Exercise 5C

1 a When $e^x = 6$
$\ln(e^x) = \ln 6$
$x = \ln 6$

b When $e^{2x} = 11$
$\ln(e^{2x}) = \ln 11$
$2x = \ln 11$
$x = \frac{1}{2}\ln 11$

c When $e^{-x+3} = 20$
$\ln(e^{-x+3}) = \ln 20$
$-x + 3 = \ln 20$
$x = 3 - \ln 20$

d When $3e^{4x} = 1$
$e^{4x} = \frac{1}{3}$
$\ln(e^{4x}) = \ln\frac{1}{3}$
$4x = \ln\frac{1}{3}$
$x = \frac{1}{4}\ln\frac{1}{3}$

e When $e^{2x+6} = 3$
$\ln(e^{2x+6}) = \ln 3$
$2x + 6 = \ln 3$
$x = \ln 3 - 6$
$x = \frac{1}{2}\ln 3 - 3$

f When $e^{5-x} = 19$
$\ln(e^{5-x}) = \ln 19$
$5 - x = \ln 19$
$x = 5 - \ln 19$

2 a When $\ln x = 2$
$e^{\ln x} = e^2$
$x = e^2$

b When $\ln(4x) = 1$
$e^{\ln(4x)} = e^1$
$4x = e^1$
$x = \frac{e}{4}$

c When $\ln(2x+3) = 4$
$e^{\ln(2x+3)} = e^4$
$2x + 3 = e^4$
$2x = e^4 - 3$
$x = \frac{1}{2}e^4 - \frac{3}{2}$

2 d When $2\ln(6x-2) = 5$
$\ln(6x-2) = \frac{5}{2}$
$e^{\ln(6x-2)} = e^{\frac{5}{2}}$
$6x - 2 = e^{\frac{5}{2}}$
$6x = e^{\frac{5}{2}} + 2$
$x = \frac{1}{6}(e^{\frac{5}{2}} + 2)$

e When $\ln(18 - x) = \frac{1}{2}$
$e^{\ln(18-x)} = e^{\frac{1}{2}}$
$18 - x = e^{\frac{1}{2}}$
$x = 18 - e^{\frac{1}{2}}$

f When $\ln(x^2 - 7x + 11) = 0$
$e^{\ln(x^2-7x+11)} = e^0$
$x^2 - 7x + 11 = 1$
$x^2 - 7x + 10 = 0$
$(x-2)(x-5) = 0$
$x = 2$ or $x = 5$

3 a $e^{2x} - 8e^x + 12 = 0$
Let $u = e^x$
$u^2 - 8u + 12 = 0$
$(u-2)(u-6) = 0$
$u = 2$ or $u = 6$
$e^x = 2$ or $e^x = 6$

When $e^x = 2$
$\ln(e^x) = \ln 2$
$x = \ln 2$
When $e^x = 6$
$\ln(e^x) = \ln 6$
$x = \ln 6$

$x = \ln 2$ or $x = \ln 6$

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3 b $e^{4x} - 3e^{2x} + 2 = 0$

Let $u = e^{2x}$

$u^2 - 3u + 2 = 0$

$(u-1)(u-2) = 0$

$u = 1$ or $u = 2$

$e^{2x} = 1$ or $e^{2x} = 2$

When $e^{2x} = 1$

$\ln(e^{2x}) = \ln 1$

$2x = 0$

$x = 0$

When $e^{2x} = 2$

$\ln(e^{2x}) = \ln 2$

$2x = \ln 2$

$x = \frac{1}{2}\ln 2$

$x = 0$ or $x = \frac{1}{2}\ln 2$

c $(\ln x)^2 + 2\ln x - 15 = 0$

Let $u = \ln x$

$u^2 + 2u - 15 = 0$

$(u+5)(u-3) = 0$

$u = -5$ or $u = 3$

When $\ln x = -5$

$e^{\ln x} = e^{-5}$

$x = e^{-5}$

When $\ln x = 3$

$e^{\ln x} = e^3$

$x = e^3$

$x = e^{-5}$ or $x = e^3$

d $e^x - 5 + 4e^{-x} = 0$

Multiply each term by e^x

$e^{2x} - 5e^x + 4 = 0$

Let $u = e^x$

$u^2 - 5u + 4 = 0$

$(u-1)(u-4) = 0$

$u = 1$ or $u = 4$

$e^x = 1$ or $e^x = 4$

When $e^x = 1$

$\ln(e^x) = \ln 1$

$x = 0$

When $e^x = 4$

$\ln(e^x) = \ln 4$

$x = \ln 4$

$x = 0$ or $x = \ln 4$

3 e $3e^{2x} - 16e^x + 5 = 0$

Let $u = e^x$

$3u^2 - 16u + 5 = 0$

$(3u-1)(u-5) = 0$

$u = \frac{1}{3}$ or $u = 5$

$e^x = \frac{1}{3}$ or $e^x = 5$

When $e^x = \frac{1}{3}$

$\ln(e^x) = \ln \frac{1}{3}$

$x = \ln \frac{1}{3}$

When $e^x = 5$

$\ln(e^x) = \ln 5$

$x = \ln 5$

$x = \ln \frac{1}{3}$ or $x = \ln 5$

f $(\ln x)^2 - 4\ln x - 12 = 0$

Let $u = \ln x$

$u^2 - 4u - 12 = 0$

$(u+2)(u-6) = 0$

$u = -2$ or $u = 6$

When $\ln x = -2$

$e^{\ln x} = e^{-2}$

$x = e^{-2}$

When $\ln x = 6$

$e^{\ln x} = e^6$

$x = e^6$

$x = e^{-2}$ or $x = e^6$

4 $e^x - 7 + 12e^{-x} = 0$

Multiply each term by e^x

$e^{2x} - 7e^x + 12 = 0$

Let $u = e^x$

$u^2 - 7u + 12 = 0$

$(u-3)(u-4) = 0$

$u = 3$ or $u = 4$

$e^x = 3$ or $e^x = 4$

When $e^x = 3$

$\ln(e^x) = \ln 3$

$x = \ln 3$

When $e^x = 4$

$\ln(e^x) = \ln 4$

$x = \ln 2^2$

$x = 2\ln 2$

$x = \ln 3$ or $x = 2\ln 2$

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5 a When $\ln(8x - 3) = 2$

$e^{\ln(8x-3)} = e^2$

$8x - 3 = e^2$

$8x = e^2 + 3$

$x = \frac{1}{8}(e^2 + 3)$

b When $e^{5(x-8)} = 3$

$\ln(e^{5(x-8)}) = \ln 3$

$5(x - 8) = \ln 3$

$x - 8 = \frac{1}{5}\ln 3$

$x = \frac{1}{5}\ln 3 + 8$

c $e^{10x} - 8e^{5x} + 7 = 0$

Let $u = e^{5x}$

$u^2 - 8u + 7 = 0$

$(u - 1)(u - 7) = 0$

$u = 1$ or $u = 7$

$e^{5x} = 1$ or $e^{5x} = 7$

When $e^{5x} = 1$

$\ln(e^{5x}) = \ln 1$

$5x = 0$

$x = 0$

When $e^{5x} = 7$

$\ln(e^{5x}) = \ln 7$

$5x = \ln 7$

$x = \frac{1}{5}\ln 7$

$x = 0$ or $x = \frac{1}{5}\ln 7$

d When $(\ln x - 1)^2 = 4$

$(\ln x)^2 - 2\ln x - 3 = 0$

Let $u = \ln x$

$u^2 - 2u - 3 = 0$

$(u + 1)(u - 3) = 0$

$u = -1$ or $u = 3$

When $\ln x = -1$

$e^{\ln x} = e^{-1}$

$x = e^{-1}$

When $\ln x = 3$

$e^{\ln x} = e^3$

$x = e^3$

$x = e^{-1}$ or $x = e^3$

6 When $3^x e^{4x-1} = 5$

$\ln(3^x e^{4x-1}) = \ln 5$

$\ln(3^x) + \ln(e^{4x-1}) = \ln 5$

$x\ln 3 + 4x - 1 = \ln 5$

$x\ln 3 + 4x = 1 + \ln 5$

$x(\ln 3 + 4) = 1 + \ln 5$

$x = \frac{1+\ln 5}{4+\ln 3}$

7 a $D = 6$ when $t = 0$ so 6 is the initial concentration of the drug in mg/l.

b $D = 6e^{\frac{-t}{10}}$

When $t = 2$

$D = 6e^{\frac{-2}{10}}$

$D = 4.91$ mg/l (3 s.f.)

c When $6e^{\frac{-t}{10}} = 3$

$e^{\frac{-t}{10}} = \frac{1}{2}$

$\ln e^{\frac{-t}{10}} = \ln\frac{1}{2}$

$-\frac{1}{10}t = \ln\frac{1}{2}$

$t = -10\ln\frac{1}{2}$

$t = 6.931471...$

$t = 6$ hours and 55.888... minutes

$t = 6$ hours and 56 minutes

8 a A is where $x = 0$

Substitute $x = 0$ into

$y = 3 + \ln(4 - x)$ to give

$y = 3 + \ln 4$

$A = (0, 3 + \ln 4)$

b B is where $y = 0$

Substitute $y = 0$ into

$y = 3 + \ln(4 - x)$ to give

$0 = 3 + \ln(4 - x)$

$-3 = \ln(4 - x)$

$e^{-3} = 4 - x$

$x = 4 - e^{-3}$

$B = (4 - e^{-3}, 0)$

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Challenge

$g(0) = Ae^{B \times 0} + C = 5$
$A + C = 5$
As $y = 2$ is an asymptote, $C = 2$
$A = 3$ and $g(6) = 3e^{B \times 6} + 2 = 10$
$3e^{6B} = 8$
$e^{6B} = \frac{8}{3}$
$\ln(e^{6B}) = \ln\frac{8}{3}$
$6B = \ln\frac{8}{3}$
$B = \frac{1}{6}\ln\frac{8}{3}$

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