Exercise 8B

1 a i $x^2 - 6x + 2 = 0$

$6x = x^2 + 2$ Add $6x$ to each side.

$x = \frac{x^2 + 2}{6}$ Divide each side by 6.

ii $x^2 - 6x + 2 = 0$

$x^2 + 2 = 6x$ Add $6x$ to each side.

$x^2 = 6x - 2$ Subtract 2 from each side.

$x = \sqrt{6x - 2}$ Take the square root of each side.

iii $x^2 - 6x + 2 = 0$

$x^2 + 2 = 6x$ Add $6x$ to each side.

$x^2 = 6x - 2$ Subtract 2 from each side.

$\frac{x^2}{x} = \frac{6x}{x} - \frac{2}{x}$ Divide each term by x.

$x = 6 - \frac{2}{x}$ Simplify.

b i $x_0 = 4 \Rightarrow x_1 = \frac{4^2 + 2}{6} = 3$

$x_2 = \frac{3^2 + 2}{6} = 1.83333$

$x_3 = \frac{1.83333^2 + 2}{6} = 0.89352$

$x_4 = \frac{0.89352^2 + 2}{6} = 0.46640$

$x_5 = \frac{0.46640^2 + 2}{6} = 0.36959$

$x_6 = \frac{0.36959^2 + 2}{6} = 0.35610$

$x_7 = \frac{0.35610^2 + 2}{6} = 0.35447$

$x_8 = \frac{0.35447^2 + 2}{6} = 0.35428$

$x = 0.354$ to 3 d.p.

ii $x_0 = 4 \Rightarrow x_1 = \sqrt{6x - 2} = 4.69042$

$x_2 = \sqrt{6 \times 4.69042 - 2} = 5.11297$

⋮

$x_{15} = \sqrt{6 \times 5.64547 - 2} = 5.6456$

$x = 5.646$ to 3 d.p.

iii $x_0 = 4 \Rightarrow x_1 = 6 - \frac{2}{x} = 5.5$

$x_2 = 5.5 - \frac{2}{x} = 5.63636$

$x_3 = 5.63636 - \frac{2}{x} = 5.64516$

$x_4 = 5.64516 - \frac{2}{x} = 5.64571$

$x = 5.646$ to 3 d.p.

c $x^2 - 6x + 2 = 0$

$x = \frac{6 \pm \sqrt{36 - 8}}{2} = 3 \pm \sqrt{7}$

$a = 3, b = 7$

2 a i $f(x) = 0$

$x^2 - 5x - 3 = 0$

$x^2 = 5x + 3$ Add $5x + 3$ to each side.

$x = \sqrt{5x + 3}$ Take the square root of each side.

ii $f(x) = 0$

$x^2 - 5x - 3 = 0$

$5x = x^2 - 3$ Add $5x$ to each side.

$x = \frac{x^2 - 3}{5}$ Divide each side by 5.

b i $x_0 = 5 \Rightarrow x_1 = \sqrt{5 \times 5 + 3} = \sqrt{28} = 5.2915$

$x_2 = \sqrt{5 \times 5.2915 + 3} = 5.4275$

$x_3 = \sqrt{5 \times 5.4275 + 3} = 5.4900$

$x_4 = \sqrt{5 \times 5.4900 + 3} = 5.5180$

$x = 5.5$ to 1 d.p.

2 b ii $x_0 = 5 \Rightarrow x_1 = \dfrac{5^2 - 3}{5} = 4.4$

$x_2 = \dfrac{4.4^2 - 3}{5} = 3.272$

$x_3 = \dfrac{3.272^2 - 3}{5} = 1.5412$

$x_4 = \dfrac{1.5412^2 - 3}{5} = -0.1249$

$x_5 = \dfrac{(-0.1249)^2 - 3}{5} = -0.5969$

$x_6 = \dfrac{(-0.5969)^2 - 3}{5} = -0.5287$

$x = -0.5$ to 1 d.p.

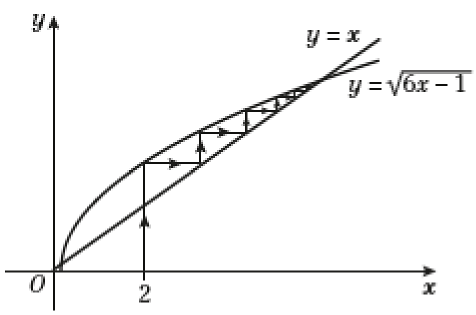
3 a $f(x) = 0$

$x^2 - 6x + 1 = 0$

$x^2 = 6x - 1$ Add $6x - 1$ to each side.

$x = \sqrt{6x-1}$ Take the square root of each side.

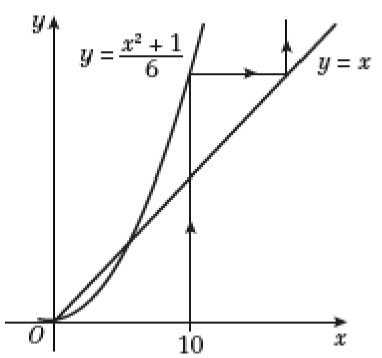
b



c The curve crosses the line twice so there are two roots.

d See diagram above.

e



4 a $f(x) = xe^{-x} - x + 2$

$f(x) = 0 \Rightarrow e^{-x} = \dfrac{x-2}{x}$

$e^x = \dfrac{x}{x-2}$

$x = \ln\left|\dfrac{x}{x-2}\right|, \ x \neq 2$

b $x_0 = -1 \Rightarrow x_1 = \ln\left|\dfrac{-1}{-3}\right| = -1.0986$

$x_2 = \ln\left|\dfrac{-1.0986}{-1.0986 - 2}\right| = -1.0369$

$x_3 = \ln\left|\dfrac{-1.0369}{-1.0369 - 2}\right| = -1.0746$

To 2 d.p., $x_1 = -1.10$, $x_2 = -1.04$, $x_3 = -1.07$.

5 a i $f(x) = 0$

$x^3 + 5x^2 - 2 = 0$

$x^3 + 5x^2 = 2$ Add 2 to each side.

$x^3 = 2 - 5x^2$ Subtract $5x^2$ from each side.

$x = \sqrt[3]{2 - 5x^2}$ Take the cube root of each side.

ii $f(x) = 0$

$x^3 + 5x^2 - 2 = 0$

$x^3 + 5x^2 = 2$ Add 2 to each side.

$x^3 = 2 - 5x^2$ Subtract $5x^2$ from each side.

$\dfrac{x^3}{x^2} = \dfrac{2}{x^2} - \dfrac{5x^2}{x^2}$ Divide each term by x^2.

$x = \dfrac{2}{x^2} - 5$ Simplify.

5 a iii $f(x) = 0$

$x^3 + 5x^2 - 2 = 0$

$x^3 + 5x^2 = 2$ Add 2 to each side.

$5x^2 = 2 - x^3$ Subtract x^3 from each side.

$x^2 = \frac{2 - x^3}{5}$ Divide each side by 5.

$x = \sqrt{\frac{2 - x^3}{5}}$ Take the square root of each side.

b $x_0 = 10 \Rightarrow x_1 = \frac{2}{10^2} - 5 = -4.98$

$x_2 = \frac{2}{4.98^2} - 5 = -4.9194$

$x_3 = \frac{2}{4.9194^2} - 5 = -4.9174$

$x_4 = \frac{2}{4.9174^2} - 5 = -4.9173$

$x = -4.917$ to 3 d.p.

c $x_0 = 1 \Rightarrow x_1 = \sqrt{\frac{2 - 1^3}{5}} = 0.4472$

$x_2 = \sqrt{\frac{2 - 0.4472^3}{5}} = 0.6182$

$x_3 = \sqrt{\frac{2 - 0.6182^3}{5}} = 0.5939$

$x_4 = \sqrt{\frac{2 - 0.5939^3}{5}} = 0.5984$

$x_5 = \sqrt{\frac{2 - 0.5984^3}{5}} = 0.5976$

$x_6 = \sqrt{\frac{2 - 0.5976^3}{5}} = 0.5978$

$x = 0.598$ to 3 d.p.

d $x_0 = 2 \Rightarrow x_1 = \sqrt{\frac{2 - 2^3}{5}} = \sqrt{-\frac{6}{5}}$

This has no real number solutions so no iteration is possible.

6 a $x^4 - 3x^3 - 6 = 0$

$3x^3 = x^4 - 6$ Add $3x^3$ to each side.

$\frac{3x^3}{3} = \frac{x^4}{3} - \frac{6}{3}$ Divide each term by 3.

$x^3 = \frac{x^4}{3} - 2$ Simplify.

$x = \sqrt[3]{\frac{x^4}{3} - 2}$ Take the cube root of each side.

So $p = \frac{1}{3}$ and $q = -2$.

b $x_0 = 0 \Rightarrow x_1 = \sqrt[3]{\frac{1}{3}0^4 - 2} = -1.25992$

$x_2 = \sqrt[3]{\frac{1}{3} \times 1.25992^4 - 2} = -1.05073$

$x_3 = \sqrt[3]{\frac{1}{3} \times 1.05073^4 - 2} = -1.16807$

To 3 d.p., $x_1 = -1.260$, $x_2 = -1.051$, $x_3 = -1.168$.

c $f(-1.1315)$

$= (-1.1315)^4 - 3 \times (-1.1315)^3 - 6$

$= -0.0148...$

$f(-1.1325)$

$= (-1.1325)^4 - 3 \times (-1.1325)^3 - 6$

$= -0.0024...$

There is a change of sign in this interval so $\alpha = 1.132$ to 3 d.p.

7 a $f(x)=3\cos(x^2)+x-2$

$f(x)=0 \Rightarrow 3\cos(x^2)=2-x$

$\cos(x^2)=\dfrac{2-x}{3}$

$x^2=\arccos\left(\dfrac{2-x}{3}\right)$

$x=\left(\arccos\left(\dfrac{2-x}{3}\right)\right)^{\frac{1}{2}}$

b $x_0=1 \Rightarrow x_1=\left(\arccos\left(\dfrac{2-1}{3}\right)\right)^{\frac{1}{2}}=1.1094$

$x_2=\left(\arccos\left(\dfrac{2-1.1094}{3}\right)\right)^{\frac{1}{2}}=1.1267$

$x_3=\left(\arccos\left(\dfrac{2-1.1267}{3}\right)\right)^{\frac{1}{2}}=1.1293$

To 3 d.p., x_1 = 1.109, x_2 = 1.127, x_3 = 1.129.

c $f(1.12975)$

$=3\cos(1.12975)^2+1.12975-2$

$=0.0004...$

$f(1.12985)$

$=3\cos(1.12985)^2+1.12985-2$

$=-0.0001...$

There is a change of sign in this interval so α = 1.1298 to 4 d.p.

8 a $f(x)=4\cot x-8x+3$

$f(0.8)=4\cot 0.8-6.4+3=0.484...$

$f(0.9)=4\cot 0.9-7.2+3=-1.025...$

There is a change of sign in the interval [0.8, 0.9], so there must be a root in this interval, since f is continuous over the interval.

b $f(x)=0$

$4\cot x-8x+3=0$

$8x=4\cot x+3=4\dfrac{\cos x}{\sin x}+3$

$x=\dfrac{\cos x}{2\sin x}+\dfrac{3}{8}$

c $x_0=0.85 \Rightarrow x_1=\dfrac{\cos 0.85}{2\sin 0.85}+\dfrac{3}{8}=0.8142$

$x_2=\dfrac{\cos 0.8142}{2\sin 0.8142}+\dfrac{3}{8}=0.8470$

$x_3=\dfrac{\cos 0.8470}{2\sin 0.8470}+\dfrac{3}{8}=0.8169$

d $f(0.8305)=4\cot 0.8305-6.644+3$

$=0.0105...$

$f(0.8315)=4\cot 0.8315-6.652+3$

$=-0.0047...$

There is a change of sign in this interval so α = 0.831 to 3 d.p.

9 a $g(x)=e^{x-1}+2x-15$

$g(x)=0 \Rightarrow e^{x-1}=15-2x$

$x-1=\ln(15-2x),\ x<\frac{15}{2}$

$x=\ln(15-2x)+1,\ x<\frac{15}{2}$

b $x_0=3 \Rightarrow x_1=\ln(15-2\times 3)+1=3.1972$

$x_2=\ln(15-2\times 3.1972)+1=3.1524$

$x_3=\ln(15-2\times 3.1524)+1=3.1628$

c $f(3.155)=e^{3.155-1}+2\times 3.155-15$

$=-0.062...$

$f(3.165)=e^{3.165-1}+2\times 3.165-15$

$=0.044...$

There is a change of sign in this interval so α = 3.16 to 2 d.p.

10 a $f(x)=xe^x-4x$

At A and B, $f(x)=0$

$xe^x-4x=0 \Rightarrow x(e^x-4)=0$

$x=0$ or $\ln 4$

Coordinates of A and B are (0, 0) and (ln 4, 0).

b $f'(x)=xe^x+e^x-4=e^x(x+1)-4$

c $f'(0.7)=e^{0.7}(0.7+1)-4=-0.5766...$

$f'(0.8)=e^{0.8}(0.8+1)-4=0.0059...$

There is a change of sign in the interval, which implies $f'(x)$ = 0, i.e. a stationary point, in this range.

10 d $f'(x) = 0 \Rightarrow e^x(x+1) - 4 = 0$

$e^x = \frac{4}{x+1}$

$x = \ln\left(\frac{4}{x+1}\right)$

e $x_0 = 0 \Rightarrow x_1 = \ln\left(\frac{4}{0+1}\right) = 1.3863$

$x_2 = \ln\left(\frac{4}{1.3863+1}\right) = 0.5166$

$x_3 = \ln\left(\frac{4}{0.5166+1}\right) = 0.9699$

$x_4 = \ln\left(\frac{4}{0.9699+1}\right) = 0.7083$

To 3 d.p., $x_1 = 1.386$, $x_2 = 0.517$, $x_3 = 0.970$, $x_4 = 0.708$.