

Chapter review

$$\begin{aligned}
 1 \text{ a } \quad & \sin 40^\circ \cos 10^\circ - \cos 40^\circ \sin 10^\circ \\
 & = \sin(40^\circ - 10^\circ) = \sin 30^\circ = \frac{1}{2}
 \end{aligned}$$

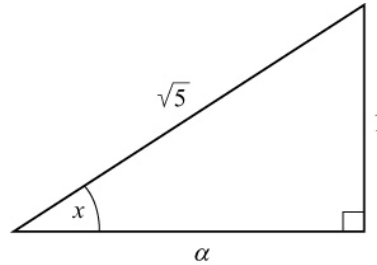
$$\begin{aligned}
 \text{b } \quad & \frac{1}{\sqrt{2}} \cos 15^\circ - \frac{1}{\sqrt{2}} \sin 15^\circ \\
 & = \cos 45^\circ \cos 15^\circ - \sin 45^\circ \sin 15^\circ \\
 & = \cos(45^\circ + 15^\circ) = \cos 60^\circ = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \quad & \frac{1 - \tan 15^\circ}{1 + \tan 15^\circ} = \frac{\tan 45^\circ - \tan 15^\circ}{1 + \tan 45^\circ \tan 15^\circ} \\
 & = \tan(45^\circ - 15^\circ) = \tan 30^\circ = \frac{\sqrt{3}}{3}
 \end{aligned}$$

$$\begin{aligned}
 2 \text{ As } \cos(x - y) &= \sin y \\
 \cos x \cos y + \sin x \sin y &= \sin y \quad (1)
 \end{aligned}$$

Draw a right-angled triangle,

$$\text{where } \sin x = \frac{1}{\sqrt{5}}$$



Using Pythagoras' theorem,

$$a^2 = (\sqrt{5})^2 - 1 = 4 \Rightarrow a = 2$$

$$\text{So } \cos x = \frac{2}{\sqrt{5}}$$

Substitute into (1):

$$\frac{2}{\sqrt{5}} \cos y + \frac{1}{\sqrt{5}} \sin y = \sin y$$

$$\Rightarrow 2 \cos y + \sin y = \sqrt{5} \sin y$$

$$\Rightarrow 2 \cos y = \sin y (\sqrt{5} - 1)$$

$$\Rightarrow \frac{2}{\sqrt{5} - 1} = \tan y \quad \left(\tan y = \frac{\sin y}{\cos y} \right)$$

$$\begin{aligned}
 \Rightarrow \tan y &= \frac{2(\sqrt{5} + 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)} \\
 &= \frac{2(\sqrt{5} + 1)}{4} = \frac{\sqrt{5} + 1}{2}
 \end{aligned}$$

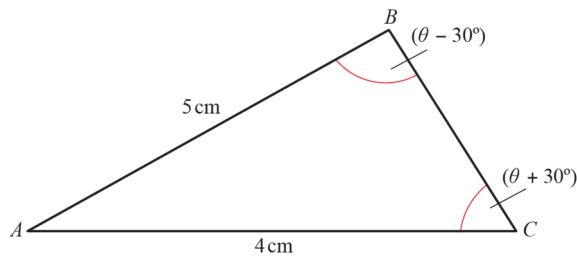
$$3 \text{ a } \tan A = 2, \tan B = \frac{1}{3} \text{ since } y = \frac{1}{3}x - \frac{1}{3}$$

b The angle required is $(A - B)$.

$$\begin{aligned}
 \text{Using } \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\
 &= \frac{2 - \frac{1}{3}}{1 + 2 \times \frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1
 \end{aligned}$$

$$\Rightarrow A - B = 45^\circ$$

4



$$\begin{aligned} \text{Using } \frac{\sin B}{b} &= \frac{\sin C}{c} \\ \Rightarrow \frac{\sin(\theta - 30^\circ)}{4} &= \frac{\sin(\theta + 30^\circ)}{5} \\ \Rightarrow 5 \sin(\theta - 30^\circ) &= 4 \sin(\theta + 30^\circ) \\ \Rightarrow 5(\sin \theta \cos 30^\circ - \cos \theta \sin 30^\circ) \\ &= 4(\sin \theta \cos 30^\circ + \cos \theta \sin 30^\circ) \\ \Rightarrow \sin \theta \cos 30^\circ &= 9 \cos \theta \sin 30^\circ \\ \Rightarrow \frac{\sin \theta}{\cos \theta} &= 9 \frac{\sin 30^\circ}{\cos 30^\circ} = 9 \tan 30^\circ \\ \Rightarrow \tan \theta &= 9 \times \frac{\sqrt{3}}{3} = 3\sqrt{3} \end{aligned}$$

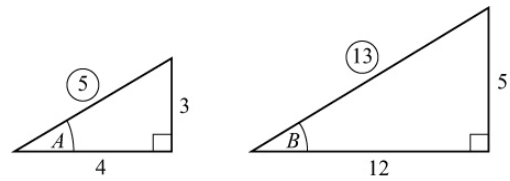
5 As the three values are consecutive terms of an arithmetic progression,

$$\begin{aligned} \sin(\theta - 30^\circ) - \sqrt{3} \cos \theta &= \sin \theta - \sin(\theta - 30^\circ) \\ \Rightarrow 2 \sin(\theta - 30^\circ) &= \sin \theta + \sqrt{3} \cos \theta \\ \Rightarrow 2(\sin \theta \cos 30^\circ - \cos \theta \sin 30^\circ) \\ &= \sin \theta + \sqrt{3} \cos \theta \\ \Rightarrow \sqrt{3} \sin \theta - \cos \theta &= \sin \theta + \sqrt{3} \cos \theta \\ \Rightarrow \sin \theta(\sqrt{3} - 1) &= \cos \theta(\sqrt{3} + 1) \\ \Rightarrow \tan \theta &= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \end{aligned}$$

$$\text{Calculator value is } \theta = \tan^{-1} \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = 75^\circ$$

No other values as θ is acute.

6 a



$$\sin A = \frac{3}{5}, \cos A = \frac{4}{5} \quad \sin B = \frac{5}{13}, \cos B = \frac{12}{13}$$

$$\begin{aligned} \text{i } \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65} \end{aligned}$$

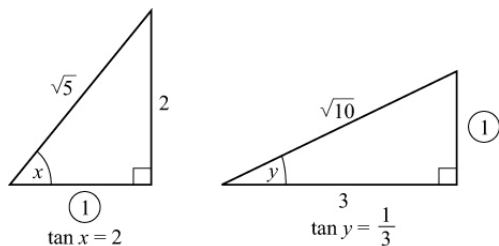
$$\begin{aligned} \text{ii } \tan 2B &= \frac{2 \tan B}{1 - \tan^2 B} \\ &= \frac{2 \times \frac{5}{12}}{1 - \left(\frac{5}{12}\right)^2} = \frac{\frac{5}{6}}{\frac{119}{144}} \\ &= \frac{5}{6} \times \frac{144}{119} = \frac{120}{119} \end{aligned}$$

$$\begin{aligned} \text{b } \cos C &= \cos(180^\circ - (A + B)) \\ &= -\cos(A + B) \\ &= -(\cos A \cos B - \sin A \sin B) \\ &= -\left(\frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}\right) \\ &= -\frac{33}{65} \end{aligned}$$

$$\begin{aligned} \text{7 a } \cos 2x &\equiv 1 - 2 \sin^2 x \\ &= 1 - 2 \left(\frac{2}{\sqrt{5}}\right)^2 = 1 - \frac{8}{5} = -\frac{3}{5} \end{aligned}$$

$$\begin{aligned} \text{b } \cos 2y &\equiv 2 \cos^2 y - 1 \\ &= 2 \left(\frac{3}{\sqrt{10}}\right)^2 - 1 = 2 \left(\frac{9}{10}\right) - 1 = \frac{4}{5} \end{aligned}$$

7 c



$$\begin{aligned} \text{i} \quad \tan(x+y) &= \frac{\tan x + \tan y}{1 - \tan x \tan y} \\ &= \frac{2 + \frac{1}{3}}{1 - \frac{2}{3}} = \frac{\frac{7}{3}}{\frac{1}{3}} = 7 \end{aligned}$$

$$\begin{aligned} \text{ii} \quad \tan(x-y) &= \frac{\tan x - \tan y}{1 + \tan x \tan y} = \frac{\frac{5}{3}}{\frac{5}{3}} = 1 \\ \text{As } x \text{ and } y \text{ are acute, and } x > y, \\ x-y \text{ is acute} \\ \text{So } x-y &= \frac{\pi}{4} \quad \left(\text{it cannot be } \frac{5\pi}{4} \right) \end{aligned}$$

$$\begin{aligned} \text{8 a} \quad \sin(x+y) &\equiv \sin x \cos y + \cos x \sin y \\ &= \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \\ 5 \sin(x-y) &\equiv 5(\sin x \cos y - \cos x \sin y) \\ &= 5\left(\frac{1}{2} - \frac{1}{3}\right) = 5 \times \frac{1}{6} = \frac{5}{6} \end{aligned}$$

$$\begin{aligned} \text{b} \quad \frac{\sin x \cos y}{\cos x \sin y} &= \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2} \\ \Rightarrow \frac{\tan x}{\tan y} &= \frac{3}{2} \\ \text{so } \tan x &= \frac{3 \tan y}{2} = \frac{3k}{2} \end{aligned}$$

$$\begin{aligned} \text{c} \quad \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} = \frac{3k}{1 - \frac{9}{4}k^2} \\ &= \frac{12k}{4 - 9k^2} \end{aligned}$$

$$\begin{aligned} \text{9 a} \quad \sqrt{3} \sin 2\theta + 2 \sin^2 \theta &= 1 \\ \sqrt{3} \sin 2\theta &= 1 - 2 \sin^2 \theta = \cos 2\theta \\ \frac{\sin 2\theta}{\cos 2\theta} &= \frac{1}{\sqrt{3}} \Rightarrow \tan 2\theta = \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \text{9 b} \quad \tan 2\theta &= \frac{1}{\sqrt{3}}, \text{ for } 0 \leq 2\theta \leq 2\pi \\ 2\theta &= \frac{\pi}{6}, \frac{7\pi}{6} \Rightarrow \theta = \frac{\pi}{12}, \frac{7\pi}{12} \end{aligned}$$

$$\begin{aligned} \text{10 a} \quad \cos 2\theta &= 5 \sin \theta \\ \Rightarrow \cos 2\theta - 5 \sin \theta &= 0 \\ \Rightarrow 1 - 2 \sin^2 \theta - 5 \sin \theta &= 0 \\ \Rightarrow 2 \sin^2 \theta + 5 \sin \theta - 1 &= 0 \\ a = 2, b = 5 \text{ and } c &= -1 \end{aligned}$$

$$\text{b} \quad 2 \sin^2 \theta + 5 \sin \theta - 1 = 0$$

Using the quadratic formula

$$\begin{aligned} \sin \theta &= \frac{-5 \pm \sqrt{5^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-5 \pm \sqrt{33}}{4} \end{aligned}$$

$$\sin \theta = 0.1861, \text{ for } -\pi \leq \theta \leq \pi$$

$\sin \theta$ is positive so solutions in the first and second quadrants

$$\theta = \sin^{-1} 0.1861, \pi - \sin^{-1} 0.1861$$

$$\theta = 0.187, 2.954 \text{ (3 d.p.)}$$

$$\begin{aligned} \text{11 a} \quad \cos(x-60^\circ) &= \cos x \cos 60^\circ + \sin x \sin 60^\circ \\ &= \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \end{aligned}$$

$$\text{So } 2 \sin x = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$$

$$\Rightarrow \left(2 - \frac{\sqrt{3}}{2}\right) \sin x = \frac{1}{2} \cos x$$

$$\Rightarrow \tan x = \frac{\frac{1}{2}}{2 - \frac{\sqrt{3}}{2}} = \frac{1}{2} \times \frac{2}{4 - \sqrt{3}} = \frac{1}{4 - \sqrt{3}}$$

$$\begin{aligned} \text{b} \quad \tan x &= \frac{1}{4 - \sqrt{3}} = 0.44 \text{ (2 d.p.)}, \text{ in the} \\ \text{interval } 0^\circ &\leq \theta \leq 360^\circ \end{aligned}$$

$\tan x$ is positive so solutions in the first and third quadrants

$$x = 23.8^\circ, 203.8^\circ$$

$$\begin{aligned}
 12 \text{ a } \quad \cos(x + 20^\circ) &= \sin(90^\circ - 20^\circ - x) \\
 &= \sin(70^\circ - x) \\
 &= \sin 70^\circ \cos x - \cos 70^\circ \sin x \quad (1) \\
 4 \sin(70^\circ + x) &= 4 \sin 70^\circ \cos x \\
 &\quad + 4 \cos 70^\circ \sin x \quad (2)
 \end{aligned}$$

As (1) = (2)

$$4 \sin 70^\circ \cos x + 4 \cos 70^\circ \sin x = \sin 70^\circ \cos x - \cos 70^\circ \sin x$$

$$5 \sin x \cos 70^\circ = -3 \sin 70^\circ \cos x$$

$$\tan x = -\frac{3}{5} \tan 70^\circ$$

$$b \quad \tan x = -\frac{3}{5} \tan 70^\circ, \text{ for } 0^\circ \leq \theta \leq 180^\circ$$

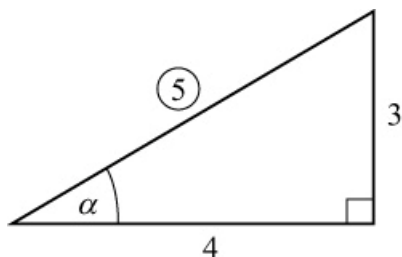
$\tan \theta$ is negative so the solution is in the second quadrant

$$x = 180^\circ + \tan^{-1}\left(-\frac{3}{5} \tan 70^\circ\right)$$

$$x = 180^\circ - \tan^{-1}(1.648)$$

$$x = 180^\circ - (-58.8^\circ) = 121.2^\circ$$

- 13 a Draw a right-angled triangle and find $\sin \alpha$ and $\cos \alpha$.



$$\Rightarrow \sin \alpha = \frac{3}{5}, \quad \cos \alpha = \frac{4}{5}$$

$$\begin{aligned}
 &3 \sin(\theta + \alpha) + 4 \cos(\theta + \alpha) \\
 &\equiv 3(\sin \theta \cos \alpha + \cos \theta \sin \alpha) \\
 &\quad + 4(\cos \theta \cos \alpha - \sin \theta \sin \alpha) \\
 &\equiv 3\left(\frac{4}{5} \sin \theta + \frac{3}{5} \cos \theta\right) \\
 &\quad + 4\left(\frac{4}{5} \cos \theta - \frac{3}{5} \sin \theta\right) \\
 &\equiv \frac{12}{5} \sin \theta + \frac{9}{5} \cos \theta + \frac{16}{5} \cos \theta - \frac{12}{5} \sin \theta \\
 &\equiv \frac{25}{5} \cos \theta \equiv 5 \cos \theta
 \end{aligned}$$

$$\begin{aligned}
 13 \text{ b } \quad \cos(x + 270^\circ) \\
 &\equiv \cos x^\circ \cos 270^\circ - \sin x^\circ \sin 270^\circ \\
 &= (-0.8)(0) - (0.6)(-1) \\
 &= 0 + 0.6 = 0.6
 \end{aligned}$$

$$\begin{aligned}
 \cos(x + 540^\circ) \\
 &\equiv \cos x^\circ \cos 540^\circ - \sin x^\circ \sin 540^\circ \\
 &= (-0.8)(-1) - (0.6)(0) \\
 &= 0.8 - 0 = 0.8
 \end{aligned}$$

- 14 a One example is sufficient to disprove a statement. Let $A = 60^\circ$, $B = 0^\circ$

$$\begin{aligned}
 \sec(A + B) &= \sec(60^\circ + 0^\circ) \\
 &= \sec 60^\circ = \frac{1}{\cos 60^\circ} = 2
 \end{aligned}$$

$$\sec A = \sec 60^\circ = \frac{1}{\cos 60^\circ} = 2$$

$$\sec B = \sec 0^\circ = \frac{1}{\cos 0^\circ} = 1$$

$$\text{So } \sec A + \sec B = 2 + 1 = 3$$

$$\text{So } \sec(60^\circ + 0^\circ) \neq \sec 60^\circ + \sec 0^\circ$$

$\Rightarrow \sin(A + B) \equiv \sec A + \sec B$ is not true for all values of A, B .

$$\begin{aligned}
 b \quad \text{LHS} &\equiv \tan \theta + \cot \theta \\
 &\equiv \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\
 &\equiv \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\
 &\equiv \frac{1}{\frac{1}{2} \sin 2\theta}
 \end{aligned}$$

Using $\sin^2 \theta + \cos^2 \theta \equiv 1$, and

$$\sin 2\theta \equiv 2 \sin \theta \cos \theta$$

$$\begin{aligned}
 \text{So LHS} &\equiv \frac{2}{\sin 2\theta} \\
 &\equiv 2 \operatorname{cosec} 2\theta \\
 &\equiv \text{RHS}
 \end{aligned}$$

15 a Using $\tan 2\theta \equiv \frac{2 \tan \theta}{1 - \tan^2 \theta}$ with $\theta = \frac{\pi}{8}$

$$\Rightarrow \tan \frac{\pi}{4} = \frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}}$$

Let $t = \tan \frac{\pi}{8}$

So $1 = \frac{2t}{1-t^2}$

$$\Rightarrow 1 - t^2 = 2t$$

$$\Rightarrow t^2 + 2t - 1 = 0$$

$$\Rightarrow t = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$= -1 \pm \sqrt{2}$$

As $\frac{\pi}{8}$ is acute, $\tan \frac{\pi}{8}$ is positive,

so $\tan \frac{\pi}{8} = \sqrt{2} - 1$

b $\tan \frac{3\pi}{8} = \tan \left(\frac{\pi}{4} + \frac{\pi}{8} \right) = \frac{\tan \frac{\pi}{4} + \tan \frac{\pi}{8}}{1 - \tan \frac{\pi}{4} \tan \frac{\pi}{8}}$

$$= \frac{1 + (\sqrt{2} - 1)}{1 - (\sqrt{2} - 1)} = \frac{\sqrt{2}}{2 - \sqrt{2}}$$

$$= \frac{\sqrt{2}(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})}$$

$$= \frac{\sqrt{2}}{2} (2 + \sqrt{2}) = \sqrt{2} + 1$$

16 a Let $\sin x - \sqrt{3} \cos x \equiv R \sin(x - \alpha)$

$$\equiv R \sin x \cos \alpha - R \cos x \sin \alpha$$

$R > 0, 0 < \alpha < 90^\circ$

Compare $\sin x$: $R \cos \alpha = 1$ (1)

Compare $\cos x$: $R \sin \alpha = \sqrt{3}$ (2)

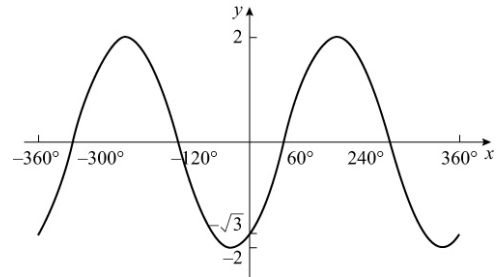
Divide (2) by (1): $\tan \alpha = \sqrt{3}$

$\Rightarrow \alpha = 60^\circ$

$$R^2 = (\sqrt{3})^2 + 1^2 = 4 \Rightarrow R = 2$$

So $\sin x - \sqrt{3} \cos x \equiv 2 \sin(x - 60^\circ)$

16 b Sketch $y = 2 \sin(x - 60^\circ)$ by first translating $y = \sin x$ by 60° to the right and then stretching the result in the y direction by scale factor 2.



Graph meets y -axis when $x = 0$,

i.e. $y = 2 \sin(-60^\circ) = -\sqrt{3}$, at $(0, -\sqrt{3})$

Graph meets x -axis when $y = 0$,

i.e. $(-300^\circ, 0)$, $(-120^\circ, 0)$,
 $(60^\circ, 0)$, $(240^\circ, 0)$

17 a Let $7 \cos 2\theta + 24 \sin 2\theta \equiv R \cos(2\theta - \alpha)$

$$\equiv R \cos 2\theta \cos \alpha + R \sin 2\theta \sin \alpha$$

$R > 0, 0 < \alpha < \frac{\pi}{2}$

Compare $\cos 2\theta$: $R \cos \alpha = 7$ (1)

Compare $\sin 2\theta$: $R \sin \alpha = 24$ (2)

Divide (2) by (1): $\tan \alpha = \frac{24}{7}$

$\Rightarrow \alpha = 1.29$

$$R^2 = 24^2 + 7^2 \Rightarrow R = 25$$

So $7 \cos 2\theta + 24 \sin 2\theta \equiv 25 \cos(2\theta - 1.29)$

b $14 \cos 2\theta + 48 \sin \theta \cos \theta$

$$\equiv 14 \left(\frac{1 + \cos 2\theta}{2} \right) + 24(2 \sin \theta \cos \theta)$$

$$\equiv 7(1 + \cos 2\theta) + 24 \sin 2\theta$$

$$\equiv 7 + 7 \cos 2\theta + 24 \sin 2\theta$$

The maximum value of

$7 \cos 2\theta + 24 \sin 2\theta$ is 25

(using (a) with $\cos(2\theta - 1.29) = 1$)

So maximum value of

$$7 + 7 \cos 2\theta + 24 \sin 2\theta = 7 + 25 = 32$$

17 c Using the answer to part a:
Solve $25\cos(2\theta - 1.29) = 12.5$

$$\cos(2\theta - 1.29) = \frac{1}{2}$$

$$2\theta - 1.29 = -\frac{\pi}{3}, \frac{\pi}{3}$$

$$\theta = 0.119902\dots, 1.167099\dots$$

$$\theta = 0.12, 1.17$$

18 a Let $1.5\sin 2x + 2\cos 2x \equiv R\sin(2x + \alpha)$
 $\equiv R\sin 2x \cos \alpha + R\cos 2x \sin \alpha$

$$R > 0, 0 < \alpha < \frac{\pi}{2}$$

$$\text{Compare } \sin 2x: R\cos \alpha = 1.5 \quad (1)$$

$$\text{Compare } \cos 2x: R\sin \alpha = 2 \quad (2)$$

$$\text{Divide (2) by (1): } \tan \alpha = \frac{4}{3}$$

$$\Rightarrow \alpha = 0.927$$

$$R^2 = 2^2 + 1.5^2 \Rightarrow R = 2.5$$

b $3\sin x \cos x + 4\cos^2 x$
 $\equiv \frac{3}{2}(2\sin x \cos x) + 4\left(\frac{1 + \cos 2x}{2}\right)$
 $\equiv \frac{3}{2}\sin 2x + 2 + 2\cos 2x$
 $\equiv \frac{3}{2}\sin 2x + 2\cos 2x + 2$

c From part (a) $1.5\sin 2x + 2\cos 2x$
 $\equiv 2.5\sin(2x + 0.927)$

So maximum value of

$$1.5\sin 2x + 2\cos 2x = 2.5 \times 1 = 2.5$$

So maximum value of

$$3\sin x \cos x + 4\cos^2 x = 2.5 + 2 = 4.5$$

19 a $\sin^2 \frac{\theta}{2} = 2\sin \theta$

$$\frac{1 - \cos \theta}{2} = 2\sin \theta$$

$$1 - \cos \theta = 4\sin \theta$$

$$4\sin \theta + \cos \theta = 1$$

$$\text{Let } 4\sin \theta + \cos \theta = R\sin(\theta + \alpha)$$

$$= R\sin \theta \cos \alpha + R\cos \theta \sin \alpha$$

$$\text{So } R\cos \alpha = 4 \text{ and } R\sin \alpha = 1$$

$$\frac{R\sin \alpha}{R\cos \alpha} = \tan \alpha = \frac{1}{4}$$

$$\alpha = \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1} 0.25 = 14.04 \text{ (2 d.p.)}$$

$$R^2 = 4^2 + 1^2 = \sqrt{17}$$

$$4\sin \theta + \cos \theta = \sqrt{17} \sin(\theta + 14.04^\circ) = 1$$

b $\sqrt{17} \sin(\theta + 14.04^\circ) = 1$, for $0^\circ \leq \theta \leq 360^\circ$

$$\sin(\theta + 14.04^\circ) = \frac{1}{\sqrt{17}} = 0.24 \text{ (2 d.p.)}$$

$$\theta + 14.04^\circ = \sin^{-1} 0.24 = 14.04^\circ, \text{ for}$$

$$14.04^\circ \leq \theta + 14.04^\circ \leq 374.04^\circ$$

$$\theta + 14.04^\circ = 14.04^\circ, 165.96^\circ, 374.04^\circ$$

$$\theta = 0^\circ, 151.9^\circ, 360^\circ$$

20 a $2\cos \theta = 1 + 3\sin \theta$

$$\text{So } 2\cos \theta - 3\sin \theta = 1$$

$$\text{Let } 2\cos \theta - 3\sin \theta = R\cos(\theta + \alpha)$$

$$= R\cos \theta \cos \alpha - R\sin \theta \sin \alpha$$

$$\text{So } R\cos \alpha = 2 \text{ and } R\sin \alpha = 3$$

$$\frac{R\sin \alpha}{R\cos \alpha} = \tan \alpha = \frac{3}{2}$$

$$\alpha = \tan^{-1}\left(\frac{3}{2}\right) = 56.3^\circ \text{ (1 d.p.)}$$

$$R^2 = 2^2 + 3^2 = 13$$

$$R = \sqrt{13}$$

$$\text{So } 2\cos \theta - 3\sin \theta = \sqrt{13} \cos(\theta + 56.3^\circ) = 1$$

$$20 \text{ b } \sqrt{13} \cos(\theta + 56.3^\circ) = 1, \text{ for } 0^\circ \leq \theta \leq 360^\circ$$

$$\cos(\theta + 56.3^\circ) = \frac{1}{\sqrt{13}},$$

$$\text{for } 56.3^\circ \leq \theta + 56.3^\circ \leq 416.3^\circ$$

$$\theta + 56.3^\circ = 73.9^\circ, 286.1^\circ \text{ (1 d.p.)}$$

$$\theta = 17.6^\circ, 229.8^\circ \text{ (1 d.p.)}$$

$$21 \text{ a } \text{LHS} \equiv \frac{1}{\cos \theta} \times \frac{1}{\sin \theta} \equiv \frac{1}{\frac{1}{2} \sin 2\theta}$$

$$\equiv \frac{2}{\sin 2\theta} \equiv 2 \operatorname{cosec} 2\theta \equiv \text{RHS}$$

$$21 \text{ b } \text{LHS} \equiv \frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x} - \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x}$$

$$\equiv \frac{1 + \tan x}{1 - \tan x} - \frac{1 - \tan x}{1 + \tan x}$$

$$\equiv \frac{(1 + \tan x)^2 - (1 - \tan x)^2}{(1 - \tan x)(1 + \tan x)}$$

$$\equiv \frac{(1 + 2 \tan x + \tan^2 x)}{1 - \tan^2 x}$$

$$- \frac{(1 - 2 \tan x + \tan^2 x)}{1 - \tan^2 x}$$

$$\equiv \frac{4 \tan x}{1 - \tan^2 x}$$

$$\equiv 2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)$$

$$\equiv 2 \tan 2x \equiv \text{RHS}$$

$$21 \text{ c } \text{LHS} \equiv (\sin x \cos y + \cos x \sin y)$$

$$\times (\sin x \cos y - \cos x \sin y)$$

$$\equiv \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$$

$$\equiv (1 - \cos^2 x) \cos^2 y$$

$$- \cos^2 x (1 - \cos^2 y)$$

$$\equiv \cos^2 y - \cos^2 x \cos^2 y$$

$$- \cos^2 x + \cos^2 x \cos^2 y$$

$$\equiv \cos^2 y - \cos^2 x = \text{RHS}$$

$$21 \text{ d } \text{LHS} \equiv 1 + 2 \cos 2\theta + (2 \cos^2 2\theta - 1)$$

$$\equiv 2 \cos 2\theta + 2 \cos^2 2\theta$$

$$\equiv 2 \cos 2\theta (1 + \cos 2\theta)$$

$$\equiv 2 \cos 2\theta (2 \cos^2 \theta)$$

$$\equiv 4 \cos^2 \theta \cos 2\theta = \text{RHS}$$

$$22 \text{ a } \text{LHS} \equiv \frac{1 - \cos 2x}{1 + \cos 2x} \equiv \frac{1 - (1 - 2 \sin^2 x)}{1 + (2 \cos^2 x - 1)}$$

$$\equiv \frac{2 \sin^2 x}{2 \cos^2 x} \equiv \tan^2 x = \text{RHS}$$

$$22 \text{ b } \tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}, \text{ for } -\pi \leq x \leq \pi$$

$$\tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3}, -\frac{2\pi}{3}$$

$$\tan x = -\sqrt{3} \Rightarrow x = -\frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = -\frac{2\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$23 \text{ a } \text{LHS} \equiv \cos^4 2\theta - \sin^4 2\theta$$

$$\equiv (\cos^2 2\theta - \sin^2 2\theta)(\cos^2 2\theta + \sin^2 2\theta)$$

$$\equiv (\cos^2 2\theta - \sin^2 2\theta)(1)$$

$$\equiv \cos 4\theta \equiv \text{RHS}$$

$$23 \text{ b } \cos 4\theta = \frac{1}{2}, \text{ for } 0^\circ \leq 4\theta \leq 720^\circ$$

$$4\theta = 60^\circ, 300^\circ, 420^\circ, 660^\circ$$

$$\theta = 15^\circ, 75^\circ, 105^\circ, 165^\circ$$

$$24 \text{ a } \text{LHS} \equiv \frac{1 - (1 - 2 \sin^2 \theta)}{2 \sin \theta \cos \theta}$$

$$\equiv \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta}$$

$$\equiv \frac{\sin \theta}{\cos \theta} \equiv \tan \theta = \text{RHS}$$

$$24 \text{ b } \text{When } \theta = 180^\circ, \sin 2\theta = \sin 360^\circ = 0$$

$$\text{and } 2 - 2 \cos 360^\circ = 2 - 2 = 0$$

$$\text{therefore } \theta = 180^\circ \text{ is a solution of the}$$

$$\text{equation } \sin 2\theta = 2 - 2 \cos 2\theta$$

24 c Rearrange $\sin 2\theta = 2 - 2 \cos 2\theta$ to give

$$\frac{2(1 - \cos 2\theta)}{\sin 2\theta} = 1$$

Using the identity in part (a) gives

$$2 \tan \theta = 1$$

$$\Rightarrow \tan \theta = \frac{1}{2}, \text{ for } 0 < \theta < 360^\circ$$

$$\theta = 26.6^\circ, 206.6^\circ \text{ (1 d.p.)}$$

Challenge

1 a Using $\cos P + \cos Q$

$$\equiv 2 \cos \left(\frac{P+Q}{2} \right) \cos \left(\frac{P-Q}{2} \right)$$

and $\sin P - \sin Q$

$$\equiv 2 \cos \left(\frac{P+Q}{2} \right) \sin \left(\frac{P-Q}{2} \right)$$

$$\begin{aligned} \text{LHS} &\equiv \frac{\cos 2\theta + \cos 4\theta}{\sin 2\theta - \sin 4\theta} \\ &\equiv \frac{2 \cos \left(\frac{6\theta}{2} \right) \cos \left(\frac{2\theta}{2} \right)}{2 \cos \left(\frac{6\theta}{2} \right) \sin \left(\frac{-2\theta}{2} \right)} \\ &\equiv \frac{2 \cos 3\theta \cos \theta}{2 \cos 3\theta \sin(-\theta)} \\ &\equiv \frac{\cos \theta}{\sin(-\theta)} \equiv -\cot \theta \end{aligned}$$

b LHS $\equiv \cos x + 2 \cos 3x + \cos 5x$

$$\equiv \cos 5x + \cos x + 2 \cos 3x$$

$$\equiv 2 \cos \left(\frac{6x}{2} \right) \cos \left(\frac{4x}{2} \right) + 2 \cos 3x$$

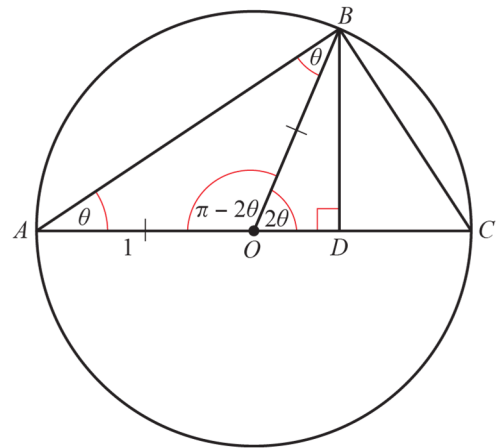
$$\equiv 2 \cos 3x \cos 2x + 2 \cos 3x$$

$$\equiv 2 \cos 3x (\cos 2x + 1)$$

$$\equiv 2 \cos 3x (2 \cos^2 x)$$

$$\equiv 4 \cos^2 x \cos 3x \equiv \text{RHS}$$

2 a As $\angle OAB = \angle OBA \Rightarrow \angle AOB = \pi - 2\theta$, so $\angle BOD = 2\theta$



$$OB = 1$$

$$OD = \cos 2\theta$$

$$BD = \sin 2\theta$$

$$AB = 2 \cos \theta$$

$$\sin \theta = \frac{BD}{AB} = \frac{BD}{2 \cos \theta}$$

$$\text{So } BD = 2 \sin \theta \cos \theta$$

$$\text{But } BD = \sin 2\theta$$

$$\text{So } \sin 2\theta \equiv 2 \sin \theta \cos \theta$$

b $AB = 2 \cos \theta$

$$AD = (2 \cos \theta) \cos \theta = 2 \cos^2 \theta$$

$$OD = 2 \cos^2 \theta - 1$$

$$\text{From part (a) } OD = \cos 2\theta$$

$$\text{So } \cos 2\theta \equiv 2 \cos^2 \theta - 1$$