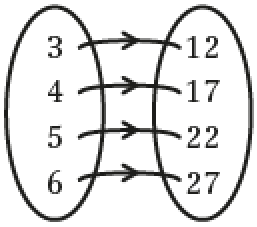
Exercise 2B

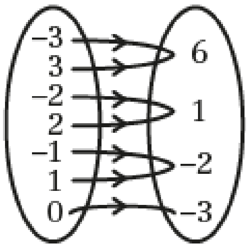
1 a i



ii Every element in set A gets mapped to one element in set B, so the mapping is one-to-one.

iii $\{f(x) = 12, 17, 22, 27\}$

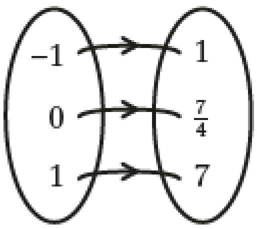
b i



ii Two elements in set A get mapped to one element in set B, so the mapping is many-to-one.

iii $\{g(x) = -3, -2, 1, 6\}$

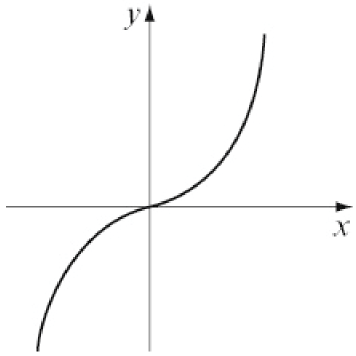
c i



ii Every element in set A gets mapped to one element in set B, so the mapping is one-to-one.

iii $\{h(x) = 1, \frac{7}{4}, 7\}$

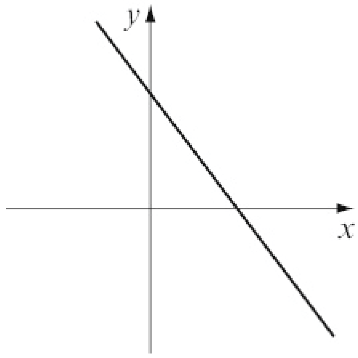
2 a



i One-to-one as each value of x is mapped to a single value of y

ii Yes, this mapping could represent a function.

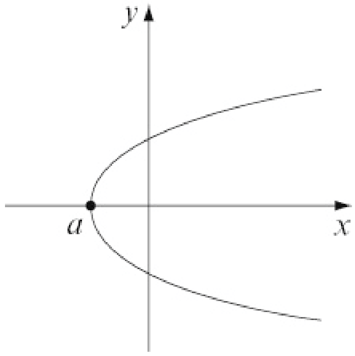
b



i One-to-one as each value of x is mapped to a single value of y

ii Yes, this mapping could represent a function.

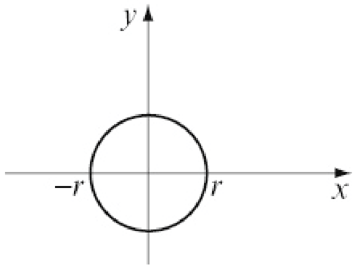
2 c



i One-to-many (see explanation in part **ii**)

ii Not a function.
Values of x which are less than a do not get mapped to a value of y.
Values of x which are greater than a get mapped to two values of y.

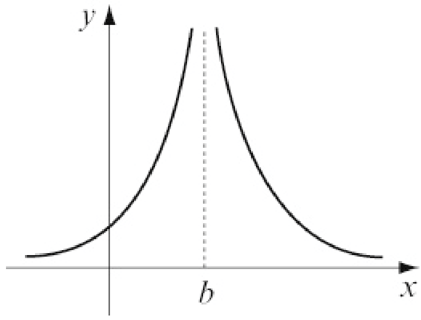
d



i One-to-many (see explanation in part **ii**)

ii Not a function.
Values of x for which $-r < x < r$ get mapped to two values of y.
Values of x for which $x < -r$ or $x > r$ don't get mapped to a value of y.

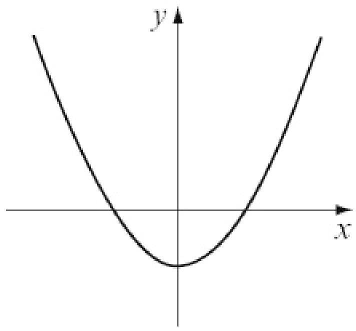
e



i One-to-one as each value of x (except for $x = b$) is mapped to a single value of y.

ii Not a function. The value $x = b$ doesn't get mapped anywhere.

f



i Many-to-one as there are two values of x which map to each value of y.

ii Yes, this mapping could represent a function.

3 a Substituting $x = a$ and $\text{p}(a) = 16$ into $\text{p}: x \mapsto 3x - 2,\ x \in \mathbb{R}$ gives:

$16 = 3a - 2$

$18 = 3a$

$a = 6$

3 b Substituting $x = b$ and $\mathrm{q}(b) = 17$ into $\mathrm{q}: x \mapsto x^2 - 3,\ x \in \mathbb{R}$ gives:

$17 = b^2 - 3$

$20 = b^2$

$b = \pm\sqrt{20}$

$b = \pm 2\sqrt{5}$

c Substituting $x = c$ and $\mathrm{r}(c) = 34$ into $\mathrm{r}: x \mapsto 2 \times 2^x + 2,\ x \in \mathbb{R}$ gives:

$34 = 2 \times 2^c + 2$

$32 = 2 \times 2^c$

$16 = 2^c$

$c = 4$

d Substituting $x = d$ and $\mathrm{s}(d) = 0$ into $\mathrm{s}: x \mapsto x^2 + x - 6,\ x \in \mathbb{R}$ gives:

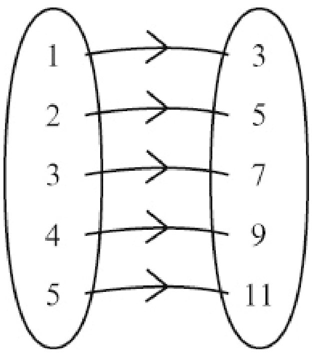
$0 = d^2 + d - 6$

$0 = (d + 3)(d - 2)$

$d = 2,\ -3$

4 a $\mathrm{f}(x) = 2x + 1$

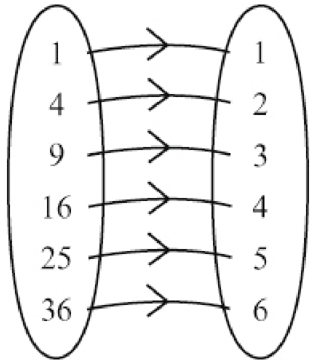
i



ii One-to-one function as each value of x maps to a single value of y.

4 b $\mathrm{g}: x \mapsto \sqrt{x}$

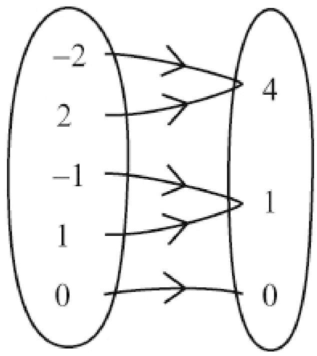
i



ii One-to-one function as each value of x maps to a single value of y.

c $\mathrm{h}(x) = x^2$

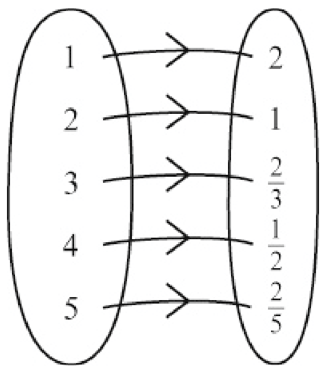
i



ii Many-to-one function as there are four values of x which map to two values of y.

d $\mathrm{j}: x \mapsto \frac{2}{x}$

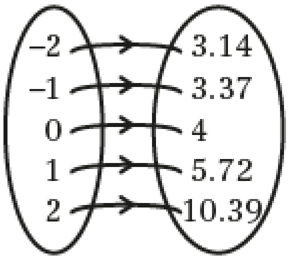
i



4 d ii One-to-one function as each value of x maps to a single value of y.

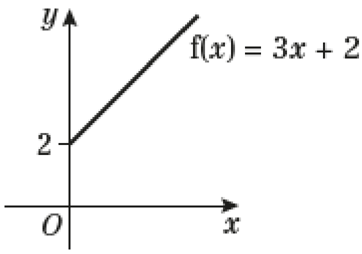
e $\text{k}(x) = \text{e}^x + 3$

i



ii Every element in set A gets mapped to one element in set B, so the mapping is one-to-one.

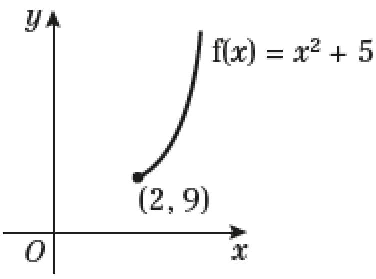
5 a i



ii Range of f(x) is $\text{f}(x) \geq 2$

iii One-to-one function as each value of x maps to a single value of y.

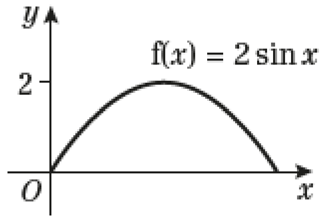
b i



ii Range of f(x) is $\text{f}(x) \geq 9$

iii One-to-one function as each value of x maps to a single value of y

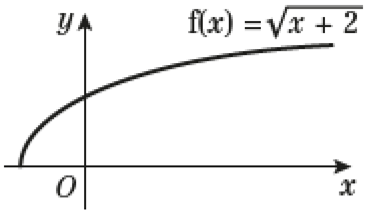
5 c i



ii Range of f(x) is $0 \leq \text{f}(x) \leq 2$

iii Many-to-one function as there are two values of x which map to a single value of y

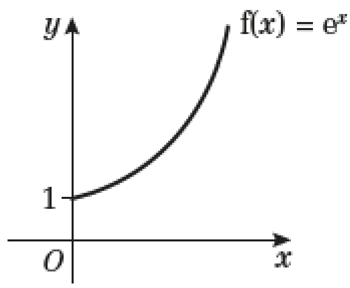
d i



ii Range of f(x) is $\text{f}(x) \geq 0$

iii One-to-one function as each value of x maps to a single value of y

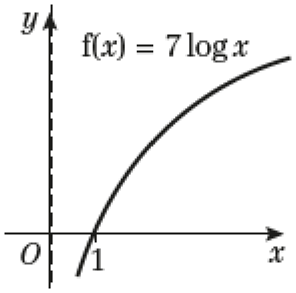
5 e i



ii Range of f(x) is $\text{f}(x) \geq 1$

iii One-to-one function as each value of x maps to a single value of y

5 f i



ii Range is $\mathrm{f}(x) \in \mathbb{R}$

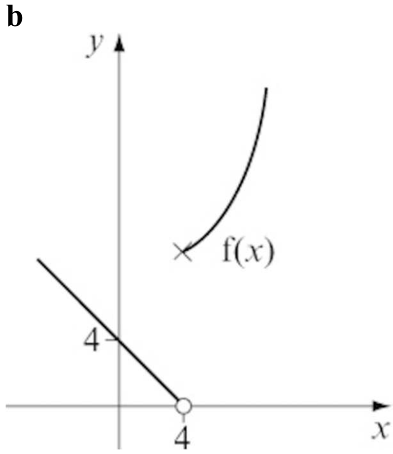
iii One-to-one function as each value of x maps to a single value of y

6 a Although g(x) is supposed to be defined on all real numbers, it does not map the element ‘4’ of the domain to any point in the range. Hence g(x) is not a function.

$\mathrm{f}(4) = 25$, so for each $x \in \mathbb{R}$ there exists a y such that $\mathrm{f}(x) = y$

Hence, $\mathrm{f}(x)$ is a function.

b



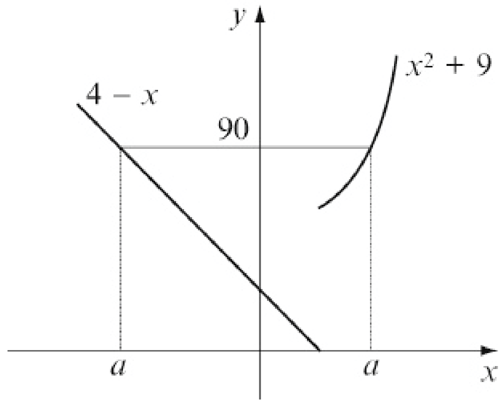
c i $\mathrm{f}(3) = 4 - 3 = 1$

(Use $4 - x$ as $3 < 4$)

ii $\mathrm{f}(10) = 10^2 + 9 = 109$

(Use $x^2 + 9$ as $10 > 4$)

6 d



The negative value of a is where

$4 - a = 90 \Rightarrow a = -86$

The positive value of a is where

$a^2 + 9 = 90$

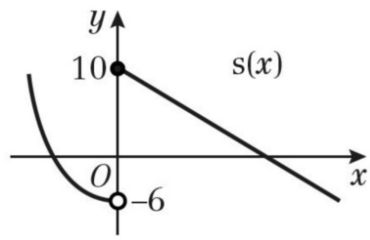
$a^2 = 81$

$a = \pm 9$

$a = 9$

The values of a are -86 and 9

7 a



b There is no solution to $10 - x = 43$ for $x \geq 0$

$\mathrm{s}(a) = 43$ only when

$x^2 - 6 = 43$

$x^2 = 49$

$x = -7$

x cannot be 7, since

$s(x) = x^2 - 6$ for $x < 0$

7 c The negative solution is where

$x^2 - 6 = x$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

As $x < 0$, $x = -2$

The positive solution is where

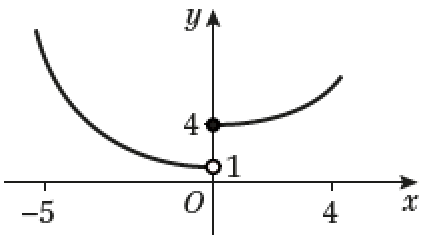
$10 - x = x$

$2x = 10$

$x = 5$

The solutions are $x = -2$ and $x = 5$

8 a



b $\mathrm{p}(a) = 50$

The negative solution is where

$e^{-a} = 50$

$-a = \ln(50)$

$a = -3.91$

The positive solution is where

$a^3 + 4 = 50$

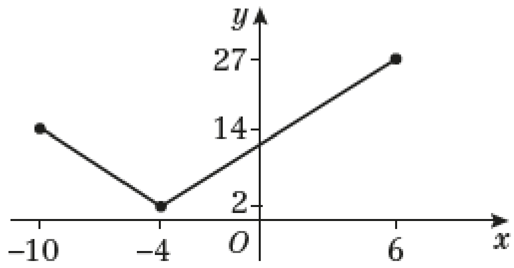
$a^3 = 46$

$a = 3.58$

The solutions are

$a = -3.91$ and $a = 3.58$

9 a



b Range of $\mathrm{h}(x)$ is $\{2 \le \mathrm{h}(x) \le 27\}$

c $\mathrm{h}(a) = 12$

One solution is for the function $\mathrm{h}(x) = -2x - 6$

$\Rightarrow -2a - 6 = 12$

$\Rightarrow a = -9$

The other solution is for the function

$\mathrm{h}(x) = \frac{5}{2}x + 12$

$\Rightarrow \frac{5}{2}a + 12 = 12$

$\Rightarrow a = 0$

The solutions are $a = -9$ and $a = 0$

10 $g(x) = cx + d$

$g(3) = 10 \Rightarrow c \times 3 + d = 10$

$\Rightarrow 3c + d = 10$ (1)

$g(8) = 12 \Rightarrow c \times 8 + d = 12$

$\Rightarrow 8c + d = 12$ (2)

(2) − (1) $\Rightarrow 5c = 2$

$\Rightarrow c = \frac{2}{5}$

Substitute $c = \frac{2}{5}$ into (1):

$3 \times \frac{2}{5} + d = 10$

$\frac{6}{5} + d = 10$

$d = \frac{44}{5}$

11 $f(x) = ax^3 + bx - 5$

$f(1) = -4 \Rightarrow a \times 1^3 + b \times 1 - 5 = -4$

$\Rightarrow a + b - 5 = -4$

$\Rightarrow a + b = 1$ (1)

$f(2) = 9 \Rightarrow a \times 2^3 + b \times 2 - 5 = 9$

$\Rightarrow 8a + 2b - 5 = 9$

$\Rightarrow 8a + 2b = 14$

$\Rightarrow 4a + b = 7$ (2)

$(2) - (1) \Rightarrow 3a = 6$

$\Rightarrow a = 2$

Substitute $a = 2$ in (1):

$2 + b = 1$

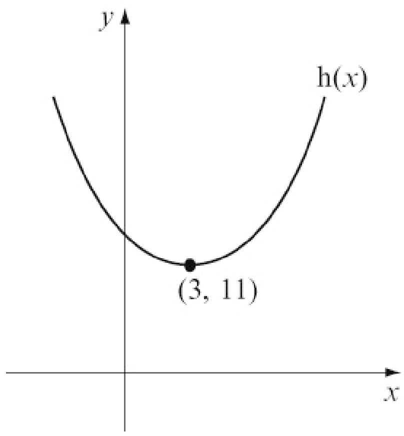
$b = -1$

12 $h(x) = x^2 - 6x + 20$

$= (x-3)^2 - 9 + 20$

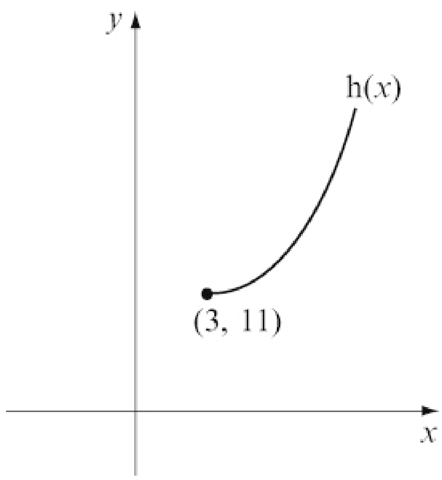
$= (x-3)^2 + 11$

This is a ∪−shaped quadratic with minimum point at (3, 11)



This is a many-to-one function.

For $h(x)$ to be one-to-one, we must restrict domain to $x \geq 3$



Hence smallest value of a is $a = 3$