times 2.6015 - 2$

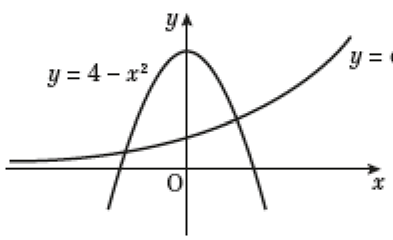
$= -0.0025...$

$f(2.6025) = 2.6025^3 - 6 \times 2.6025 - 2$

$= 0.0117...$

There is a change of sign in this interval so $\alpha = 2.602$ correct to 3 d.p.

**2 a**




**b** There is one positive and one negative root of the equation p($x$) = q($x$) at the points of intersection.

$p(x) = q(x) \Rightarrow 4 - x^2 = e^x$

i.e. $x^2 + e^x - 4 = 0$

**c** $x^2 = 4 - e^x$

$x = \pm\left(4 - e^x\right)^{\frac{1}{2}}$

**d** $x_{n+1} = -\left(4 - e^{x_n}\right)^{\frac{1}{2}}$

$x_0 = -2 \Rightarrow x_1 = -\left(4 - e^{-2}\right)^{\frac{1}{2}} = -1.96587...$

$x_2 = -\left(4 - e^{-1.96587...}\right)^{\frac{1}{2}} = -1.96467...$

$x_3 = -\left(4 - e^{-1.96467...}\right)^{\frac{1}{2}} = -1.96463...$

$x_4 = -\left(4 - e^{-1.96463...}\right)^{\frac{1}{2}} = -1.96463...$

To 4 d.p., the values are $x_1 = -1.9659$, $x_2 = -1.9647$, $x_3 = -1.9646$, $x_4 = -1.9646$.

**e** $x_0 = 1.4 \Rightarrow 4 - e^{1.4} < 0$

There can be no square root of a negative number.

**3 a** $g(x) = x^5 - 5x - 6$

$g(1) = 1 - 5 - 6 = -10$

$g(2) = 32 - 10 - 6 = 16$

There is a change of sign in the interval, so there must be a root in the interval, since f is continuous over the interval.

**b** $g(x) = 0 \Rightarrow x^5 = 5x + 6$

$x = \left(5x + 6\right)^{\frac{1}{5}}$

$p = 5,\ q = 6,\ r = 5$

**c** $x_{n+1} = \left(5x_n + 6\right)^{\frac{1}{5}}$

$x_0 = 1 \Rightarrow x_1 = \left(5 + 6\right)^{\frac{1}{5}} = 1.61539...$

$x_2 = \left(5 \times 1.61539... + 6\right)^{\frac{1}{5}} = 1.69707...$

$x_3 = \left(5 \times 1.69707... + 6\right)^{\frac{1}{5}} = 1.70681...$

To 4 d.p., the values are $x_1 = 1.6154$, $x_2 = 1.6971$, $x_3 = 1.7068$.

**d** $g(1.7075) = 1.7075^5 - 5 \times 1.7075 - 6$

$= -0.0229...$

$g(1.7085) = 1.7085^5 - 5 \times 1.7085 - 6$

$= 0.0146...$

The sign change implies there is a root in this interval so $\alpha = 1.708$ correct to 3 d.p.

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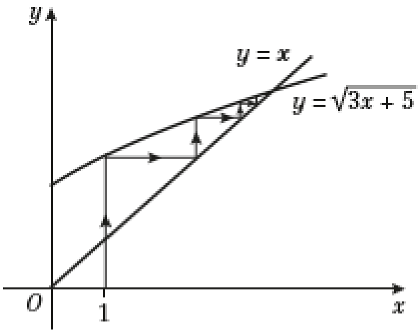
**4 a** $g(x) = x^2 - 3x - 5$

$g(x) = 0 \Rightarrow x^2 - 3x - 5 = 0$

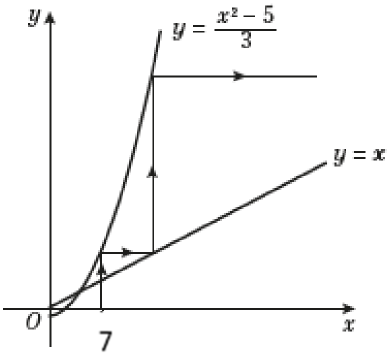
$x^2 = 3x + 5$

$x = \sqrt{3x+5}$

**b, c**




**d**




$g(x) = 0 \Rightarrow x^2 - 3x - 5 = 0$

$3x = x^2 - 5$

$x = \dfrac{x^2 - 5}{3}$

**5 a** $f(x) = 5x - 4\sin x - 2$

$f(1.1) = 5(1.1) - 4\sin(1.1) - 2$

$= -0.0648...$

$f(1.15) = 5(1.15) - 4\sin(1.15) - 2$

$= -0.0989...$

$f(1.1) < 0$ and $f(1.15) > 0$ so there is a change of sign, which implies there is a root between $x = 1.1$ and $x = 1.15$.

**b** $5x - 4\sin x - 2 = 0$

$5x - 2 = 4\sin x$ Add 4 sin $x$ to each side.

$5x = 4\sin x + 2$ Add 2 to each side.

$\dfrac{5x}{5} = \dfrac{4\sin x}{5} + \dfrac{2}{5}$ Divide each term by 5.

$x = \frac{4}{5}\sin x + \frac{2}{5}$ Simplify.

So $p = \frac{4}{5}$ and $q = \frac{2}{5}$.

**5 c** $x_0 = 1.1 \Rightarrow$

$x_1 = 0.8\sin(1.1) + 0.4 = 1.1129...$

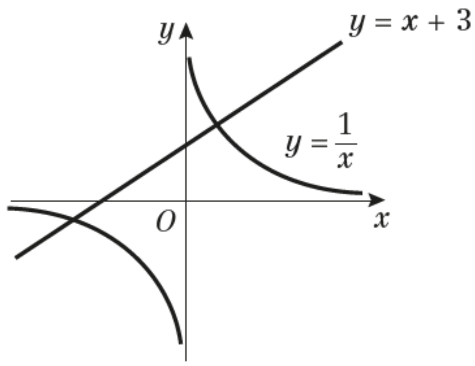
$x_2 = 0.8\sin(1.1129...) + 0.4 = 1.1176...$

$x_3 = 0.8\sin(1.1176...) + 0.4 = 1.1192...$

$x_4 = 0.8\sin(1.1192...) + 0.4 = 1.1198...$

To 3 d.p., the values are $x_1 = 1.113$, $x_2 = 1.118$, $x_3 = 1.119$, $x_4 = 1.120$.

**6 a**




**b** The line meets the curve at two points, so there are two values of $x$ that satisfy the equation $\dfrac{1}{x} = x + 3$.

So $\dfrac{1}{x} = x + 3$ has two roots.

**c** $\dfrac{1}{x} = x + 3 \Rightarrow 0 = x + 3 - \dfrac{1}{x}$

Let $f(x) = x + 3 - \dfrac{1}{x}$

$f(0.30) = (0.30) + 3 - \dfrac{1}{0.30} = -0.0333...$

$f(0.31) = (0.31) + 3 - \dfrac{1}{0.31} = 0.0841...$

$f(0.30) < 0$ and $f(0.31) > 0$ so there is a change of sign, which implies there is a root between $x = 0.30$ and $x = 0.31$.

**d** $\dfrac{1}{x} = x + 3$

$\dfrac{1}{x} \times x = x \times x + 3 \times x$ Multiply by $x$.

$1 = x^2 + 3x$

So $x^2 + 3x - 1 = 0$

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**6 e** Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ with

$a = 1, b = 3, c = -1$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9+4}}{2} = \frac{-3 \pm \sqrt{13}}{2}$$

So $x = \frac{-3+\sqrt{13}}{2} = 0.3027...$

The positive root is 0.303 to 3 d.p.

**Challenge**

**a** $f(x) = x^6 + x^3 - 7x^2 - x + 3$

$f'(x) = 6x^5 + 3x^2 - 14x - 1$

$f''(x) = 30x^4 + 6x - 14$

**i** $f''(x) = 0 \Rightarrow 6x = 14 - 30x^4$

$3x = 7 - 15x^4$

$$x = \frac{7 - 15x^4}{3}$$

**ii** $f''(x) = 0 \Rightarrow 6x = 14 - 30x^4$

$15x^4 + 3x = 7$

$x(15x^3 + 3) = 7$

$$x = \frac{7}{15x^3 + 3}$$

**iii** $f''(x) = 0 \Rightarrow 6x = 14 - 30x^4$

$15x^4 = 7 - 3x$

$$x^4 = \frac{7 - 3x}{15}$$

$$x = \sqrt[4]{\frac{7 - 3x}{15}}$$

**b** As $B$ is a point of inflection $f''(x) = 0$.

Using $x_0 = 1$ in part **iii**

$$x_1 = \sqrt[4]{\frac{4}{15}} = 0.7186...$$

$$x_2 = \sqrt[4]{\frac{7 - 3\times 0.7186...}{15}} = 0.7538...$$

$$x_3 = \sqrt[4]{\frac{7 - 3\times 0.7538...}{15}} = 0.7496...$$

$$x_4 = \sqrt[4]{\frac{7 - 3\times 0.7496...}{15}} = 0.7501...$$

$$x_5 = \sqrt[4]{\frac{7 - 3\times 0.7501...}{15}} = 0.7501...$$

Correct to 3 d.p., an approximation for the $x$-coordinate of $B$ is 0.750.

**c** $A$ has a negative $x$-coordinate. Formula **iii** gives the positive fourth root, so cannot be used to find a negative root.

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