C3 Paper K – Marking Guide

1. $I = [\frac{1}{2}\ln|4x-1|]_1^7$ — M1 A1

$= \frac{1}{2}(\ln 27 - \ln 3)$ — M1

$= \frac{1}{2}\ln 9 = \ln 9^{\frac{1}{2}} = \ln 3$ — A1 **(4)**

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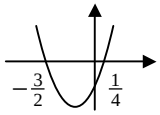
2. $(3x+1)^2 \le (x-2)^2$ — M1

$8x^2 + 10x - 3 \le 0$ — A1

$(4x-1)(2x+3) \le 0$ — M1

$-\frac{3}{2} \le x \le \frac{1}{4}$ — A2 **(5)**



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3. $\sec^2\theta - 1 + \sec\theta = 1$ — M1

$\sec^2\theta + \sec\theta - 2 = 0$

$(\sec\theta + 2)(\sec\theta - 1) = 0$ — M1

$\sec\theta = -2$ or 1 — A1

$\cos\theta = -\frac{1}{2}$ or 1 — M1

$\theta = 180 - 60, -180 + 60$ or 0

$\theta = -120, 0, 120$ — A2 **(6)**

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4. *(i)* $4x - 3 = \ln 2$ — M1

$x = \frac{1}{4}(3 + \ln 2)$ — A1

(ii) $\ln(2y-1) - \ln(3-y) = \ln\frac{2y-1}{3-y} = 1$ — M1

$\frac{2y-1}{3-y} = e$ — A1

$2y - 1 = e(3-y)$, $\quad y(e+2) = 3e+1$ — M1

$y = \frac{3e+1}{e+2}$ — A1 **(6)**

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5. *(i)* if $\theta = \frac{\pi}{2}$, $\sin\theta = 1$, $\operatorname{cosec}\theta = 1$ — M1

$\therefore \operatorname{cosec}\theta - \sin\theta = 1 - 1 = 0$

$\therefore$ statement is false — A1

(ii) $1 - \sin^2\theta = 2\sin\theta$ — M1

$\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = \frac{-2 \pm \sqrt{4+4}}{2} = -1 - \sqrt{2}$ (no solutions) or $-1 + \sqrt{2}$ — M1 A1

$\theta = 0.4271, \pi - 0.4271$

$\theta = 0.43, 2.71$ (2dp) — A2 **(7)**

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6. *(i)* $\frac{dy}{dx} = 2x - 5 + \frac{2}{x}$ — M1

$x = 3$, $y = -6$, grad $= \frac{5}{3}$ — A1

grad of normal $= -\frac{3}{5}$ — M1

$\therefore\ y + 6 = -\frac{3}{5}(x-3)$ — M1

$5y + 30 = -3x + 9$

$3x + 5y + 21 = 0$ — A1

(ii) SP: $2x - 5 + \frac{2}{x} = 0$, $\quad 2x^2 - 5x + 2 = 0$ — M1

$(2x-1)(x-2) = 0$ — M1

$x = \frac{1}{2}, 2$ — A1 **(8)**

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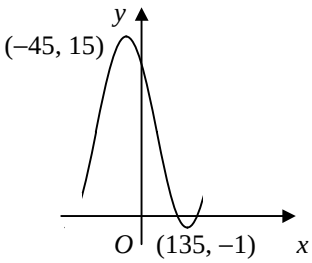
7. *(i)*



M2 A1

(ii) $2\sqrt{2}\cos x - 2\sqrt{2}\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = 2\sqrt{2}$, $R\sin\alpha = 2\sqrt{2}$ M1

$\therefore\ R = \sqrt{8+8} = 4$ A1

$\tan\alpha = 1$, $\alpha = 45$ A1

$\therefore\ f(x) = A + 4\cos(x + 45)°$

(iii) 3 B1

(iv) $3 + 4\cos(x + 45) = 0$

$\cos(x + 45) = -\frac{3}{4}$ M1

$x + 45 = 180 - 41.4, 180 + 41.4 = 138.6, 221.4$ M1

$x = 93.6, 176.4$ (1dp) A2 **(11)**

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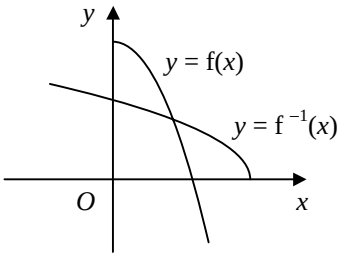
8. *(i)* $f(x) \leq 3$ B1

(ii)



B3

(iii) $y = 3 - x^2$, $x^2 = 3 - y$, $x = \pm\sqrt{3-y}$ M1

$f^{-1}(x) = \sqrt{3-x}$, $x \in \mathbb{R}$, $x \leq 3$ A2

(iv) $= f(\frac{4}{3}) = \frac{11}{9}$ M1 A1

(v) $\sqrt{3-x} = \frac{8}{3-x}$, $3 - x = \frac{64}{(3-x)^2}$ M1

$(3 - x)^3 = 64$, $3 - x = 4$ M1

$x = -1$ A1 **(12)**

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9. *(i)* $\frac{dy}{dx} = 2 \times e^{-x} + (2x + 3) \times (-e^{-x}) = -(2x + 1)e^{-x}$ M1 A1

SP: $-(2x + 1)e^{-x} = 0$

$x = -\frac{1}{2}$ $\therefore\ (-\frac{1}{2}, 2e^{\frac{1}{2}})$ M1 A1

(ii) $x = 0$, $y = 3$, grad $= -1$, grad of normal $= 1$ M1

$\therefore\ y = x + 3$ A1

(iii) $x + 3 = (2x + 3)e^{-x}$, $x + 3 - (2x + 3)e^{-x} = 0$ M1

let $f(x) = x + 3 - (2x + 3)e^{-x}$

$f(-2) = 8.4$, $f(-1) = -0.72$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

(iv) $x_1 = -1.1619$, $x_2 = -1.2218$, $x_3 = -1.2408$, $x_4 = -1.2465 = -1.25$ (2dp) M1 A1

(v) $f(-1.255) = 0.026$, $f(-1.245) = -0.016$ M1

sign change, $f(x)$ continuous $\therefore$ root A1 **(13)**

Total **(72)**