# Core Mathematics 3 Paper K

**1.** Show that

$$\int_1^7 \frac{2}{4x-1}\,dx = \ln 3. \qquad [4]$$

**2.** Find the set of values of $x$ such that

$$|3x + 1| \leq |x - 2|. \qquad [5]$$

**3.** Find all values of $\theta$ in the interval $-180 < \theta < 180$ for which

$$\tan^2 \theta° + \sec \theta° = 1. \qquad [6]$$

**4.** Solve each equation, giving your answers in exact form.

*(i)* $e^{4x-3} = 2$ [2]

*(ii)* $\ln(2y - 1) = 1 + \ln(3 - y)$ [4]

**5.** *(i)* Prove, by counter-example, that the statement

"$\text{cosec}\,\theta - \sin\theta > 0$ for all values of $\theta$ in the interval $0 < \theta < \pi$"

is false. [2]

*(ii)* Find the values of $\theta$ in the interval $0 < \theta < \pi$ such that

$$\text{cosec}\,\theta - \sin\theta = 2,$$

giving your answers to 2 decimal places. [5]

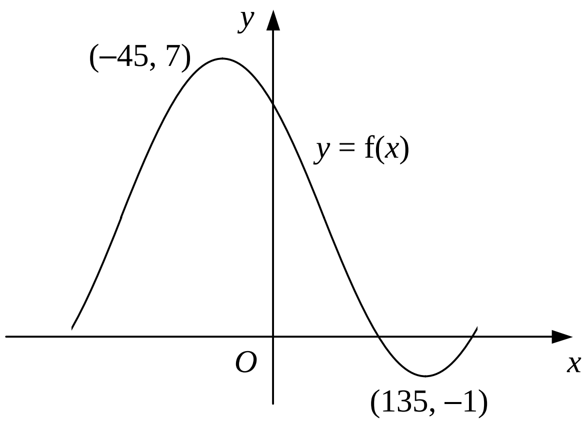
**6.** The curve $C$ has the equation $y = x^2 - 5x + 2\ln\frac{x}{3}$, $x > 0$.

*(i)* Show that the normal to $C$ at the point where $x = 3$ has the equation

$$3x + 5y + 21 = 0. \qquad [5]$$

*(ii)* Find the $x$-coordinates of the stationary points of $C$. [3]

**7.**




The diagram shows the curve $y = \mathrm{f}(x)$ which has a maximum point at (–45, 7) and a minimum point at (135, –1).

*(i)* Showing the coordinates of any stationary points, sketch the curve with equation $y = 1 + 2\mathrm{f}(x)$. [3]

Given that

$$\mathrm{f}(x) = A + 2\sqrt{2}\cos x° - 2\sqrt{2}\sin x°, \quad x \in \mathbb{R}, \quad -180 \le x \le 180,$$

where $A$ is a constant,

*(ii)* show that f($x$) can be expressed in the form

$$\mathrm{f}(x) = A + R\cos(x + \alpha)°,$$

where $R > 0$ and $0 < \alpha < 90$, [3]

*(iii)* state the value of $A$, [1]

*(iv)* find, to 1 decimal place, the $x$-coordinates of the points where the curve $y = \mathrm{f}(x)$ crosses the $x$-axis. [4]

***Turn over***

**8.** The function f is defined by

$$f(x) \equiv 3 - x^2, \quad x \in \mathbb{R}, \quad x \geq 0.$$

*(i)* State the range of f. [1]

*(ii)* Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. [3]

*(iii)* Find an expression for $f^{-1}(x)$ and state its domain. [3]

The function g is defined by

$$g(x) \equiv \frac{8}{3-x}, \quad x \in \mathbb{R}, \quad x \neq 3.$$

*(iv)* Evaluate fg(–3). [2]

*(v)* Solve the equation

$$f^{-1}(x) = g(x).$$ [3]

**9.** A curve has the equation $y = (2x + 3)\,e^{-x}$.

*(i)* Find the exact coordinates of the stationary point of the curve. [4]

The curve crosses the $y$-axis at the point $P$.

*(ii)* Find an equation for the normal to the curve at $P$. [2]

The normal to the curve at $P$ meets the curve again at $Q$.

*(iii)* Show that the $x$-coordinate of $Q$ lies between –2 and –1. [3]

*(iv)* Use the iterative formula

$$x_{n+1} = \frac{3-3e^{x_n}}{e^{x_n}-2},$$

with $x_0 = -1$, to find $x_1$, $x_2$, $x_3$ and $x_4$. Give the value of $x_4$ to 2 decimal places. [2]

*(v)* Show that your value for $x_4$ is the $x$-coordinate of $Q$ correct to 2 decimal places. [2]