Must work in rads

C3 Paper J – Marking Guide

$h = {}^{3}/_{4} = 0.75$

1.

x	0	0.75	1.5	2.25	3	
$e^{\cos x}$	2.7183	2.0786	1.0733	0.5336	0.3716	M1 A1

$I \approx \frac{1}{3} \times 0.75 \times [2.7183 + 0.3716 + 4(2.0786 + 0.5336) + 2(1.0733)]$ M1

$= 3.92$ (3sf) A1 **(4)**

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2. $1+\tan^2 2\theta = \sec^2 2\theta$

$5(\sec^2 2\theta - 1) - 13 \sec 2\theta = 1$ M1

$5 \sec^2 2\theta - 13 \sec 2\theta - 6 = 0$

$(5 \sec 2\theta + 2)(\sec 2\theta - 3) = 0$ M1

$\sec 2\theta = -\frac{2}{5}$ or 3 A1

$\cos 2\theta = -\frac{5}{2}$ (no solutions) or $\frac{1}{3}$

$2\theta = 70.529, 360 - 70.529, 360 + 70.529, 720 - 70.529$ M1

$= 70.529, 289.471, 430.529, 649.471$

$\theta = 35.3°, 144.7°, 215.3°, 324.7°$ (1dp) A3 **(7)**

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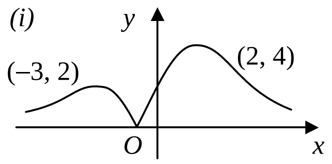
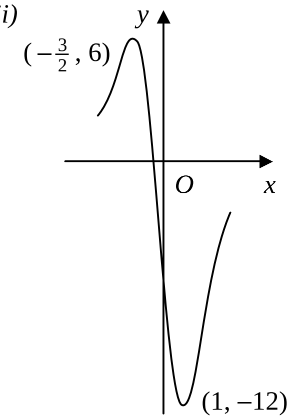
3. *(a)* *(i)*



(ii)



M1 A1

M2 A1

(b) $a = 4$, $b = 2$ B2 **(7)**

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4. $\frac{\tan x + \tan 45}{1 - \tan x \tan 45} - \tan x = 4$ M1

$\frac{\tan x + 1}{1 - \tan x} = 4 + \tan x$

$\tan x + 1 = (4 + \tan x)(1 - \tan x)$ M1

$\tan x + 1 = 4 - 3 \tan x - \tan^2 x$

$\tan^2 x + 4 \tan x - 3 = 0$ A1

$\tan x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$ M1

$x = 180 - 77.9, -77.9$ or $32.9, -180 + 32.9$

$x = -147.1, -77.9, 32.9, 102.1$ (1dp) A3 **(7)**

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5. *(i)* $= \int_{\frac{2}{3}}^{3} \sqrt[3]{3x-1}\ dx$

$= [\frac{1}{4}(3x-1)^{\frac{4}{3}}]_{\frac{2}{3}}^{3}$ M1 A1

$= \frac{1}{4}(16 - 1) = \frac{15}{4}$ M1 A1

(ii) $= \pi \int_{\frac{2}{3}}^{3} (3x-1)^{\frac{2}{3}}\ dx$ M1

$= \pi[\frac{1}{5}(3x-1)^{\frac{5}{3}}]_{\frac{2}{3}}^{3}$ A1

$= \frac{1}{5}\pi(32 - 1) = \frac{31}{5}\pi$ M1 A1 **(8)**

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6. (i) $y = 1 - ax$, $x = \frac{1-y}{a}$ — M1

$f^{-1}(x) = \frac{1-x}{a}$ — A1

(ii) $g(x) = (x + a)^2 - a^2 + 2$ — M1 A1

or use y`=2x+2a=0 for TP x = -a so y = 2-2a² y>2-2a²

$\therefore$ $g(x) \geq 2 - a^2$ — A1

(iii) $gf(3) = g(1 - 3a) = (1 - 3a)^2 + 2a(1 - 3a) + 2$ — M1

$\therefore$ $1 - 6a + 9a^2 + 2a - 6a^2 + 2 = 7$

$3a^2 - 4a - 4 = 0$ — A1

$(3a + 2)(a - 2) = 0$ — M1

$a = -\frac{2}{3}, 2$ — A1 **(9)**

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7. (i) $(4, 0)$ — B1

(ii) $\frac{dy}{dx} = \frac{5}{2}x^{\frac{3}{2}} \times \ln \frac{x}{4} + x^{\frac{5}{2}} \times \frac{1}{x} = \frac{1}{2}x^{\frac{3}{2}}(5 \ln \frac{x}{4} + 2)$ — M1 A1

grad = 8, grad of normal = $-\frac{1}{8}$ — A1

$\therefore$ $y - 0 = -\frac{1}{8}(x - 4)$ — M1

at Q, $x = 0$, $y = \frac{1}{2}$ — M1

area = $\frac{1}{2} \times \frac{1}{2} \times 4 = 1$ — A1

(iii) $\frac{1}{2}x^{\frac{3}{2}}(5 \ln \frac{x}{4} + 2) = 0$

$x > 0$ $\therefore$ $\ln \frac{x}{4} = -\frac{2}{5}$ — M1

$x = 4e^{-\frac{2}{5}}$ — A1 **(9)**

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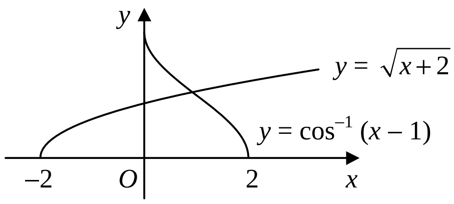
8. (i) $\cos^{-1}\theta = \frac{\pi}{3}$, $\theta = \cos \frac{\pi}{3} = \frac{1}{2}$ — M1 A1

(ii)



— B3

(iii) let $f(x) = \cos^{-1}(x - 1) - \sqrt{x+2}$

$f(0) = 1.7$, $f(1) = -0.16$ — M1

sign change, $f(x)$ continuous $\therefore$ root — A1

(iv) $x_1 = 0.83944$, $x_2 = 0.88598$, $x_3 = 0.87233$,

$x_4 = 0.87632$, $x_5 = 0.87515$, $x_6 = 0.87549$ — M1 A1

$\therefore$ $\alpha = 0.875$ (3dp) — A1 **(10)**

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9. (i) $t = 3$, $N = 18\,000$ $\Rightarrow$ $18\,000 = 2000e^{3k}$

$e^{3k} = 9$ — M1

$k = \frac{1}{3}\ln 9 = 0.732$ (3sf) — M1 A1

(ii) $4000 = 2000e^{0.7324t}$ — B1

$t = \frac{1}{0.7324}\ln 2 = 0.9464$ hours — M2

$\therefore$ doubles in 57 minutes (nearest minute) — A1

(iii) $N = 2000e^{0.7324t}$, $\frac{dN}{dt} = 0.7324 \times 2000e^{0.7324t} = 1465e^{0.7324t}$ — M1 A1

when $t = 3$, $\frac{dN}{dt} = 13\,200$ $\therefore$ increasing at rate of 13 200 per hour (3sf) — M1 A1 **(11)**

Total **(72)**