Chapter review 1

1 a $\frac{3x^4 - 21x}{3x} = \frac{3x(x^3 - 7)}{3x}$

$= x^3 - 7$

b $\frac{x^2 - 2x - 24}{x^2 - 7x + 6} = \frac{(x-6)(x+4)}{(x-6)(x-1)}$

$= \frac{(x+4)}{(x-1)}$

c $\frac{2x^2 + 7x - 4}{2x^2 + 9x + 4} = \frac{(2x-1)(x+4)}{(2x+1)(x+4)}$

$= \frac{(2x-1)}{(2x+1)}$

2 $\frac{3x^3 + 12x^2 + 5x + 20}{x+4}$

$$\begin{array}{r} 3x^2 + 0x + 5 \\ x+4\overline{)3x^3 + 12x^2 + 5x + 20} \\ \underline{3x^3 + 12x^2} \\ 0 + 5x + 20 \\ \underline{5x + 20} \\ 0 \end{array}$$

So, $3x^3 + 12x^2 + 5x + 20$ divided by $x + 4$ is

$3x^2 + 5$

3 $\frac{2x^3 + 3x + 5}{x+1}$

$$\begin{array}{r} 2x^2 - 2x + 5 \\ x+1\overline{)2x^3 + 0x^2 + 3x + 5} \\ \underline{2x^3 + 2x^2} \\ -2x^2 + 3x \\ \underline{-2x^2 - 2x} \\ 5x + 5 \\ \underline{5x + 5} \\ 0 \end{array}$$

$\frac{2x^3 + 3x + 5}{x+1} = 2x^2 - 2x + 5$

4 a $\frac{x-4}{6}\times\frac{2x+8}{x^2-16}=\frac{x-4}{6}\times\frac{2(x+4)}{(x-4)(x+4)}$

$=\frac{1}{3}$

b $\frac{x^2-3x-10}{3x^2-21}\times\frac{6x^2+24}{x^2+6x+8}=\frac{(x-5)(x+2)}{3(x^2-7)}\times\frac{6(x^2+4)}{(x+2)(x+4)}$

$=\frac{2(x^2+4)(x-5)}{(x^2-7)(x+4)}$

c $\frac{4x^2+12x+9}{x^2+6x}\div\frac{4x^2-9}{2x^2+9x-18}=\frac{4x^2+12x+9}{x^2+6x}\times\frac{2x^2+9x-18}{4x^2-9}$

$=\frac{(2x+3)^2}{x(x+6)}\times\frac{(2x-3)(x+6)}{(2x-3)(2x+3)}$

$=\frac{2x+3}{x}$

5 a $\frac{4x^2-8x}{x^2-3x-4}\times\frac{x^2+6x+5}{2x^2+10x}=\frac{4x(x-2)}{(x-4)(x+1)}\times\frac{(x+1)(x+5)}{2x(x+5)}$

$=\frac{2(x-2)}{x-4}$

$=\frac{2x-4}{x-4}$

b $6=\ln\left(\left(4x^2-8x\right)\left(x^2+6x+5\right)\right)-\ln\left(\left(x^2-3x-4\right)\left(2x^2+10x\right)\right)$

$=\ln\left(\frac{\left(4x^2-8x\right)\left(x^2+6x+5\right)}{\left(x^2-3x-4\right)\left(2x^2+10x\right)}\right)$

$=\ln\left(\frac{4x(x-2)(x+1)(x+5)}{(x-4)(x+1)2x(x+5)}\right)$

$=\ln\left(\frac{2x-4}{x-4}\right)$

5 b

$$\frac{2x-4}{x-4} = e^6$$
$$2x-4 = xe^6 - 4e^6$$
$$4e^6 - 4 = xe^6 - 2x$$
$$4(e^6-1) = x(e^6-2)$$
$$x = \frac{4(e^6-1)}{e^6-2}$$

6 a

$$g(x) = \frac{4x^3-9x^2-9x}{32x+24} \div \frac{x^2-3x}{6x^2-13x-5}$$
$$= \frac{4x^3-9x^2-9x}{32x+24} \times \frac{6x^2-13x-5}{x^2-3x}$$
$$= \frac{x(4x+3)(x-3)}{8(4x+3)} \times \frac{(3x+1)(2x-5)}{x(x-3)}$$
$$= \frac{(3x+1)(2x-5)}{8}$$
$$= \frac{6x^2-13x-5}{8}$$
$$= \frac{3}{4}x^2 - \frac{13}{8}x - \frac{5}{8}$$

$$a = \frac{3}{4},\ b = -\frac{13}{8},\ c = -\frac{5}{8}$$

b

$$g'(x) = \frac{3}{2}x - \frac{13}{8}$$
$$g'(-2) = \frac{3}{2}(-2) - \frac{13}{8}$$
$$= -\frac{37}{8}$$

7

$$\frac{6x+1}{x-5}+\frac{5x+3}{x^2-3x-10}=\frac{6x+1}{x-5}+\frac{5x+3}{(x-5)(x+2)}$$

$$=\frac{(6x+1)(x+2)}{(x-5)(x+2)}+\frac{5x+3}{(x-5)(x+2)}$$

$$=\frac{6x^2+13x+2}{(x-5)(x+2)}+\frac{5x+3}{(x-5)(x+2)}$$

$$=\frac{6x^2+13x+2+5x+3}{(x-5)(x+2)}$$

$$=\frac{6x^2+18x+5}{x^2-3x-10}$$

8 $x+\frac{3}{x-1}-\frac{12}{x^2+2x-3}$

$$=\frac{x(x+3)(x-1)}{(x+3)(x-1)}+\frac{3(x+3)}{(x+3)(x-1)}-\frac{12}{(x+3)(x-1)}$$

$$=\frac{x(x+3)(x-1)+3x-3}{(x+3)(x-1)}$$

$$=\frac{(x-1)\left[x(x+3)+3\right]}{(x+3)(x-1)}$$

$$=\frac{x^2+3x+3}{x+3}$$

9

$$\begin{array}{r|l} & x^2-4x+3 \\ x-2 & x^3-6x^2+11x+2 \\ & \underline{x^3-2x^2} \\ & -4x^2+11x \\ & \underline{-4x^2+8x} \\ & 3x+2 \\ & \underline{3x-6} \\ & 8 \end{array}$$

$x^3-6x^2+11x+2\equiv(x-2)(x^2-4x+3)+8$

$A = 1$, $B = -4$, $C = 3$ and $D = 8$

10

$$\begin{array}{r}
2x^2 - 4x + 6 \\
2x+1 \overline{)\, 4x^3 - 6x^2 + 8x - 5} \\
\underline{4x^3 + 2x^2} \qquad\qquad \\
-8x^2 + 8x \qquad \\
\underline{-8x^2 - 4x} \qquad \\
12x - 5 \\
\underline{12x + 6} \\
-11
\end{array}$$

$$\frac{4x^3 - 6x^2 + 8x - 5}{2x+1} \equiv 2x^2 - 4x + 6 - \frac{11}{2x+1}$$

$A = 2$, $B = -4$, $C = 6$ and $D = -11$

11

$$\begin{array}{r}
x^2 + 1 \\
x^2 - 1 \overline{)\, x^4 + 0x^2 + 2} \\
\underline{x^4 - x^2} \qquad\quad \\
x^2 + 2 \\
\underline{x^2 - 1} \\
3
\end{array}$$

$$\frac{x^4 + 2}{x^2 - 1} \equiv x^2 + 1 + \frac{3}{x^2 - 1}$$

So $A = 1$, $B = 0$, $C = 1$, and $D = 3$

Challenge

1 $\dfrac{6x^3-7x^2+3}{3x^2+x-10}=Ax+B+\dfrac{C}{3x-5}+\dfrac{D}{x+2}$

$3x^2+x-10=(3x-5)(x+2)$

$$6x^3-7x^2+3=Ax(3x^2+x-10)+B(3x^2+x-10)+C(x+2)+D(3x-5)$$
$$=3Ax^3+Ax^2-10Ax+3Bx^2+Bx-10B+Cx+2C+3Dx-5D$$
$$=3Ax^3+(A+3B)x^2+(-10A+B+C+3D)x+(-10B+2C-5D)$$

Comparing coefficients

For x^3:

$3A=6\Rightarrow A=2$

For x^2:

$A+3B=-7$

$2+3B=-7\Rightarrow B=-3$

For x:

$-10A+B+C+3D=0$

$-10(2)+(-3)+C+3D=0$

$C+3D=23$ **(1)**

For constant:

$-10B+2C-5D=3$

$-10(-3)+2C-5D=3$

$2C-5D=-27$ **(2)**

$2\times$ equation (**1**) $-$ equation (**2**)

$2(C+3D)-(2C-5D)=2(23)-(-27)$

$2C+6D-2C+5D=46+27$

$11D=73$

$D=\dfrac{73}{11}$

Since $D=\dfrac{73}{11}$ equation **(1)** becomes:

$C+3\left(\dfrac{73}{11}\right)=23$

$C=\dfrac{34}{11}$

So $A=2$, $B=-3$, $C=\dfrac{34}{11}$ and $D=\dfrac{73}{11}$

2 $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$

$(ax^3 + bx^2 + cx + d) \div (x - p)$

$$
\begin{array}{r|l}
 & ax^2 + (b+ap)x + (c+bp+ap^2) \\
\hline
x - p & ax^3 + bx^2 + cx + d \\
 & \underline{ax^3 - apx^2} \\
 & (b+ap)x^2 + cx \\
 & \underline{(b+ap)x^2 - (bp+ap^2)x} \\
 & (c+bp+ap^2)x + d \\
 & \underline{(c+bp+ap^2)x - (cp+bp^2+ap^3)} \\
 & (d+cp+bp^2+ap^3)
\end{array}
$$

So, $\mathrm{f}(x) \div (x - p) = ax^2 + x(b + ap) + (c + bp + ap^2)$

with a remainder of $ap^3 + bp^2 + cp + d$

$\mathrm{f}(p) = ap^3 + bp^2 + cp + d = 0$

which matches the remainder,
so $(x - p)$ is a factor of $\mathrm{f}(x)$.

3 a $\mathrm{f}(x) = 2x^3 + 9x^2 + 10x + 3$

By the factor theorem if -3 is a root of $\mathrm{f}(x)$ then $\mathrm{f}(-3) = 0$.

$\mathrm{f}(-3) = 2(-3)^3 + 9(-3)^2 + 10(-3) + 3$

$= -54 + 81 - 30 + 3$

$= 0$

Therefore -3 is a root of $\mathrm{f}(x)$.

3 b $\frac{10}{\text{f}(x)} = \frac{10}{2x^3 + 9x^2 + 10x + 3}$

from part **a**, $(x + 3)$ is a factor of $2x^3 + 9x^2 + 10x + 3$

$$\begin{array}{r} 2x^2 + 3x + 1 \\ x+3 \overline{)\, 2x^3 + 9x^2 + 10x + 3} \\ \underline{2x^3 + 6x^2} \qquad\qquad \\ 3x^2 + 10x \qquad \\ \underline{3x^2 + 9x} \qquad \\ x + 3 \\ \underline{x + 3} \\ 0 \end{array}$$

So

$$\begin{aligned} 2x^3 + 9x^2 + 10x + 3 &= \left(2x^2 + 3x + 1\right)(x+3) \\ &= (2x+1)(x+1)(x+3) \end{aligned}$$

and

$$\frac{10}{2x^3 + 9x^2 + 10x + 3} = \frac{10}{(2x+1)(x+1)(x+3)}$$

$$\frac{10}{(2x+1)(x+1)(x+3)} = \frac{A}{(2x+1)} + \frac{B}{(x+1)} + \frac{C}{(x+3)}$$

$$10 = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$$

Taking $x = -1$:

$$10 = A(0)(-1+3) + B(-2+1)(-1+3) + C(-2+1)(0)$$

$$10 = B(-1)(2)$$

$$\therefore B = -5$$

Taking $x = -3$:

$$10 = A(-3+1)(0) + B(-6+1)(0) + C(-6+1)(-3+1)$$

$$10 = C(-5)(-2)$$

$$\therefore C = 1$$

Equating coefficients of x^2:

$$0 = A + 2B + 2C$$

$$0 = A + 2(-5) + 2(1)$$

$$A = 8$$

$$\frac{10}{(2x+1)(x+1)(x+3)} = \frac{8}{(2x+1)} - \frac{5}{(x+1)} + \frac{1}{(x+3)}$$