

C3 Paper L – Marking Guide

1.	(i)	$= 3x^2 \times \ln x + x^3 \times \frac{1}{x} = x^2(3 \ln x + 1)$	M1 A1
	(ii)	$\frac{dx}{dy} = \frac{1 \times (3-2y) - (y+1) \times (-2)}{(3-2y)^2} = \frac{5}{(3-2y)^2}$	M1 A1
		$\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{1}{5}(3-2y)^2$	M1 A1 (6)
<hr/>			
2.	(i)	A (0, 5), B (0, e ²)	B2
	(ii)	$3 + 2e^x = e^{x+2} = e^2 e^x$	M1
		$3 = e^x(e^2 - 2), \quad e^x = \frac{3}{e^2 - 2}$	M1
		$x = \ln \frac{3}{e^2 - 2}$	A1
		$\therefore y = e^2 e^x = e^2 \times \frac{3}{e^2 - 2} = \frac{3e^2}{e^2 - 2}$	M1 A1 (7)
<hr/>			
3.	(i)	$= g(5) = \log_2 16 = 4$	M1 A1
	(ii)	$y = \log_2(3x + 1), \quad 3x + 1 = 2^y$	M1
		$x = \frac{1}{3}(2^y - 1), \quad g^{-1}(x) = \frac{1}{3}(2^x - 1)$	A1
	(iii)	$fg^{-1}(x) = f[\frac{1}{3}(2^x - 1)] = 2(2^x - 1) - 1 = 2(2^x) - 3$	M1
		$\therefore 2(2^x) - 3 = 2, \quad 2^x = \frac{5}{2}$	A1
		$x = \frac{\ln \frac{5}{2}}{\ln 2} \text{ or } \frac{\ln 5 - \ln 2}{\ln 2}$	M1 A1 (8)
<hr/>			
4.	(i)	$\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$ let $A = B = x$ $\cos 2x \equiv \cos^2 x - \sin^2 x$	M1
		$\cos 2x \equiv \cos^2 x - (1 - \cos^2 x)$ $\cos 2x \equiv 2 \cos^2 x - 1$	A1
	(ii)	$\text{LHS} \equiv 2 \cos x - \frac{1}{\cos x} \equiv \frac{2 \cos^2 x - 1}{\cos x}$	M1
		$\equiv \frac{\cos 2x}{\cos x} \equiv \sec x \cos 2x \equiv \text{RHS}$	M1 A1
	(iii)	$\sec x \cos 2x = 2 \cos 2x$ $\cos 2x(\sec x - 2) = 0$ $\cos 2x = 0$ or $\sec x = 2$ $2x = 90, 270$ or $\cos x = \frac{1}{2}$ $x = 45^\circ, 60^\circ, 135^\circ$	M1 A1 A2 (9)
<hr/>			
5.	(i)	$2 \sin x = -\frac{1}{\cos(x + \frac{\pi}{6})}, \quad 2 \sin x \cos(x + \frac{\pi}{6}) = -1$	M1
		$2 \sin x [\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6}] = -1$	M1
		$2 \sin x [\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x] = -1$ $\sqrt{3} \sin x \cos x - \sin^2 x = -1$	A1
		$\sqrt{3} \sin x \cos x - (1 - \cos^2 x) = -1$	M1
		$\sqrt{3} \sin x \cos x + \cos^2 x = 0$	A1
	(ii)	$\cos x (\sqrt{3} \sin x + \cos x) = 0$ $\cos x = 0$ or $\tan x = -\frac{1}{\sqrt{3}}$ $x = \frac{\pi}{2}, \frac{5\pi}{6}$	M1 M1 A2 (9)

6. (i)

x	0	0.5	1	1.5	2	2.5	3	
y	0.5774	0.8660	0.7071	0.7746	0.8165	0.8452	0.8660	M1 A1

$$\text{area} \approx \frac{1}{3} \times 0.5 \times [0 + 0.8660 + 4(0.5774 + 0.7746 + 0.8452) + 2(0.7071 + 0.8165)]$$

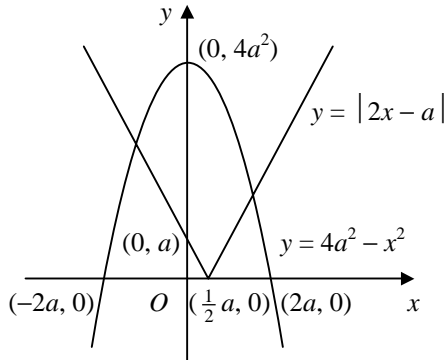
$$= 2.12 \text{ (3sf)} \quad \text{M1}$$

(ii)
$$= \pi \int_0^3 \frac{x}{x+1} dx \quad \text{M1}$$

$$= \pi \int_0^3 \frac{x+1-1}{x+1} dx = \pi \int_0^3 \left(1 - \frac{1}{x+1}\right) dx \quad \text{M1}$$

$$= \pi [x - \ln|x+1|]_0^3 \quad \text{M1 A1}$$

$$= \pi \{(3 - \ln 4) - (0)\} = \pi(3 - \ln 4) \quad \text{M1 A1} \quad \mathbf{(10)}$$

7. (i) 

B3

B2

(ii)
$$4 - x^2 = 2x - 1 \quad \text{M1}$$

$$x^2 + 2x - 5 = 0$$

$$x = \frac{-2 \pm \sqrt{4 + 20}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} \quad \text{M1}$$

$$x > \frac{1}{2} \therefore x = -1 + \sqrt{6} \quad \text{A1}$$

$$4 - x^2 = -(2x - 1) \quad \text{M1}$$

$$x^2 - 2x - 3 = 0$$

$$(x + 1)(x - 3) = 0 \quad \text{M1}$$

$$x < \frac{1}{2} \therefore x = -1, \quad x = -1, -1 + \sqrt{6} \quad \text{A1} \quad \mathbf{(11)}$$

8. (i)
$$\frac{dy}{dx} = -e^2 x^{-2} + e^x \quad \text{M1 A1}$$

(ii) SP: $-e^2 x^{-2} + e^x = 0 \quad \text{M1}$
let $f(x) = -e^2 x^{-2} + e^x$
 $f(1.3) = -0.70, f(1.4) = 0.29 \quad \text{M1}$
sign change, $f(x)$ continuous \therefore root A1

(iii) $x = 2, y = \frac{3}{2} e^2, \text{grad} = \frac{3}{4} e^2 \quad \text{M1}$
 $\therefore y - \frac{3}{2} e^2 = \frac{3}{4} e^2 (x - 2) \quad \text{M1 A1}$
 $y = \frac{3}{4} e^2 x$
 $\therefore x = 0 \Rightarrow y = 0$ so passes through origin A1

(iv) $x_1 = -1.125589, x_2 = -1.125803, x_3 = -1.125804$ (7sf) M1 A1
 \therefore x-coordinate of B = -1.1258 (5sf) $\text{A1} \quad \mathbf{(12)}$

Total $\mathbf{(72)}$