C3 Paper I – Marking Guide

1. $\dfrac{dV}{dt} = 80$ B1

$V = \frac{4}{3}\pi r^3 \ \therefore\ \dfrac{dV}{dr} = 4\pi r^2, \qquad r = 6 \ \therefore\ \dfrac{dV}{dr} = 144\pi$ M1 A1

$\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt} \ \therefore\ 80 = 144\pi \times \dfrac{dr}{dt}$ M1

$\dfrac{dr}{dt} = \dfrac{80}{144\pi} = \dfrac{5}{9\pi} = 0.177$ (3sf)

radius is increasing at rate of 0.177 cm per second A1 **(5)**

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2. $\dfrac{3}{\sin\theta} = -8\cos\theta$ M1

$3 = -8\sin\theta\cos\theta = -4\sin 2\theta$ M1

$\sin 2\theta = -\frac{3}{4}$ A1

$2\theta = 180 + 48.590,\ 360 - 48.590 = 228.590,\ 311.410$ M1

$\theta = 114.3,\ 155.7$ (1dp) A2 **(6)**

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3. *(a)* *(i)* $\ln\dfrac{x^2}{e} = \ln x^2 - \ln e = 2\ln x - 1 = 2y - 1$ M1 A1

(ii) let $t = \log_2 x \Rightarrow x = 2^t$ M1

$\ln x = t\ln 2$ M1

$t = \dfrac{\ln x}{\ln 2} \qquad \therefore\ \log_2 x = \dfrac{y}{\ln 2}$ A1

(b) $\dfrac{y}{\ln 2} = 4 - (2y - 1), \qquad y = (5 - 2y)\ln 2$

$y(2\ln 2 + 1) = 5\ln 2$ M1

$y = \dfrac{5\ln 2}{2\ln 2 + 1}$ M1

$x = e^y = 4.27$ (2dp) A1 **(8)**

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4. *(i)* when $x = 1,\ (x-1)^2 = 0$ and $2 - \frac{2}{x} = 0 \ \therefore$ intersect B1

when $x = 2,\ (x-1)^2 = 1$ and $2 - \frac{2}{x} = 1 \ \therefore$ intersect B1

(ii) $= \pi\int_1^2 (2 - \frac{2}{x})^2\ dx - \pi\int_1^2 (x-1)^4\ dx$ M1

$= \pi\int_1^2 (4 - 8x^{-1} + 4x^{-2})\ dx - \pi\int_1^2 (x-1)^4\ dx$ M1

$= \pi[4x - 8\ln|x| - 4x^{-1}]_1^2 - \pi[\frac{1}{5}(x-1)^5]_1^2$ M1 A2

$= \pi[(8 - 8\ln 2 - 2) - (4 - 0 - 4)] - \pi[\frac{1}{5} - 0]$ M1

$= \pi(5\frac{4}{5} - 8\ln 2)$ A1 **(9)**

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5. *(i)* $f(x) > 5$ B1

(ii) $y = 5 + e^{2x-3}$

$2x - 3 = \ln(y - 5)$ M1

$x = \frac{1}{2}[3 + \ln(y-5)]$

$\therefore\ f^{-1}(x) = \frac{1}{2}[3 + \ln(x-5)],\ x \in \mathbb{R},\ x > 5$ A2

(iii) $x = f^{-1}(7) = \frac{1}{2}(3 + \ln 2)$ M1 A1

(iv) $f'(x) = 2e^{2x-3}$ M1

grad = 4 A1

$\therefore\ y - 7 = 4[x - \frac{1}{2}(3 + \ln 2)] \qquad [\ y = 4x + 1 - 2\ln 2\ ]$ M1 A1 **(10)**

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6. *(i)* $\sqrt{3}\sin\theta + \cos\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

$R\cos\alpha = \sqrt{3}$, $R\sin\alpha = 1$ M1

$\therefore\ R = \sqrt{3+1} = 2$ A1

$\tan\alpha = \frac{1}{\sqrt{3}}$, $\alpha = \frac{\pi}{6}$ A1

$\therefore\ \sqrt{3}\sin\theta + \cos\theta = 2\sin(\theta + \frac{\pi}{6})$

(ii) maximum = 2 B1

occurs when $\theta + \frac{\pi}{6} = \frac{\pi}{2}$, $\theta = \frac{\pi}{3}$ M1 A1

(iii) $2\sin(\theta + \frac{\pi}{6}) + \sqrt{3} = 0$

$\sin(\theta + \frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$ M1

$\theta + \frac{\pi}{6} = -\frac{\pi}{3}, -\pi + \frac{\pi}{3} = -\frac{\pi}{3}, -\frac{2\pi}{3}$ M1

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{2}$ A2 **(10)**

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7. *(i)* $f'(x) = \frac{2x\times(4x+1)-(x^2+3)\times 4}{(4x+1)^2}$ M1 A1

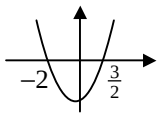
$= \frac{4x^2+2x-12}{(4x+1)^2}$ A1

(ii) $\frac{4x^2+2x-12}{(4x+1)^2} \geq 0$

for $x \neq -\frac{1}{4}$, $(4x+1)^2 > 0$ $\therefore\ 4x^2 + 2x - 12 \geq 0$ M1 A1

$2(2x-3)(x+2) \geq 0$ M1

$x \leq -2$ or $x \geq \frac{3}{2}$ A1



(iii)

x	0	1	2	3	4	5	6
f(x)	3	$\frac{4}{5}$	$\frac{7}{9}$	$\frac{12}{13}$	$\frac{19}{17}$	$\frac{28}{21}$	$\frac{39}{25}$

M1

$I \approx \frac{1}{3}\times 1\times[3 + \frac{39}{25} + 4(\frac{4}{5} + \frac{12}{13} + \frac{28}{21}) + 2(\frac{7}{9} + \frac{19}{17})]$ M1

= 6.86 (3sf) A1 **(10)**

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8. *(i)* f(x) ≥ 0 B1

(ii) = f(0) = 5 M1 A1

(iii) fg(x) = f [ln (x + 3)] = $|2\ln(x+3) - 5|$ M1

$\therefore\ |2\ln(x+3) - 5| = 3$

$2\ln(x+3) = 2, 8$ M1

$\ln(x+3) = 1, 4$ A1

$x = e - 3, e^4 - 3$ M1 A1

(iv) let h(x) = f(x) – g(x)

h(3) = –0.79, f(4) = 1.1 M1

sign change, h(x) continuous $\therefore$ root A1

(v) $x_1 = 3.396$, $x_2 = 3.428$, $x_3 = 3.430$, $x_4 = 3.431$ M1 A1

(vi) h(3.4305) = –0.000052, f(3.4315) = 0.0018 M1

sign change, h(x) continuous $\therefore$ root $\therefore\ \alpha = x_4$ to 4sf A1 **(14)**

Total **(72)**