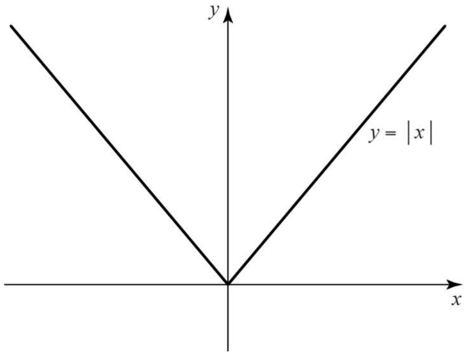
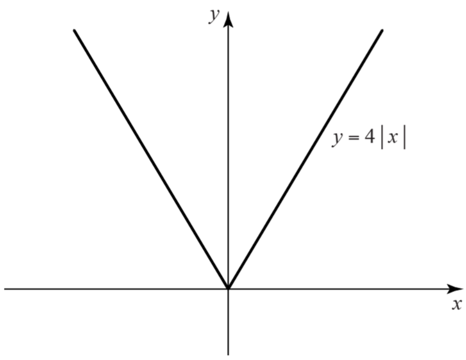
Exercise 2G

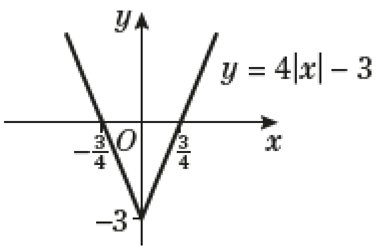
1 a i Start with $y=|x|$



$y=4|x|$ is a vertical stretch by scale factor 4

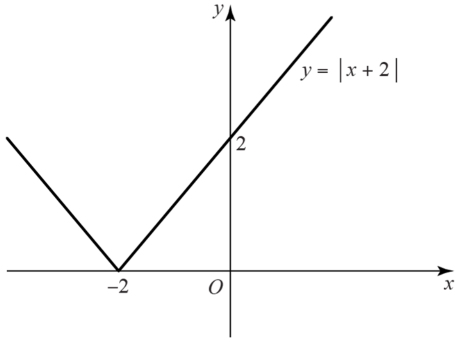


$y=4|x|-3$ is a horizontal translation by –3



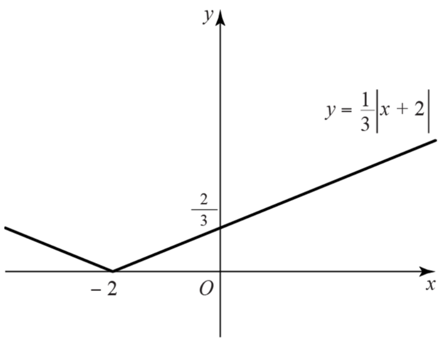
ii The range is $f(x)\geq -3$

b I Start with $y=|x|$

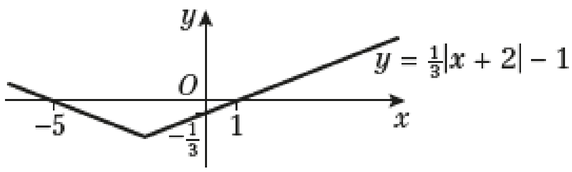


$y=|x+2|$ is a horizontal translation by –2

$y=\frac{1}{3}|x+2|$ is a vertical stretch by scale factor $\frac{1}{3}$

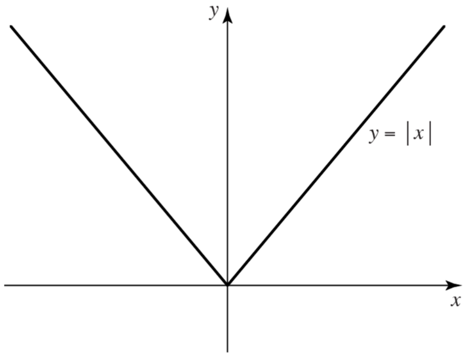


$y=\frac{1}{3}|x+2|-1$ is a vertical translation by –1

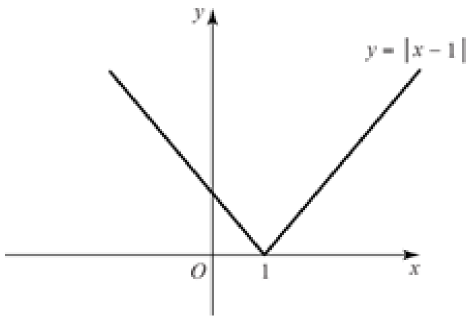


ii The range is $f(x)\geq -1$

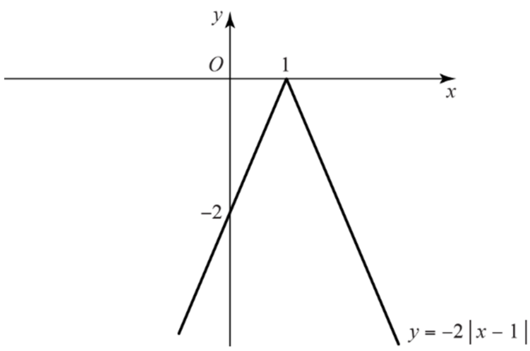
1 c i Start with $y = |x|$



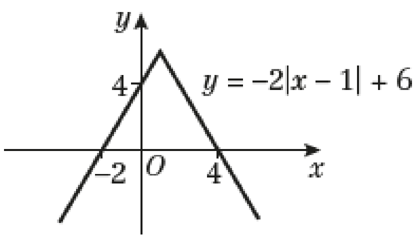
$y = |x-1|$ is a horizontal translation by +1



$y = -2|x-1|$ is a vertical stretch by scale factor –2

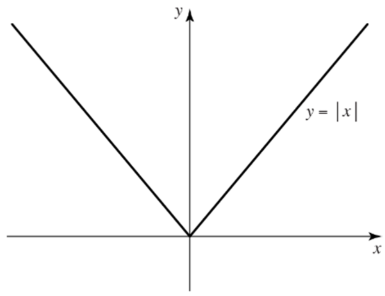


$y = -2|x-1| + 6$ is a vertical translation by +6

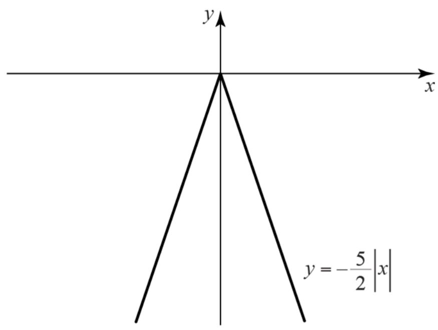


1 c ii The range is $f(x) \leq 6$

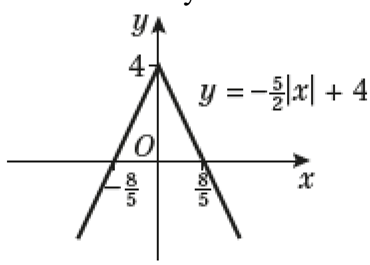
d i Start with $y = |x|$



$y = -\frac{5}{2}|x|$ is a vertical stretch by scale factor $-\frac{5}{2}$



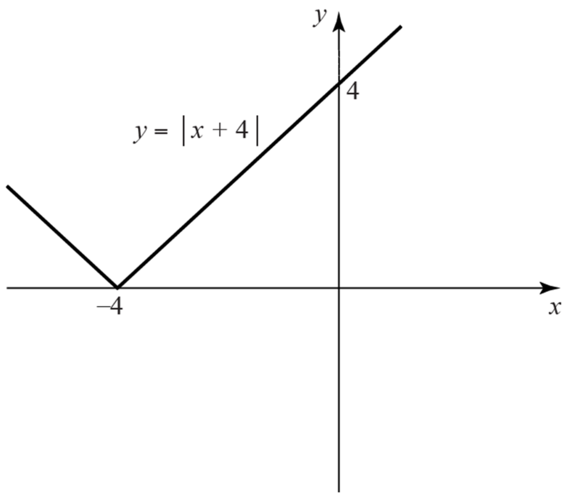
$y = -\frac{5}{2}|x| + 4$ is a horizontal translation by –3



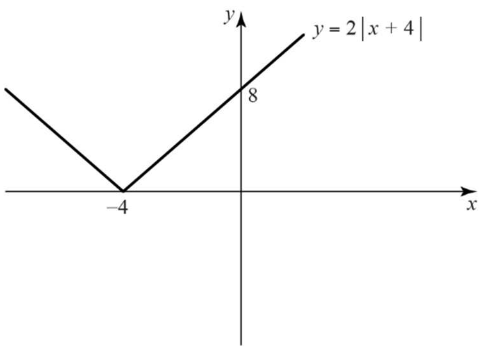
d ii The range is $f(x) \leq 4$

2 a Start with $y=|x|$

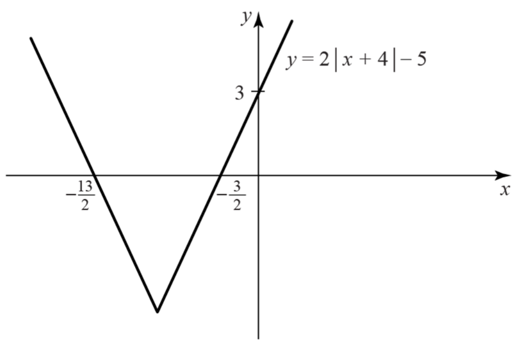
$y=|x+4|$ is a horizontal translation of –4



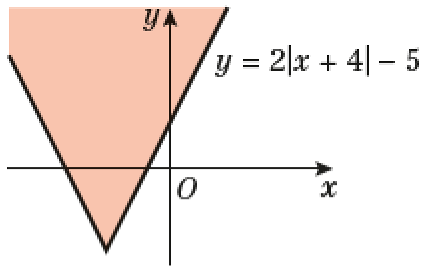
$y=2|x+4|$ is a vertical stretch scale factor 2



$y=2|x+4|-5$ is a vertical translation of –5

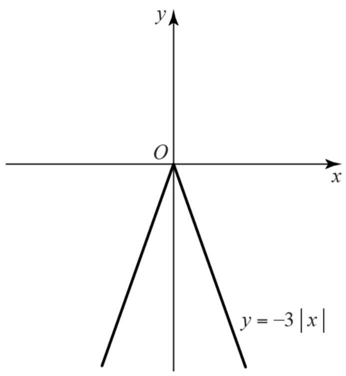


2 b The region where $y \geq \text{p}(x)$ is the region which lies on and above the line $y=2|x+4|-5$

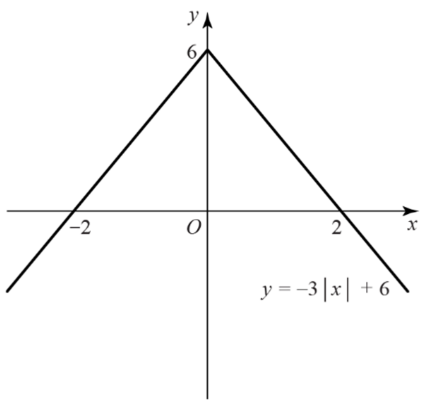


3 a Start with $y=|x|$

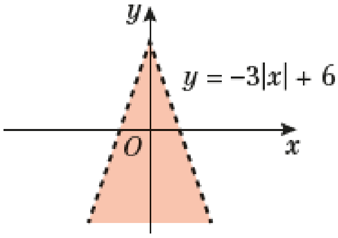
$y=-3|x|$ is a vertical stretch scale factor –3



$y=-3|x|+6$ is a vertical translation of +6

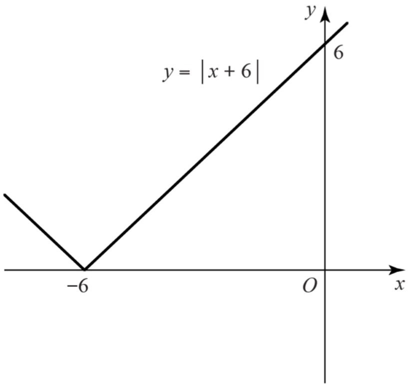


3 b The region where $y < \mathrm{q}(x)$ is the region which lies below the line $y = -3|x| + 6$

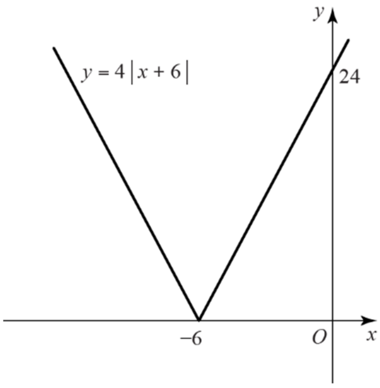


4 a Start with $y = |x|$

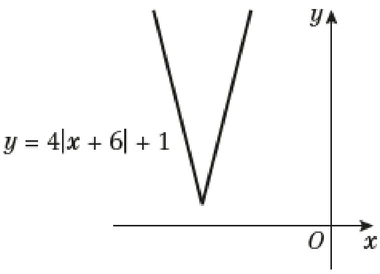
$y = |x+6|$ is a horizontal translation of -6



$y = 4|x+6|$ is a vertical stretch scale factor 4



$y = 4|x+6|+1$ is a vertical translation of $+1$



4 b The range is $\mathrm{f}(x) \geq 1$

c At one point of intersection:

$$-4(x+6)+1 = -\frac{1}{2}x+1$$
$$-4x-23 = -\frac{1}{2}x+1$$
$$-8x-46 = -x+2$$
$$-48 = 7x$$
$$x = -\frac{48}{7}$$

At other point of intersection:

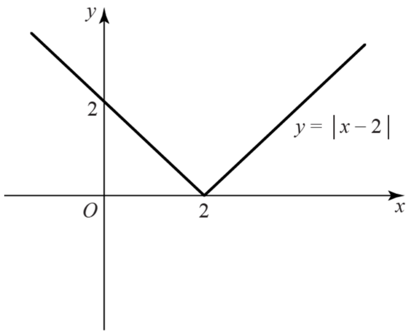
$$4(x+6)+1 = -\frac{1}{2}x+1$$
$$4x+25 = -\frac{1}{2}x+1$$
$$8x+50 = -x+2$$
$$9x = -48$$
$$x = -\frac{16}{3}$$

So the solutions are

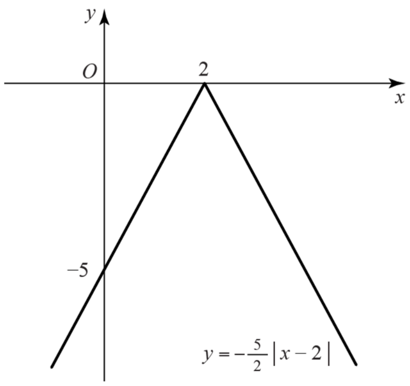
$x = -\frac{48}{7}$ and $x = -\frac{16}{3}$

5 a Start with $y = |x|$

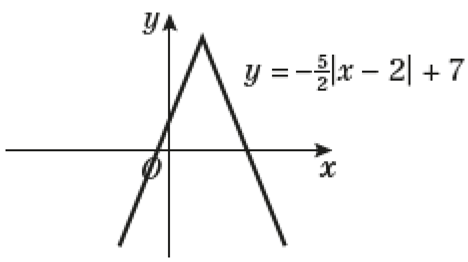
$y = |x-2|$ is a horizontal translation of +2



$y = -\frac{5}{2}|x-2|$ is a vertical stretch scale factor $-\frac{5}{2}$



$y = -\frac{5}{2}|x-2|+7$ is a vertical translation of +7



b The range is $g(x) \leq 7$

5 c At one point of intersection:

$$-\frac{5}{2}(x-2)+7 = x+1$$
$$-\frac{5}{2}x+12 = x+1$$
$$-5x+24 = 2x+2$$
$$22 = 7x$$
$$x = \frac{22}{7}$$

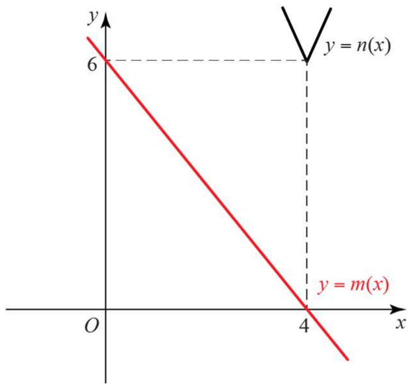
At other point of intersection:

$$\frac{5}{2}(x-2)+7 = x+1$$
$$\frac{5}{2}x+2 = x+1$$
$$5x+4 = 2x+2$$
$$3x = -2$$
$$x = -\frac{2}{3}$$

So the solutions are

$x = -\frac{2}{3}$ and $x = \frac{22}{7}$

6 For the equation m(x) = n(x) to have no real roots, it must be the case that y = m(x) and y = n(x) do not intersect.



The least value of

$y = \mathrm{n}(x) = 3|x-4|+6$ is

$y = 6$ when $x = 4$

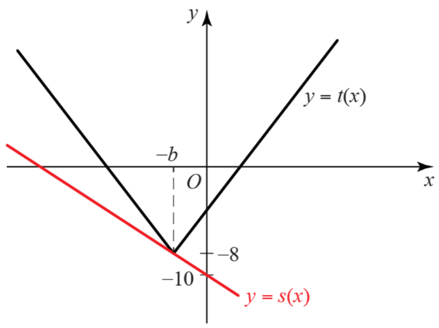
Hence, we need m(4) < 6 to avoid intersection

So $-2(4) + k < 6$

$-8 + k < 6$

$k < 14$

7 For the equation s(x) = t(x) to have exactly one real root, it must be the case that y = s(x) and y = t(x) intersect at the minimum point of t(x).



The least value of

$y = \mathrm{t}(x) = 2|x+b|-8$ is

$y = -8$ when $x = -b$

Hence, we need s(–b) = –8 to ensure one intersection

$\Rightarrow -8 = -10-(-b)$

$b = 2$

8 a The range is $\mathrm{h}(x) \geq -7$

b h(x) is many-to-one, therefore $\mathrm{h}^{-1}(x)$ would be one-to-many, and so would not be a function.

c At one point of intersection:

$$-\frac{2}{3}(x-1)-7=-6$$

$$2x-2+21=18$$

$$2x=-1$$

$$x=-\frac{1}{2}$$

At other point of intersection:

$$\frac{2}{3}(x-1)-7=-6$$

$$2x-2-21=-18$$

$$2x=5$$

$$x=\frac{5}{2}$$

So the solutions are

$x=-\frac{1}{2}$ and $x=\frac{5}{2}$

h(x) < –6 between the two points of intersection, so the solution to the inequality h(x) < −6 is

$$-\frac{1}{2}<x<\frac{5}{2}$$

d Since $\mathrm{h}(x) \geq -7$ and $\mathrm{h}(1) = -7$,

then for the equation $\mathrm{h}(x) = \frac{2}{3}x + k$

to have no solutions, we require

$$\frac{2}{3}(1)+k<-7$$

$$\Rightarrow k<-\frac{23}{3}$$

9 a We can write h as

$$h(x)=\begin{cases} a+2(x+3), & x\le -3 \\ a-2(x+3), & x\ge -3 \end{cases}$$

The line which has gradient –2 and passes through (0, 4) is $y=-2x+4$

So, for $x\ge -3$

$-2(x+3)+a=-2x+4$

$-2x-6+a=-2x+4$

$a=10$

b At P, h(x) = 10 (from part **a**)

So $10=10-2(x+3)$

$-2x-6=0$

$x=-3$

At Q, h(x) = 0

So $0=10-2(x+3)$

$4-2x=0$

$x=2$

$P(-3, 10)$ and $Q(2, 0)$

c $h(x)=\frac{1}{3}x+6$

At one point of intersection:

$10-2(x+3)=\frac{1}{3}x+6$

$4-2x=\frac{1}{3}x+6$

$12-6x=x+18$

$7x=-6$

$x=-\frac{6}{7}$

At other point of intersection:

$10+2(x+3)=\frac{1}{3}x+6$

$16+2x=\frac{1}{3}x+6$

$48+6x=x+18$

$5x=-30$

$x=-6$

So the solutions are

$x=-6$ and $x=-\frac{6}{7}$

10 a The range of m(x) is $m(x)\le 7$

b $m(x)=\frac{3}{5}x+2$

At one point of intersection:

$-4(x+3)+7=\frac{3}{5}x+2$

$-4x-5=\frac{3}{5}x+2$

$-20x-25=3x+10$

$-23x=35$

$x=-\frac{35}{23}$

At other point of intersection:

$4(x+3)+7=\frac{3}{5}x+2$

$4x+19=\frac{3}{5}x+2$

$20x+95=3x+10$

$17x=-85$

$x=-5$

So the solutions are $x=-5$ and $x=-\frac{35}{23}$

c For two distinct roots, there are two points of intersection, so $m(x) < 7$. Therefore, $k < 7$.

Challenge

1 a At A:

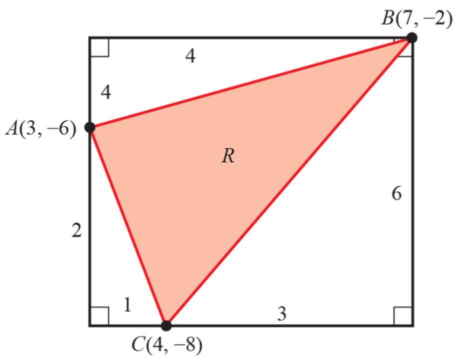
$$-2(x-4)-8=x-9$$
$$-2x=x-9$$
$$-3x=-9$$
$$x=3$$
$$y=3-9=-6$$

At B:

$$2(x-4)-8=x-9$$
$$2x-16=x-9$$
$$x=7$$
$$y=7-9=-2$$

$A(3, -6)$ and $B(7, -2)$

b Taking the shaded triangle R and enclosing it in a rectangle looks like:



$$R=(4\times 6)-\left(\frac{1}{2}\times 4\times 4\right)-\left(\frac{1}{2}\times 6\times 3\right)-\left(\frac{1}{2}\times 2\times 1\right)$$
$$R=24-8-9-1$$
$$R=6\text{ units}^2$$

2 At the first point of intersection:

$$x-3+10=-2(x-3)+2$$
$$x+7=-2x+8$$
$$3x=1$$
$$x=\frac{1}{3}$$

At the other point of intersection:

$$-(x-3)+10=2(x-3)+2$$
$$-x+13=2x-4$$
$$-3x=-17$$
$$x=\frac{17}{3}$$

Maximum point of f(x) is
f(x) = 10 when x = 3, so at (3, 10)
Minimum point of g(x) is
g(x) = 2 when x = 3, so at (3, 2)

$$\text{Area of a kite} = \frac{1}{2}\times\text{width}\times\text{height}$$
$$=\frac{1}{2}\times\left(\frac{17}{3}-\frac{1}{3}\right)\times(10-2)$$
$$=\frac{1}{2}\times\frac{16}{3}\times 8$$
$$=\frac{64}{3}\text{ units}^2$$