Exercise 7A

1 a $\int\left(3\sec^2 x+\frac{5}{x}+\frac{2}{x^2}\right)dx$

$=\int\left(3\sec^2 x+\frac{5}{x}+2x^{-2}\right)dx$

$=3\tan x+5\ln|x|-\frac{2}{x}+c$

b $\int(5e^x-4\sin x+2x^3)dx$

$=5e^x+4\cos x+\frac{2x^4}{4}+c$

$=5e^x+4\cos x+\frac{x^4}{2}+c$

c $\int 2(\sin x-\cos x+x)dx$

$=\int(2\sin x-2\cos x+2x)dx$

$=-2\cos x-2\sin x+x^2+c$

d $\int\left(3\sec x\tan x-\frac{2}{x}\right)dx$

$=3\sec x-2\ln|x|+c$

e $\int\left(5e^x+4\cos x-\frac{2}{x^2}\right)dx$

$=\int(5e^x+4\cos x-2x^{-2})dx$

$=5e^x+4\sin x+\frac{2}{x}+c$

f $\int\left(\frac{1}{2x}+2\operatorname{cosec}^2 x\right)dx$

$=\int\left(\frac{1}{2}\times\frac{1}{x}+2\operatorname{cosec}^2 x\right)dx$

$=\frac{1}{2}\ln|x|-2\cot x+c$

g $\int\left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)dx$

$=\int\left(\frac{1}{x}+x^{-2}+x^{-3}\right)dx$

$=\ln|x|+\frac{x^{-1}}{-1}+\frac{x^{-2}}{-2}+c$

$=\ln|x|-\frac{1}{x}-\frac{1}{2x^2}+c$

h $\int(e^x+\sin x+\cos x)dx$

$=e^x-\cos x+\sin x+c$

i $\int(2\operatorname{cosec} x\cot x-\sec^2 x)dx$

$=-2\operatorname{cosec} x-\tan x+c$

j $\int\left(e^x+\frac{1}{x}-\operatorname{cosec}^2 x\right)dx$

$=e^x+\ln|x|+\cot x+c$

2 a $\int\left(\frac{1}{\cos^2 x}+\frac{1}{x^2}\right)dx$

$=\int(\sec^2 x+x^{-2})dx$

$=\tan x-\frac{1}{x}+c$

b $\int\left(\frac{\sin x}{\cos^2 x}+2e^x\right)dx$

$=\int(\tan x\sec x+2e^x)dx$

$=\sec x+2e^x+c$

c $\int\left(\frac{1+\cos x}{\sin^2 x}+\frac{1+x}{x^2}\right)dx$

$=\int(\operatorname{cosec}^2 x+\cot x\operatorname{cosec} x+x^{-2}+x^{-1})dx$

$=-\cot x-\operatorname{cosec} x-\frac{1}{x}+\ln|x|+c$

d $\int\left(\frac{1}{\sin^2 x}+\frac{1}{x}\right)dx$

$=\int\left(\operatorname{cosec}^2 x+\frac{1}{x}\right)dx$

$=-\cot x+\ln|x|+c$

2 e $\int \sin x(1+\sec^2 x)\mathrm{d}x$

$=\int (\sin x+\sin x\sec^2 x)\mathrm{d}x$

$=\int (\sin x+\tan x\sec x)\mathrm{d}x$

$=-\cos x+\sec x+c$

f $\int \cos x(1+\mathrm{cosec}^2 x)\mathrm{d}x$

$=\int (\cos x+\cos x\,\mathrm{cosec}^2 x)\mathrm{d}x$

$=\int (\cos x+\cot x\,\mathrm{cosec}\,x)\mathrm{d}x$

$=\sin x-\mathrm{cosec}\,x+c$

g $\int \mathrm{cosec}^2 x(1+\tan^2 x)\mathrm{d}x$

$=\int (\mathrm{cosec}^2 x+\mathrm{cosec}^2 x\tan^2 x)\mathrm{d}x$

$=\int (\mathrm{cosec}^2 x+\sec^2 x)\mathrm{d}x$

$=-\cot x+\tan x+c$

h $\int \sec^2 x(1-\cot^2 x)\mathrm{d}x$

$=\int (\sec^2 x-\sec^2 x\cot^2 x)\mathrm{d}x$

$=\int (\sec^2 x-\mathrm{cosec}^2 x)\mathrm{d}x$

$=\tan x+\cot x+c$

i $\int \sec^2 x(1+\mathrm{e}^x\cos^2 x)\mathrm{d}x$

$=\int (\sec^2 x+\mathrm{e}^x\cos^2 x\sec^2 x)\mathrm{d}x$

$=\int (\sec^2 x+\mathrm{e}^x)\mathrm{d}x$

$=\tan x+\mathrm{e}^x+c$

j $\int \left(\frac{1+\sin x}{\cos^2 x}+\cos^2 x\sec x\right)\mathrm{d}x$

$=\int (\sec^2 x+\tan x\sec x+\cos x)\mathrm{d}x$

$=\tan x+\sec x+\sin x+c$

3 a $\int_3^7 2\mathrm{e}^x\mathrm{d}x=\left[2\mathrm{e}^x\right]_3^7=2\mathrm{e}^7-2\mathrm{e}^3$

b $\int_1^6 \frac{1+x}{x^3}\mathrm{d}x=\int_1^6\left(\frac{1}{x^3}+\frac{1}{x^2}\right)\mathrm{d}x$

$=\left[-\frac{1}{2x^2}-\frac{1}{x}\right]_1^6=\left(-\frac{1}{72}-\frac{1}{6}\right)-\left(-\frac{1}{2}-1\right)$

$=-\frac{13}{72}+\frac{108}{72}=\frac{95}{72}$

c $\int_{\frac{\pi}{2}}^{\pi} -5\sin x\mathrm{d}x=\left[5\cos x\right]_{\frac{\pi}{2}}^{\pi}$

$=5\cos\pi-5\cos\frac{\pi}{2}=-5-0=-5$

d $\int_{-\frac{\pi}{4}}^{0} \sec x(\sec x+\tan x)\mathrm{d}x$

$=\int_{-\frac{\pi}{4}}^{0}(\sec^2 x+\sec x\tan x)\mathrm{d}x$

$=\left[\tan x+\sec x\right]_{-\frac{\pi}{4}}^{0}=(0+1)-\left(-1+\sqrt{2}\right)$

$=2-\sqrt{2}$

4 $\int_a^{2a}\frac{3x-1}{x}\mathrm{d}x=\int_a^{2a}3-\frac{1}{x}\mathrm{d}x=\left[3x-\ln|x|\right]_a^{2a}$

$=\left(6a-\ln 2a\right)-\left(3a-\ln a\right)$ (a is positive)

$=3a-\ln 2a+\ln a$

$=3a-(\ln 2+\ln a)+\ln a$

$=3a-\ln 2$

$=3a+\ln\left(\frac{1}{2}\right)=6+\ln\left(\frac{1}{2}\right)$

so $a=2$.

5 $\int_{\ln 1}^{\ln a}\mathrm{e}^x+\mathrm{e}^{-x}\mathrm{d}x=\left[\mathrm{e}^x-\mathrm{e}^{-x}\right]_{\ln 1}^{\ln a}$

$=(\mathrm{e}^{\ln a}-\mathrm{e}^{-\ln a})-(\mathrm{e}^{\ln 1}-\mathrm{e}^{-\ln 1})$

$=\left(a-\frac{1}{a}\right)-(1-1)$

So $a-\frac{1}{a}=\frac{48}{7}$

$7a^2-48a-7=0$

$(7a+1)(a-7)=0$

$a=7$ since $a>0$.

6 $\int_2^b(3\mathrm{e}^x+6\mathrm{e}^{-2x})\mathrm{d}x=\left[3\mathrm{e}^x-3\mathrm{e}^{-2x}\right]_2^b$

$=3((\mathrm{e}^b-\mathrm{e}^{-2b})-(\mathrm{e}^2-\mathrm{e}^{-4}))=0$

$(\mathrm{e}^b-\mathrm{e}^{-2b})-(\mathrm{e}^2-\mathrm{e}^{-4})=0$

so $b=2$.

7 a $\mathrm{f}(x)=\frac{1}{8}x^{\frac{3}{2}}-\frac{4}{x}=0$

$\frac{1}{8}x^{\frac{5}{2}}-4=0$ since $x\neq 0$

$x^{\frac{5}{2}}=32\Rightarrow x=4$

7 b $\int\left(\frac{1}{8}x^{\frac{3}{2}}-\frac{4}{x}\right)\mathrm{d}x=\frac{x^{\frac{5}{2}}}{20}-4\ln|x|+c$

c $\int_1^4 \mathrm{f}(x)\mathrm{d}x=\left[\frac{x^{\frac{5}{2}}}{20}-4\ln|x|\right]_1^4$

$=\left(\frac{32}{20}-4\ln 4\right)-\left(\frac{1}{20}-4\ln 1\right)$

$=\frac{31}{20}-4\ln 4$