Exercise 2C

1 a $pq(-8) = p\left(\frac{-8}{4}\right)$
$= p(-2)$
$= 1 - 3(-2)$
$= 7$

b $qr(5) = q[(5-2)^2]$
$= q(9)$
$= \frac{9}{4}$

c $rq(6) = r\left(\frac{6}{4}\right)$
$= r\left(\frac{3}{2}\right)$
$= \left(\frac{3}{2} - 2\right)^2$
$= \frac{1}{4}$

d $p^2(-5) = p(1 - 3(-5))$
$= p(16)$
$= 1 - 3(16)$
$= -47$

e $pqr(8) = pq[(8-2)^2]$
$= pq(36)$
$= p\left(\frac{36}{4}\right)$
$= p(9)$
$= 1 - 3(9)$
$= -26$

2 a $fg(x) = f(x^2 - 4)$
$= 4(x^2 - 4) + 1$
$= 4x^2 - 15$

b $gf(x) = g(4x+1)$
$= (4x+1)^2 - 4$
$= 16x^2 + 8x - 3$

c $gh(x) = g\left(\frac{1}{x}\right)$
$= \left(\frac{1}{x}\right)^2 - 4$
$= \frac{1}{x^2} - 4$

d $fh(x) = f\left(\frac{1}{x}\right)$
$= 4 \times \left(\frac{1}{x}\right) + 1$
$= \frac{4}{x} + 1$

e $f^2(x) = ff(x)$
$= f(4x+1)$
$= 4(4x+1) + 1$
$= 16x + 5$

3 a $fg(x) = f(x^2)$
$= 3x^2 - 2$

b $gf(x) = g(3x-2)$
$= (3x-2)^2$

When $fg(x) = gf(x)$ then
$3x^2 - 2 = (3x-2)^2$
$3x^2 - 2 = 9x^2 - 12x + 4$
$0 = 6x^2 - 12x + 6$
$0 = x^2 - 2x + 1$
$0 = (x-1)^2$
Hence $x = 1$

4 a $\text{qp}(x) = \text{q}\left(\frac{1}{x-2}\right)$

$= 3 \times \left(\frac{1}{x-2}\right) + 4$

$= \frac{3}{x-2} + \frac{4(x-2)}{x-2}$

$= \frac{4x-5}{x-2}$

b If $\text{qp}(m) = 16$ then

$3\left(\frac{1}{m-2}\right) + 4 = 16$

$\frac{3}{m-2} = 12$

$3 = 12(m-2)$

$\frac{3}{12} = m-2$

$\frac{1}{4} = m-2$

$m = \frac{9}{4}$

5 a $\text{fg}(6) = \text{f}\left(\frac{3(6)-2}{2}\right)$

$= \text{f}(8)$

$= |9-4(8)|$

$= |-23|$

$= 23$

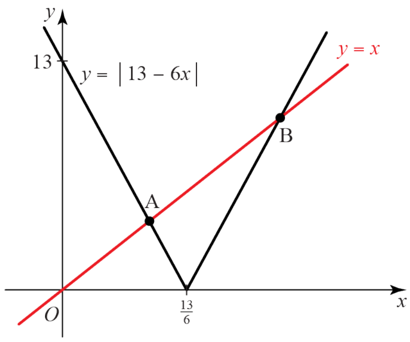
5 b $\text{fg}(x) = \text{f}\left(\frac{3x-2}{2}\right)$

$= \left|9-4\left(\frac{3x-2}{2}\right)\right|$

$= |9-6x+4|$

$= |13-6x|$

Now $\text{fg}(x) = x$ when $|13-6x| = x$



At A: $13 - 6x = x$

$13 = 7x$

$x = \frac{13}{7}$

At B: $-(13 - 6x) = x$

$5x = 13$

$x = \frac{13}{5}$

The solutions are

$x = \frac{13}{7}$ and $x = \frac{13}{5}$

6 a $f^2(x) = f\left(\frac{1}{x+1}\right)$
$= \left(\frac{1}{\left(\frac{1}{x+1}\right)+1}\right)$
$= \left(\frac{1}{\left(\frac{1+x+1}{x+1}\right)}\right)$
$= \left(\frac{x+1}{x+2}\right)$

b $f^3(x) = f\left(\frac{x+1}{x+2}\right)$
$= \left(\frac{1}{\left(\frac{x+1}{x+2}\right)+1}\right)$
$= \left(\frac{1}{\left(\frac{x+1+x+2}{x+2}\right)}\right)$
$= \left(\frac{x+2}{2x+3}\right)$

7 a $st(x) = s(x+3)$
$= 2^{x+3}$

b $ts(x) = t(2^x)$
$= 2^x + 3$

8 a $gf(x) = g(e^{5x})$
$= 4\ln(e^{5x})$
$= 4(5x)$
$= 20x$

b $fg(x) = f(4\ln x)$
$= e^{5(4\ln x)}$
$= e^{\ln x^{20}}$
$= x^{20}$

9 a $qp(x) = q(\ln(x+3))$
$= e^{3(\ln(x+3))} - 1$
$= e^{\ln(x+3)^3} - 1$
$= (x+3)^3 - 1$

Since $x > -3$, so $qp(x) > -1$

b $qp(7) = (7+3)^3 - 1$
$= 999$

c From part **a**
$qp(x) = (x+3)^3 - 1$
When $qp(x) = 124$
$(x+3)^3 - 1 = 124$
$(x+3)^3 = 125$
$x+3 = 5$
$x = 2$

10 $t^2(x) = t(5-2x)$
$= 5 - 2(5-2x)$
$= 5 - 10 + 4x$
$= -5 + 4x$

$t^2(x) - (t(x))^2 = 0$
$-5 + 4x - (5-2x)^2 = 0$
$-5 + 4x - 25 + 20x - 4x^2 = 0$
$-4x^2 + 24x - 30 = 0$
$2x^2 - 12x + 15 = 0$

Using the formula:
$x = \frac{12 \pm \sqrt{(-12)^2 - 4\times2\times15}}{2\times2}$
$= \frac{12\pm\sqrt{24}}{4}$
$= \frac{12\pm2\sqrt{6}}{4}$
$= 3 \pm \frac{\sqrt{6}}{2}$

11 a Range of g is $-8 \leq x \leq 12$

b From the graph,

$g(x) = -\frac{1}{2}x + 12$ for $0 \leq x \leq 14$

and $g(0) = 12$

So $gg(0) = g(12)$

$= -\frac{1}{2}(12) + 12$

$= 6$

c $gh(7) = g\left(\frac{2(7)-5}{10-7}\right)$

$= g(3)$

$= -\frac{1}{2}(3) + 12$

$= 10.5$