

Exercise 1B

$$1 \text{ a } \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} \\ = \frac{7}{12}$$

$$\text{b } \frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20} \\ = \frac{7}{20}$$

$$\text{c } \frac{1}{p} + \frac{1}{q} = \frac{q}{pq} + \frac{p}{pq} \\ = \frac{p+q}{pq}$$

$$\text{d } \frac{3}{4x} + \frac{1}{8x} = \frac{6}{8x} + \frac{1}{8x} \\ = \frac{7}{8x}$$

$$\text{e } \frac{3}{x^2} - \frac{1}{x} = \frac{3}{x^2} - \frac{x}{x^2} \\ = \frac{3-x}{x^2}$$

$$\text{f } \frac{a}{5b} - \frac{3}{2b} = \frac{2a}{10b} - \frac{15}{10b} \\ = \frac{2a-15}{10b}$$

$$2 \text{ a } \frac{3}{x} - \frac{2}{x+1} = \frac{3(x+1)}{x(x+1)} - \frac{2x}{x(x+1)} \\ = \frac{3x+3-2x}{x(x+1)} \\ = \frac{x+3}{x(x+1)}$$

$$\text{b } \frac{2}{x-1} - \frac{3}{x+2} \\ = \frac{2(x+2)}{(x-1)(x+2)} - \frac{3(x-1)}{(x-1)(x+2)} \\ = \frac{2(x+2)-3(x-1)}{(x-1)(x+2)} \\ = \frac{2x+4-3x+3}{(x-1)(x+2)} \\ = \frac{-x+7}{(x-1)(x+2)}$$

$$\text{c } \frac{4}{2x+1} + \frac{2}{x-1} \\ = \frac{4(x-1)}{(2x+1)(x-1)} + \frac{2(2x+1)}{(2x+1)(x-1)} \\ = \frac{4(x-1)+2(2x+1)}{(2x+1)(x-1)} \\ = \frac{4x-4+4x+2}{(2x+1)(x-1)} \\ = \frac{8x-2}{(2x+1)(x-1)}$$

$$\text{d } \frac{1}{3}(x+2) - \frac{1}{2}(x+3) \\ = \frac{2}{6}(x+2) - \frac{3}{6}(x+3) \\ = \frac{2(x+2)-3(x+3)}{6} \\ = \frac{2x+4-3x-9}{6} \\ = \frac{-x-5}{6}$$

$$\text{e } \frac{3x}{(x+4)^2} - \frac{1}{x+4} \\ = \frac{3x}{(x+4)^2} - \frac{x+4}{(x+4)^2} \\ = \frac{3x-x-4}{(x+4)^2} \\ = \frac{2x-4}{(x+4)^2}$$

$$\begin{aligned}
 2 \text{ f } & \frac{5}{2(x+3)} + \frac{4}{3(x-1)} \\
 &= \frac{15(x-1)}{6(x+3)(x-1)} + \frac{8(x+3)}{6(x+3)(x-1)} \\
 &= \frac{15(x-1) + 8(x+3)}{6(x+3)(x-1)} \\
 &= \frac{15x - 15 + 8x + 24}{6(x+3)(x-1)} \\
 &= \frac{23x + 9}{6(x+3)(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ a } & \frac{2}{x^2 + 2x + 1} + \frac{1}{x+1} \\
 &= \frac{2}{(x+1)^2} + \frac{1}{x+1} \\
 &= \frac{2}{(x+1)^2} + \frac{x+1}{(x+1)^2} \\
 &= \frac{2+x+1}{(x+1)^2} \\
 &= \frac{x+3}{(x+1)^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } & \frac{7}{x^2 - 4} + \frac{3}{x+2} \\
 &= \frac{7}{(x+2)(x-2)} + \frac{3}{x+2} \\
 &= \frac{7}{(x+2)(x-2)} + \frac{3(x-2)}{(x+2)(x-2)} \\
 &= \frac{7+3(x-2)}{(x+2)(x-2)} \\
 &= \frac{7+3x-6}{(x+2)(x-2)} \\
 &= \frac{3x+1}{(x+2)(x-2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{c } & \frac{2}{x^2 + 6x + 9} - \frac{3}{x^2 + 4x + 3} \\
 &= \frac{2}{(x+3)^2} - \frac{3}{(x+3)(x+1)} \\
 &= \frac{2(x+1)}{(x+3)^2(x+1)} - \frac{3(x+3)}{(x+3)^2(x+1)} \\
 &= \frac{2(x+1) - 3(x+3)}{(x+3)^2(x+1)} \\
 &= \frac{2x+2-3x-9}{(x+3)^2(x+1)} \\
 &= \frac{-x-7}{(x+3)^2(x+1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{d } & \frac{2}{y^2 - x^2} + \frac{3}{y-x} \\
 &= \frac{2}{(y+x)(y-x)} + \frac{3}{y-x} \\
 &= \frac{2}{(y+x)(y-x)} + \frac{3(y+x)}{(y+x)(y-x)} \\
 &= \frac{2+3(y+x)}{(y+x)(y-x)} \\
 &= \frac{3x+3y+2}{(y+x)(y-x)}
 \end{aligned}$$

$$\begin{aligned}
 \text{e } & \frac{3}{x^2 + 3x + 2} - \frac{1}{x^2 + 4x + 4} \\
 &= \frac{3}{(x+1)(x+2)} - \frac{1}{(x+2)^2} \\
 &= \frac{3(x+2)}{(x+1)(x+2)^2} - \frac{(x+1)}{(x+1)(x+2)^2} \\
 &= \frac{3(x+2) - (x+1)}{(x+1)(x+2)^2} \\
 &= \frac{3x+6-x-1}{(x+1)(x+2)^2} \\
 &= \frac{2x+5}{(x+1)(x+2)^2}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad \text{f} \quad & \frac{x+2}{x^2-x-12} - \frac{x+1}{x^2+5x+6} \\
 &= \frac{x+2}{(x-4)(x+3)} - \frac{x+1}{(x+2)(x+3)} \\
 &= \frac{(x+2)^2}{(x-4)(x+2)(x+3)} - \frac{(x+1)(x-4)}{(x-4)(x+2)(x+3)} \\
 &= \frac{(x+2)^2 - (x+1)(x-4)}{(x-4)(x+2)(x+3)} \\
 &= \frac{x^2+4x+4-x^2+3x+4}{(x-4)(x+2)(x+3)} \\
 &= \frac{7x+8}{(x-4)(x+2)(x+3)}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad & \frac{6x+1}{x^2+2x-15} - \frac{4}{x-3} \\
 &= \frac{6x+1}{(x+5)(x-3)} - \frac{4}{x-3} \\
 &= \frac{6x+1}{(x+5)(x-3)} - \frac{4(x+5)}{(x+5)(x-3)} \\
 &= \frac{6x+1-4(x+5)}{(x+5)(x-3)} \\
 &= \frac{6x+1-4x-20}{(x+5)(x-3)} \\
 &= \frac{2x-19}{(x+5)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \text{a} \quad & \frac{3}{x} + \frac{2}{x+1} + \frac{1}{x+2} \\
 &= \frac{3(x+1)(x+2)}{x(x+1)(x+2)} + \frac{2x(x+2)}{x(x+1)(x+2)} \\
 &\quad + \frac{x(x+1)}{x(x+1)(x+2)} \\
 &= \frac{3(x+1)(x+2) + 2x(x+2) + x(x+1)}{x(x+1)(x+2)} \\
 &= \frac{3x^2+9x+6+2x^2+4x+x^2+x}{x(x+1)(x+2)} \\
 &= \frac{6x^2+14x+6}{x(x+1)(x+2)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b} \quad & \frac{4}{3x} - \frac{2}{x-2} + \frac{1}{2x+1} \\
 &= \frac{4(x-2)(2x+1)}{3x(x-2)(2x+1)} - \frac{6x(2x+1)}{3x(x-2)(2x+1)} \\
 &\quad + \frac{3x(x-2)}{3x(x-2)(2x+1)} \\
 &= \frac{4(x-2)(2x+1) - 6x(2x+1) + 3x(x-2)}{3x(x-2)(2x+1)} \\
 &= \frac{8x^2-12x-8-12x^2-6x+3x^2-6x}{3x(x-2)(2x+1)} \\
 &= \frac{-x^2-24x-8}{3x(x-2)(2x+1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{c} \quad & \frac{3}{x-1} + \frac{2}{x+1} + \frac{4}{x-3} \\
 &= \frac{3(x+1)(x-3)}{(x-1)(x+1)(x-3)} + \frac{2(x-1)(x-3)}{(x-1)(x+1)(x-3)} \\
 &\quad + \frac{4(x-1)(x+1)}{(x-1)(x+1)(x-3)} \\
 &= \frac{3(x+1)(x-3) + 2(x-1)(x-3) + 4(x-1)(x+1)}{(x-1)(x+1)(x-3)} \\
 &= \frac{3x^2-6x-9+2x^2-8x+6+4x^2-4}{(x-1)(x+1)(x-3)} \\
 &= \frac{9x^2-14x-7}{(x-1)(x+1)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 6 \quad & \frac{4(2x-1)}{36x^2-1} + \frac{7}{6x-1} \\
 &= \frac{4(2x-1)}{(6x-1)(6x+1)} + \frac{7}{6x-1} \\
 &= \frac{4(2x-1)}{(6x-1)(6x+1)} + \frac{7(6x+1)}{(6x-1)(6x+1)} \\
 &= \frac{4(2x-1) + 7(6x+1)}{(6x-1)(6x+1)} \\
 &= \frac{8x-4+42x+7}{(6x-1)(6x+1)} \\
 &= \frac{50x+3}{(6x-1)(6x+1)}
 \end{aligned}$$

$$\begin{aligned}
 7 \text{ a } g(x) &= x + \frac{6}{x+2} + \frac{36}{x^2 - 2x - 8} \\
 &= x + \frac{6}{x+2} + \frac{36}{(x-4)(x+2)} \\
 &= \frac{x(x+2)(x-4)}{(x+2)(x-4)} + \frac{6(x-4)}{(x+2)(x-4)} \\
 &\quad + \frac{36}{(x+2)(x-4)} \\
 &= \frac{x(x+2)(x-4) + 6(x-4) + 36}{(x+2)(x-4)} \\
 &= \frac{x^3 - 2x^2 - 8x + 6x - 24 + 36}{(x+2)(x-4)} \\
 &= \frac{x^3 - 2x^2 - 2x + 12}{(x+2)(x-4)}
 \end{aligned}$$

b Using the factor theorem,

$$(-2)^3 - 2(-2)^2 - 2(-2) + 12 = 0$$

So $(x+2)$ is a factor of

$$x^3 - 2x^2 - 2x + 12$$

Hence, you can write

$$x^3 - 2x^2 - 2x + 12 = (x+2) \times p(x)$$

for some quadratic polynomial

$p(x)$.

You can find $p(x)$ by long division:

$$\begin{array}{r}
 \overline{x^2 - 4x + 6} \\
 x+2 \overline{) x^3 - 2x^2 - 2x + 12} \\
 \underline{x^3 + 2x^2} \\
 -4x^2 - 2x \\
 \underline{-4x^2 - 8x} \\
 6x + 12 \\
 \underline{6x + 12} \\
 0
 \end{array}$$

Hence, $p(x) = x^2 - 4x + 6$ and so

$$\begin{aligned}
 g(x) &= \frac{(x+2)(x^2 - 4x + 6)}{(x+2)(x-4)} \\
 &= \frac{x^2 - 4x + 6}{x-4}
 \end{aligned}$$