## Chapter review

**1 a** $I=\int(2x-3)^7\,\mathrm{d}x$

Consider $y=(2x-3)^8$

$\frac{\mathrm{d}y}{\mathrm{d}x}=16(2x-3)^7$

$I=\frac{(2x-3)^8}{16}+c$

**b** $I=\int x\sqrt{4x-1}\,\mathrm{d}x$

Let $u=4x-1\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=4$

$I=\int\frac{u+1}{16}\sqrt{u}\,\mathrm{d}u$

$=\frac{2u^{\frac{5}{2}}}{80}+\frac{2u^{\frac{3}{2}}}{48}+c$

$=\frac{(4x-1)^{\frac{5}{2}}}{40}+\frac{(4x-1)^{\frac{3}{2}}}{24}+c$

**c** $I=\int\sin^2 x\cos x\,\mathrm{d}x$

Consider $y=\sin^3 x\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=3\sin^2 x\cos x$

$I=\frac{1}{3}\sin^3 x+c$

**d** $I=\int x\ln x\,\mathrm{d}x$

Let $u=\ln x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$

$\frac{\mathrm{d}v}{\mathrm{d}x}=x\Rightarrow v=\frac{x^2}{2}$

$I=\frac{x^2}{2}\ln x-\int\frac{x^2}{2}\times\frac{1}{x}\,\mathrm{d}x$

$=\frac{x^2}{2}\ln x-\frac{x^2}{4}+c$

**e** $I=\int\frac{4\sin x\cos x}{4-8\sin^2 x}\,\mathrm{d}x$

$I=\int\frac{2\sin 2x}{4(1-2\sin^2 x)}\,\mathrm{d}x$

$I=\int\frac{2\sin 2x}{4\cos 2x}\,\mathrm{d}x$

$=-\frac{1}{4}\ln|\cos 2x|+c$

**f** $I=\int\frac{1}{3-4x}\,\mathrm{d}x$

$=-\frac{1}{4}\ln|3-4x|+c$

**2 a** $I=\int_{-3}^{0}x(x^2+3)^5\,\mathrm{d}x$

Consider $y=(x^2+3)^6$

$\frac{\mathrm{d}y}{\mathrm{d}x}=12x(x^2+3)^5$

$I=\left[\frac{1}{12}(x^2+3)^6\right]_{-3}^{0}$

$=\frac{1}{12}(729-2985984)$

$=-\frac{995085}{4}$

**b** $I=\int_0^{\frac{\pi}{4}}x\sec^2 x\,\mathrm{d}x$

Let $u=x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=1$

$\frac{\mathrm{d}v}{\mathrm{d}x}=\sec^2 x\Rightarrow v=\tan x$

$I=[x\tan x]_0^{\frac{\pi}{4}}-\int_0^{\frac{\pi}{4}}\tan x\,\mathrm{d}x$

$=\frac{\pi}{4}+[\ln|\cos x|]_0^{\frac{\pi}{4}}=\frac{\pi}{4}+\ln\frac{1}{\sqrt{2}}$

$=\frac{\pi}{4}-\frac{1}{2}\ln 2$

**c** $I=\int_1^4\left(16x^{\frac{3}{2}}-\frac{2}{x}\right)\mathrm{d}x$

$=\left[\frac{32}{5}x^{\frac{5}{2}}-2\ln|x|\right]_1^4$

$=\frac{1024}{5}-2\ln 4-\frac{32}{5}$

$=\frac{992}{5}-2\ln 4$

**2 d** $I=\int_{\frac{\pi}{12}}^{\frac{\pi}{3}}(\cos x+\sin x)(\cos x-\sin x)\ \mathrm{d}x$

$=\int_{\frac{\pi}{12}}^{\frac{\pi}{3}}(\cos^2 x-\sin^2 x)\ \mathrm{d}x$

$=\int_{\frac{\pi}{12}}^{\frac{\pi}{3}}\cos 2x\ \mathrm{d}x$

$=\left[\tfrac{1}{2}\sin 2x\right]_{\frac{\pi}{12}}^{\frac{\pi}{3}}$

$=\frac{\sqrt{3}}{4}-\frac{1}{4}$

$=\frac{\sqrt{3}-1}{4}$

**e** $I=\int_1^4\frac{4}{16x^2+8x-3}\ \mathrm{d}x$

$\frac{4}{16x^2+8x-3}=\frac{4}{(4x+3)(4x-1)}$

$\frac{4}{(4x+3)(4x-1)}=\frac{A}{4x+3}+\frac{B}{4x-1}$

$4=A(4x-1)+B(4x+3)$

$x=\tfrac{1}{4}\Rightarrow 4=4B\Rightarrow B=1$

$x=-\frac{3}{4}\Rightarrow 4=-4A\Rightarrow A=-1$

$I=\int_1^4\frac{1}{4x-1}-\frac{1}{4x+3}\ \mathrm{d}x$

$=\tfrac{1}{4}\left[\ln|4x-1|-\ln|4x+3|\right]_1^4$

$=\tfrac{1}{4}(\ln 15-\ln 19-\ln 3+\ln 7)$

$=\tfrac{1}{4}\ln\tfrac{105}{57}$

$=\tfrac{1}{4}\ln\tfrac{35}{19}$

**f** $I=\int_0^{\ln 2}\frac{1}{1+\mathrm{e}^x}\ \mathrm{d}x$

Let $u=1+\mathrm{e}^x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x=u-1$

$I=\int_2^3\frac{1}{(u-1)u}\ \mathrm{d}u$

$I=\int_2^3\left(\frac{1}{(u-1)}-\frac{1}{u}\right)\ \mathrm{d}u$

$=\left[\ln|u-1|-\ln|u|\right]_2^3$

$=\ln 2-\ln 3-\ln 1+\ln 2$

$=\ln\tfrac{4}{3}$

**3 a** $I=\int_1^{\mathrm{e}}\frac{1}{x^2}\ln x\,\mathrm{d}x$

$u=\ln x\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$

$\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{1}{x^2}\Rightarrow v=-\frac{1}{x}$

$\therefore I=\left[-\frac{1}{x}\ln x\right]_1^{\mathrm{e}}-\int_1^{\mathrm{e}}\left(-\frac{1}{x^2}\right)\mathrm{d}x$

$=\left(-\frac{1}{\mathrm{e}}\right)-(0)+\left[-\frac{1}{x}\right]_1^{\mathrm{e}}$

$=-\frac{1}{\mathrm{e}}+\left(-\frac{1}{\mathrm{e}}\right)-(-1)$

$=1-\frac{2}{\mathrm{e}}$

**b** $\frac{1}{(x+1)(2x-1)}\equiv\frac{A}{x+1}+\frac{B}{2x-1}$

$\Rightarrow 1\equiv A(2x-1)+B(x+1)$

$x-\tfrac{1}{2}\Rightarrow 1=\tfrac{3}{2}B\Rightarrow B=\tfrac{2}{3}$

$x=-1\Rightarrow 1=-3A\Rightarrow A=-\tfrac{1}{3}$

$\therefore\int_1^p\frac{1}{(x+1)(2x-1)}\mathrm{d}x=\int_1^p\left(\frac{\frac{2}{3}}{2x-1}+\frac{-\frac{1}{3}}{x+1}\right)\mathrm{d}x$

$=\left[\tfrac{1}{3}\ln|2x-1|-\tfrac{1}{3}\ln|x+1|\right]_1^p$

$=\left[\frac{1}{3}\ln\left|\frac{2x-1}{x+1}\right|\right]_1^p$

$=\frac{1}{3}\ln\left(\frac{2p-1}{p+1}\right)-\left(\frac{1}{3}\ln\frac{1}{2}\right)$

$=\frac{1}{3}\ln\left(\frac{4p-2}{p+1}\right)$

**4** $\int_{\frac{1}{2}}^{b}\left(\frac{2}{x^3}-\frac{1}{x^2}\right)\ \mathrm{d}x=\frac{9}{4}$

$\left[-\frac{1}{x^2}+\frac{1}{x}\right]_{\frac{1}{2}}^{b}=\frac{9}{4}$

$-\frac{1}{b^2}+\frac{1}{b}+4-2=\frac{9}{4}$

$\frac{b-1}{b^2}=\frac{1}{4}$

$b^2-4b+4=0$

$(b-2)^2=0$

$b=2$

**5** $I = \int_0^{\theta} \cos x \sin^3 x \, dx = \frac{9}{64}$

$\left[\frac{\sin^4 x}{4}\right]_0^{\theta} = \frac{9}{64}$

$\sin^4 \theta = \frac{9}{16}$

$\sin \theta = \pm \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

**Challenge**

$\int_{\frac{\pi}{4k}}^{\frac{\pi}{3k}} (1 - \pi \sin kx) dx = \left[x + \frac{\pi}{k} \cos kx\right]_{\frac{\pi}{4k}}^{\frac{\pi}{3k}}$

$= \left(\frac{\pi}{3k} + \frac{\pi}{k} \cos \frac{\pi}{3}\right) - \left(\frac{\pi}{4k} + \frac{\pi}{k} \cos \frac{\pi}{4}\right)$

$= \left(\frac{\pi}{3k} + \frac{\pi}{2k}\right) - \left(\frac{\pi}{4k} + \frac{\pi}{\sqrt{2}k}\right)$

$= \frac{\pi}{k}\left(\frac{1}{3} + \frac{1}{2}\right) - \left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = \frac{\pi}{k}\left(\frac{7}{12} - \frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{k}\left(\frac{7}{12} - \frac{\sqrt{2}}{2}\right) = \pi(7 - 6\sqrt{2})$

$\frac{\pi}{k}\left(\frac{7 - 6\sqrt{2}}{12}\right) = \pi(7 - 6\sqrt{2})$

$k = \frac{1}{12}$