Exercise 5D

1 a When $S = 4 \times 7^x$
$\log S = \log (4 \times 7^x)$
$\log S = \log 4 + \log 7^x$
$\log S = \log 4 + x \log 7$

b $\log S = x \log 7 + \log 4$
Gradient = $\log 7$
Intercept = $\log 4$

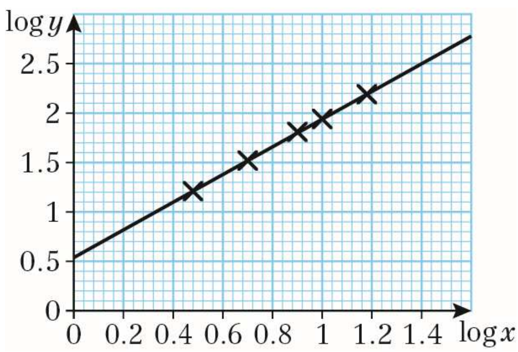
2 a When $A = 6x^4$
$\log A = \log (6x^4)$
$\log A = \log 6 + \log x^4$
$\log A = \log 6 + 4 \log x$

b $\log A = 4 \log x + \log 6$
Gradient = 4
Intercept = $\log 6$

3 a

$\log x$	0.48	0.70	0.90	1	1.18
$\log y$	1.21	1.52	1.81	1.94	2.19

b



c $y = ax^n$
$\log y = \log (ax^n)$
$\log y = \log a + \log x^n$
$\log y = \log a + n \log x$
$\log y = n \log x + \log a$
Gradient = n
Intercept = $\log a$
Calculating the gradient from the table,

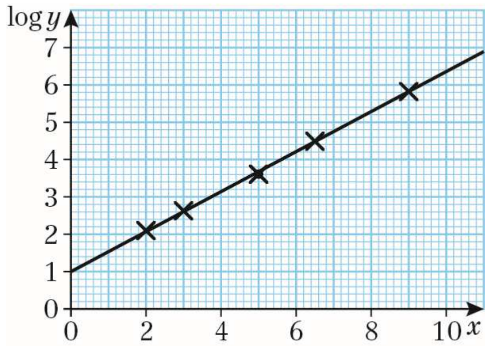
$$n = \frac{2.19 - 1.21}{1.18 - 0.48} = \frac{0.98}{0.7} = 1.4$$

Reading the intercept from the graph,
$\log a = 0.55$
$a = 10^{0.55} = 3.548...$
$a = 3.5$, $n = 1.4$

4 a

x	2	3	5	6.5	9
$\log y$	2.10	2.63	3.61	4.49	5.82

b



c $y = ab^x$
$\log y = \log (ab^x)$
$\log y = \log a + \log b^x$
$\log y = \log a + x \log b$
$\log y = x \log b + \log a$
Gradient = $\log b$
Intercept = $\log a$
Calculating the gradient from the table and the graph,

$$\log b = \frac{5.82 - 1}{9 - 0} = \frac{4.82}{9} = 0.53555...$$

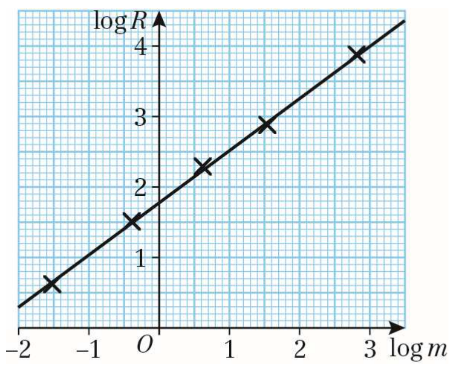
$$b = 10^{0.53555...} = 3.43...$$

Reading the intercept from the graph,
$\log a = 1$
$a = 10^1 = 10$
$a = 10$, $b = 3.4$

5 a

$\log m$	−1.52	−0.39	0.62	1.54	2.81
$\log R$	0.62	1.51	2.29	2.88	3.88

b



5 c $R = am^b$

$\log R = \log (am^b)$

$\log R = \log a + \log m^b$

$\log R = \log a + b \log m$

Gradient = b

Intercept = $\log a$

Calculating the gradient from the table,

$$b = \frac{3.88 - 0.62}{2.81 - (-1.52)} = \frac{3.26}{4.33} = 0.75288...$$

Reading the intercept from the graph,

$\log a = 1.78$

$a = 10^{1.78} = 60.255...$

$a = 60,\ b = 0.75$

d $R = 60m^{0.75}$

When $m = 80$

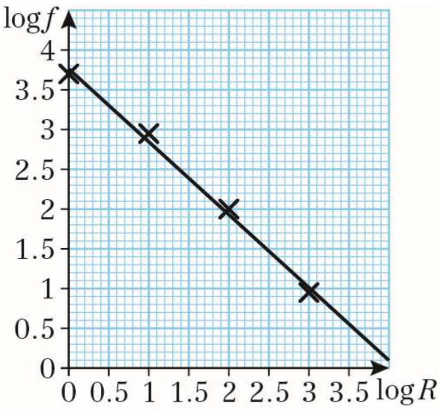
$R = 60(80)^{0.75} = 1604.97...$

1605 kcal/day

6 a

$\log R$	0	1	2	3
$\log f$	3.69	2.94	1.96	0.95

b



c $f = AR^b$

$\log f = \log (AR^b)$

$\log f = \log A + \log R^b$

$\log f = \log A + b \log R$

$\log y = b \log R + \log A$

Gradient = b

Intercept = $\log A$

Calculating the gradient from the table,

$$b = \frac{0.95 - 3.69}{3 - 0} = \frac{-2.74}{3} = -0.91...$$

Reading the intercept from the graph,

$\log A = 3.76$

$A = 10^{3.76} = 5754.39...$

$A = 5800,\ b = -0.9$

6 d $f = 5800R^{-0.9}$ per 100 000 words

When $R = 57$

$f = 152.45...$

For 455 125 words, $4.55125 \times f = 693.85...$

694 times

7 a

t	0	10	20	30	40	50
$\log P$	0.88	0.98	1.08	1.13	1.26	1.37

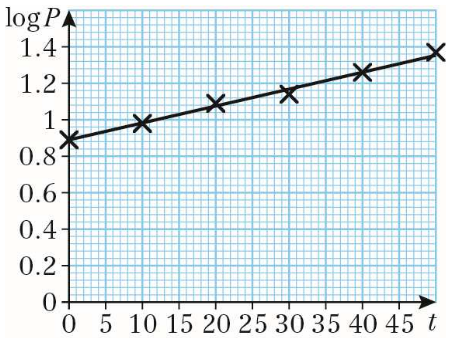
b When $P = ab^t$

$\log P = \log (ab^t)$

$\log P = \log a + \log b^t$

$\log P = \log a + t \log b$

c



d Gradient = $\log b$

Intercept = $\log a$

Calculating the gradient from the table,

$$\log b = \frac{1.37 - 0.88}{50 - 0} = \frac{0.49}{50} = 0.0098$$

$$b = 10^{0.0098} = 1.0228...$$

Reading the intercept from the graph,

$\log a = 0.88$

$a = 10^{0.88} = 7.5857...$

$a = 7.59,\ b = 1.03$

e The rate of growth is often proportional to the size of the population

8 a $N = ab^t$

$\log N = \log (ab^t)$

$\log N = \log a + \log b^t$

$\log N = \log a + t \log b$

$$\text{Gradient} = \frac{2.55 - 1.6}{10 - 0} = \frac{0.95}{10} = 0.095$$

Intercept = 1.6

$\log N = 0.095t + 1.6$

8 **b** $\log a = 1.6$
$a = 10^{1.6} = 39.8...$
$\log b = 0.095$
$b = 10^{0.095} = 1.2445...$
$a = 40$, $b = 1.2$

c a is the initial number of sick people

d $N = ab^t$
$N = 40(1.2)^{30} = 9495.052 = 9500$ (2 s.f.)
After 30 days people may start to recover, or the disease may stop spreading as quickly.

9 **a** $A = pw^q$
$\log A = m \log w + c$
Intercept = −0.1049
Gradient = 2
$\log A = 2 \log w - 0.1049$

b $A = pw^q$
$\log A = \log (pw^q)$
$\log A = \log p + \log w^q$
$\log A = \log p + q \log w$
Equating coefficients
$q = 2$
$\log p = -0.1049$
$p = 10^{-0.1049}$
$p = 0.785416...$

c The shapes are circles.
Multiply p by 4
$4p = 3.1416... \approx \pi$
So p is approximately $\frac{1}{4}$ of π

So $A = \dfrac{\pi}{4} w^2$

The width is the diameter of the circle

so $A = \dfrac{\pi}{4}(2r)^2 = \pi r^2$