

## Exercise 2C

$$\begin{aligned}
 1 \quad \mathbf{a} \quad pq(-8) &= p\left(\frac{-8}{4}\right) \\
 &= p(-2) \\
 &= 1 - 3(-2) \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad qr(5) &= q[(5-2)^2] \\
 &= q(9) \\
 &= \frac{9}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} \quad rq(6) &= r\left(\frac{6}{4}\right) \\
 &= r\left(\frac{3}{2}\right) \\
 &= \left(\frac{3}{2} - 2\right)^2 \\
 &= \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad p^2(-5) &= p(1 - 3(-5)) \\
 &= p(16) \\
 &= 1 - 3(16) \\
 &= -47
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad pqr(8) &= pq[(8-2)^2] \\
 &= pq(36) \\
 &= p\left(\frac{36}{4}\right) \\
 &= p(9) \\
 &= 1 - 3(9) \\
 &= -26
 \end{aligned}$$

$$\begin{aligned}
 2 \quad \mathbf{a} \quad fg(x) &= f(x^2 - 4) \\
 &= 4(x^2 - 4) + 1 \\
 &= 4x^2 - 15
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad gf(x) &= g(4x + 1) \\
 &= (4x + 1)^2 - 4 \\
 &= 16x^2 + 8x - 3
 \end{aligned}$$

$$\begin{aligned}
 2 \quad \mathbf{c} \quad gh(x) &= g\left(\frac{1}{x}\right) \\
 &= \left(\frac{1}{x}\right)^2 - 4 \\
 &= \frac{1}{x^2} - 4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} \quad fh(x) &= f\left(\frac{1}{x}\right) \\
 &= 4 \times \left(\frac{1}{x}\right) + 1 \\
 &= \frac{4}{x} + 1
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} \quad f^2(x) &= ff(x) \\
 &= f(4x + 1) \\
 &= 4(4x + 1) + 1 \\
 &= 16x + 5
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \mathbf{a} \quad fg(x) &= f(x^2) \\
 &= 3x^2 - 2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} \quad gf(x) &= g(3x - 2) \\
 &= (3x - 2)^2
 \end{aligned}$$

When  $fg(x) = gf(x)$  then

$$3x^2 - 2 = (3x - 2)^2$$

$$3x^2 - 2 = 9x^2 - 12x + 4$$

$$0 = 6x^2 - 12x + 6$$

$$0 = x^2 - 2x + 1$$

$$0 = (x - 1)^2$$

Hence  $x = 1$

$$\begin{aligned}
 4 \text{ a } qp(x) &= q\left(\frac{1}{x-2}\right) \\
 &= 3 \times \left(\frac{1}{x-2}\right) + 4 \\
 &= \frac{3}{x-2} + \frac{4(x-2)}{x-2} \\
 &= \frac{4x-5}{x-2}
 \end{aligned}$$

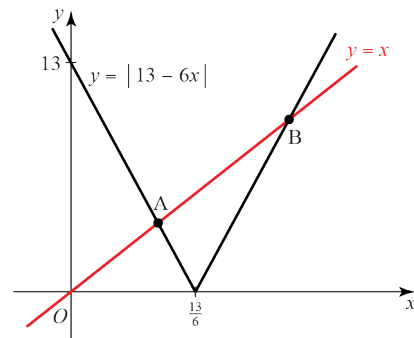
**b** If  $qp(m) = 16$  then

$$\begin{aligned}
 3\left(\frac{1}{m-2}\right) + 4 &= 16 \\
 \frac{3}{m-2} &= 12 \\
 3 &= 12(m-2) \\
 \frac{3}{12} &= m-2 \\
 \frac{1}{4} &= m-2 \\
 m &= \frac{9}{4}
 \end{aligned}$$

$$\begin{aligned}
 5 \text{ a } fg(6) &= f\left(\frac{3(6)-2}{2}\right) \\
 &= f(8) \\
 &= |9-4(8)| \\
 &= |-23| \\
 &= 23
 \end{aligned}$$

$$\begin{aligned}
 5 \text{ b } fg(x) &= f\left(\frac{3x-2}{2}\right) \\
 &= \left|9-4\left(\frac{3x-2}{2}\right)\right| \\
 &= |9-6x+4| \\
 &= |13-6x|
 \end{aligned}$$

Now  $fg(x) = x$  when  $|13-6x| = x$



$$\begin{aligned}
 \text{At A: } 13 - 6x &= x \\
 13 &= 7x \\
 x &= \frac{13}{7}
 \end{aligned}$$

$$\begin{aligned}
 \text{At B: } -(13 - 6x) &= x \\
 5x &= 13 \\
 x &= \frac{13}{5}
 \end{aligned}$$

The solutions are  
 $x = \frac{13}{7}$  and  $x = \frac{13}{5}$

$$\begin{aligned}
 6 \text{ a } f^2(x) &= f\left(\frac{1}{x+1}\right) \\
 &= \left(\frac{1}{\left(\frac{1}{x+1}\right)+1}\right) \\
 &= \left(\frac{1}{\left(\frac{1+x+1}{x+1}\right)}\right) \\
 &= \left(\frac{x+1}{x+2}\right)
 \end{aligned}$$

$$\begin{aligned}
 6 \text{ b } f^3(x) &= f\left(\frac{x+1}{x+2}\right) \\
 &= \left(\frac{1}{\left(\frac{x+1}{x+2}\right)+1}\right) \\
 &= \left(\frac{1}{\left(\frac{x+1+x+2}{x+2}\right)}\right) \\
 &= \left(\frac{x+2}{2x+3}\right)
 \end{aligned}$$

$$7 \text{ a } st(x) = s(x+3) = 2^{x+3}$$

$$7 \text{ b } ts(x) = t(2^x) = 2^x + 3$$

$$\begin{aligned}
 8 \text{ a } gf(x) &= g(e^{5x}) \\
 &= 4 \ln(e^{5x}) \\
 &= 4(5x) \\
 &= 20x
 \end{aligned}$$

$$\begin{aligned}
 8 \text{ b } fg(x) &= f(4 \ln x) \\
 &= e^{5(4 \ln x)} \\
 &= e^{\ln x^{20}} \\
 &= x^{20}
 \end{aligned}$$

$$\begin{aligned}
 9 \text{ a } qp(x) &= q(\ln(x+3)) \\
 &= e^{3(\ln(x+3))} - 1 \\
 &= e^{\ln(x+3)^3} - 1 \\
 &= (x+3)^3 - 1
 \end{aligned}$$

Since  $x > -3$ , so  $qp(x) > -1$

$$9 \text{ b } qp(7) = (7+3)^3 - 1 = 999$$

$$\begin{aligned}
 9 \text{ c } \text{ From part a } \\
 qp(x) &= (x+3)^3 - 1 \\
 \text{When } qp(x) &= 124
 \end{aligned}$$

$$\begin{aligned}
 (x+3)^3 - 1 &= 124 \\
 (x+3)^3 &= 125 \\
 x+3 &= 5 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 10 \text{ } t^2(x) &= t(5-2x) \\
 &= 5 - 2(5-2x) \\
 &= 5 - 10 + 4x \\
 &= -5 + 4x
 \end{aligned}$$

$$\begin{aligned}
 t^2(x) - (t(x))^2 &= 0 \\
 -5 + 4x - (5-2x)^2 &= 0 \\
 -5 + 4x - 25 + 20x - 4x^2 &= 0 \\
 -4x^2 + 24x - 30 &= 0 \\
 2x^2 - 12x + 15 &= 0
 \end{aligned}$$

Using the formula:

$$\begin{aligned}
 x &= \frac{12 \pm \sqrt{(-12)^2 - 4 \times 2 \times 15}}{2 \times 2} \\
 &= \frac{12 \pm \sqrt{24}}{4} \\
 &= \frac{12 \pm 2\sqrt{6}}{4} \\
 &= 3 \pm \frac{\sqrt{6}}{2}
 \end{aligned}$$

**11 a** Range of  $g$  is  $-8 \leq x \leq 12$

**b** From the graph,

$$g(x) = -\frac{1}{2}x + 12 \text{ for } 0 \leq x \leq 14$$

$$\text{and } g(0) = 12$$

$$\text{So } gg(0) = g(12)$$

$$= -\frac{1}{2}(12) + 12$$

$$= 6$$

$$\text{c } gh(7) = g\left(\frac{2(7)-5}{10-7}\right)$$

$$= g(3)$$

$$= -\frac{1}{2}(3) + 12$$

$$= 10.5$$