# Core Mathematics 3 Paper I

**1.** A balloon is filled with air at a constant rate of 80 $cm^3$ per second.

Assuming that the balloon is spherical as it is filled, find to 3 significant figures the rate at which its radius is increasing at the instant when its radius is 6 cm. [5]

**2.** Solve the equation

$$3\,\text{cosec}\,\theta^\circ + 8\cos\theta^\circ = 0$$

for $\theta$ in the interval $0 \leq \theta \leq 180$, giving your answers to 1 decimal place. [6]

**3.** *(a)* Given that $y = \ln x$,

*(i)* find an expression for $\ln \frac{x^2}{e}$ in terms of $y$, [2]

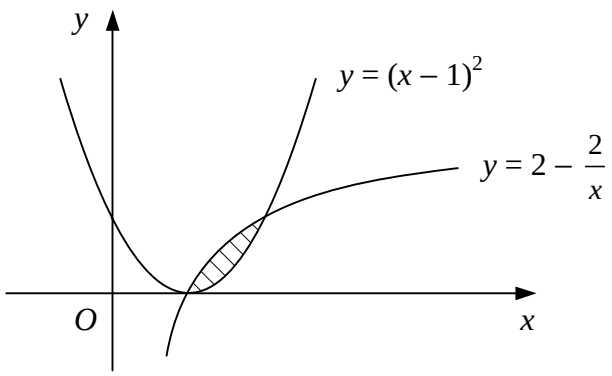
*(ii)* show that $\log_2 x = \frac{y}{\ln 2}$. [3]

*(b)* Hence, or otherwise, solve the equation

$$\log_2 x = 4 - \ln \frac{x^2}{e},$$

giving your answer to 2 decimal places. [3]

**4.**




The diagram shows the curves $y = (x-1)^2$ and $y = 2 - \frac{2}{x}$, $x > 0$.

*(i)* Verify that the two curves meet at the points where $x = 1$ and where $x = 2$. [2]

The shaded region bounded by the two curves is rotated completely about the $x$-axis.

*(ii)* Find the exact volume of the solid formed. [7]

**5.** $f(x) = 5 + e^{2x-3}, \quad x \in \mathbb{R}$.

*(i)* State the range of f. [1]

*(ii)* Find an expression for $f^{-1}(x)$ and state its domain. [3]

*(iii)* Solve the equation $f(x) = 7$. [2]

*(iv)* Find an equation for the tangent to the curve $y = f(x)$ at the point where $y = 7$. [4]

**6.** *(i)* Express $\sqrt{3}\sin\theta + \cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

*(ii)* State the maximum value of $\sqrt{3}\sin\theta + \cos\theta$ and the smallest positive value of $\theta$ for which this maximum value occurs. [3]

*(iii)* Solve the equation

$$\sqrt{3}\sin\theta + \cos\theta + \sqrt{3} = 0,$$

for $\theta$ in the interval $-\pi \leq \theta \leq \pi$, giving your answers in terms of $\pi$. [4]

**7.** $f(x) = \dfrac{x^2+3}{4x+1}, \quad x \in \mathbb{R}, \quad x \neq -\frac{1}{4}$.

*(i)* Find and simplify an expression for $f'(x)$. [3]

*(ii)* Find the set of values of $x$ for which $f(x)$ is increasing. [4]

*(iii)* Use Simpson's rule with six strips to find an approximate value for

$$\int_0^6 f(x)\,dx.$$ [3]

***Turn over***

**8.** The functions f and g are defined by

$$\mathrm{f} : x \to |2x - 5|, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \to \ln (x + 3), \quad x \in \mathbb{R}, \quad x > -3.$$

*(i)* State the range of f. [1]

*(ii)* Evaluate fg(−2). [2]

*(iii)* Solve the equation

$$\mathrm{fg}(x) = 3,$$

giving your answers in exact form. [5]

*(iv)* Show that the equation

$$\mathrm{f}(x) = \mathrm{g}(x)$$

has a root, $\alpha$, in the interval [3, 4]. [2]

*(v)* Use the iterative formula

$$x_{n+1} = \tfrac{1}{2}[5 + \ln (x_n + 3)],$$

with $x_0 = 3$, to find $x_1$, $x_2$, $x_3$ and $x_4$, giving your answers to 4 significant figures. [2]

*(vi)* Show that your answer for $x_4$ is the value of $\alpha$ correct to 4 significant figures. [2]