Core Mathematics 3 Paper L

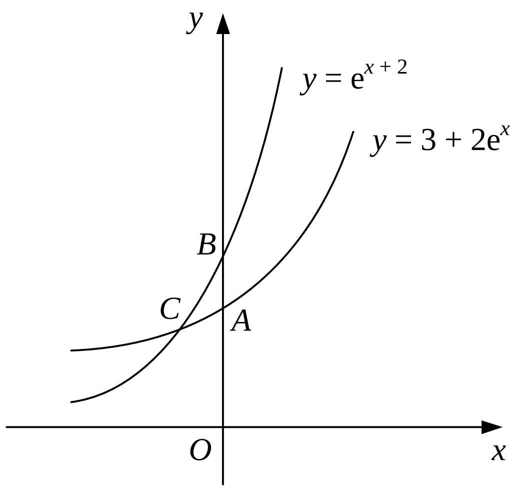
1. *(i)* Differentiate $x^3 \ln x$ with respect to x. [2]

(ii) Given that

$$x = \frac{y+1}{3-2y},$$

find and simplify an expression for $\frac{dy}{dx}$ in terms of y. [4]

2.



The diagram shows the curves $y = 3 + 2e^x$ and $y = e^{x+2}$ which cross the y-axis at the points A and B respectively.

(i) Write down the coordinates of A and B. [2]

The two curves intersect at the point C.

(ii) Find an expression for the x-coordinate of C and show that the y-coordinate of C is $\frac{3e^2}{e^2-2}$. [5]

3. The functions f and g are defined by

$$f(x) \equiv 6x - 1, \quad x \in \mathbb{R},$$

$$g(x) \equiv \log_2 (3x + 1), \quad x \in \mathbb{R}, \quad x > -\tfrac{1}{3}.$$

(i) Evaluate gf(1). [2]

(ii) Find an expression for $g^{-1}(x)$. [2]

(iii) Find, in terms of natural logarithms, the solution of the equation

$$fg^{-1}(x) = 2.$$ [4]

4. *(i)* Use the identity for $\cos(A + B)$ to prove that

$$\cos 2x \equiv 2\cos^2 x - 1.$$ [2]

(ii) Prove that, for $\cos x \neq 0$,

$$2\cos x - \sec x \equiv \sec x \cos 2x.$$ [3]

(iii) Hence, or otherwise, find the values of x in the interval $0 \leq x \leq 180°$ for which

$$2\cos x - \sec x \equiv 2\cos 2x.$$ [4]

5. *(i)* Show that the equation

$$2\sin x + \sec(x + \tfrac{\pi}{6}) = 0$$

can be written as

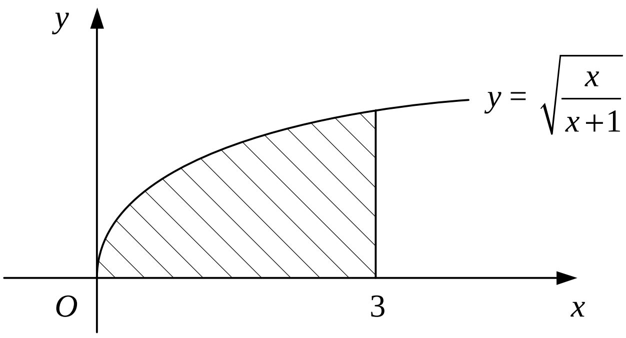
$$\sqrt{3}\sin x\cos x + \cos^2 x = 0.$$ [5]

(ii) Hence, or otherwise, find in terms of π the solutions of the equation

$$2\sin x + \sec(x + \tfrac{\pi}{6}) = 0$$

for x in the interval $0 \leq x \leq \pi$. [4]

6.



The diagram shows the curve with equation $y = \sqrt{\dfrac{x}{x+1}}$.

The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(i) Use Simpson's rule with six strips to estimate the area of the shaded region. [4]

The shaded region is rotated through four right angles about the x-axis.

(ii) Show that the volume of the solid formed is $\pi(3 - \ln 4)$. [6]

Turn over

7. *(i)* Sketch on the same diagram the graphs of $y = 4a^2 - x^2$ and $y = |2x - a|$, where a is a positive constant. Show, in terms of a, the coordinates of any points where each graph meets the coordinate axes. [5]

(ii) Find the exact solutions of the equation

$$4 - x^2 = |2x - 1|.$$ [6]

8. A curve has the equation $y = \frac{e^2}{x} + e^x, \quad x \neq 0$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Show that the curve has a stationary point in the interval [1.3, 1.4]. [3]

The point A on the curve has x-coordinate 2.

(iii) Show that the tangent to the curve at A passes through the origin. [4]

The tangent to the curve at A intersects the curve again at the point B.

The x-coordinate of B is to be estimated using the iterative formula

$$x_{n+1} = -\tfrac{2}{3}\sqrt{3 + 3x_n e^{x_n - 2}},$$

with $x_0 = -1$.

(iv) Find x_1, x_2 and x_3 to 7 significant figures and hence state the x-coordinate of B to 5 significant figures. [3]