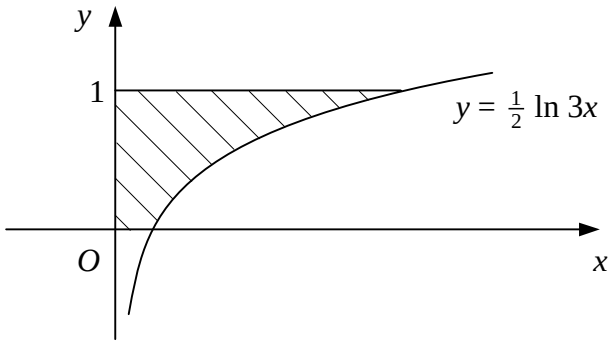
Core Mathematics 3 Paper H

1. $f(x) = \dfrac{4x-1}{2x+1}$.

Find an equation for the tangent to the curve $y = f(x)$ at the point where $x = -2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

2.



The diagram shows the curve with equation $y = \frac{1}{2} \ln 3x$.

(i) Express the equation of the curve in the form $x = f(y)$. [2]

The shaded region is bounded by the curve, the coordinate axes and the line $y = 1$.

(ii) Find, in terms of π and e, the volume of the solid formed when the shaded region is rotated through four right angles about the y-axis. [5]

3. *(i)* Use the identity for $\sin(A + B)$ to show that

$$\sin 3x \equiv 3 \sin x - 4 \sin^3 x.$$ [4]

(ii) Hence find, in terms of π, the solutions of the equation

$$\sin 3x - \sin x = 0,$$

for x in the interval $0 \leq x < 2\pi$. [4]

4. The function f is defined by

$$f(x) \equiv x^2 - 2ax, \quad x \in \mathbb{R},$$

where a is a positive constant.

(i) Showing the coordinates of any points where the graph meets the axes, sketch the graph of $y = |f(x)|$. [3]

The function g is defined by

$$g(x) \equiv 3ax, \quad x \in \mathbb{R}.$$

(ii) Find $fg(a)$ in terms of a. [2]

(iii) Solve the equation

$$gf(x) = 9a^3.$$ [4]

5. *(i)* Find, as natural logarithms, the solutions of the equation

$$e^{2x} - 8e^x + 15 = 0.$$ [3]

(ii) Use proof by contradiction to prove that $\log_2 3$ is irrational. [6]

6.
$$f(x) = 2x^2 + 3\ln(2 - x), \quad x \in \mathbb{R}, \quad x < 2.$$

(i) Show that the equation $f(x) = 0$ can be written in the form

$$x = 2 - e^{kx^2},$$

where k is a constant to be found. [3]

The root, α, of the equation $f(x) = 0$ is 1.9 correct to 1 decimal place.

(ii) Use the iterative formula

$$x_{n+1} = 2 - e^{kx_n^2},$$

with $x_0 = 1.9$ and your value of k, to find α correct to 3 decimal places.

You should show the result of each iteration. [3]

(iii) Solve the equation $f'(x) = 0$. [5]

Turn over

7. *(i)* Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 1 - 2 \sin^2 \tfrac{x}{2}.$$ [2]

(ii) Prove that, for $\sin x \neq 0$,

$$\frac{1-\cos x}{\sin x} \equiv \tan \tfrac{x}{2}.$$ [3]

(iii) Find the values of x in the interval $0 \leq x \leq 360°$ for which

$$\frac{1-\cos x}{\sin x} = 2 \sec^2 \tfrac{x}{2} - 5,$$

giving your answers to 1 decimal place where appropriate. [6]

8. $$f(x) = x^2 - 2x + 5, \quad x \in \mathbb{R}, \quad x \geq 1.$$

(i) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the range of f. [1]

(iii) Find an expression for $f^{-1}(x)$. [2]

(iv) Describe fully two transformations that would map the graph of $y = f^{-1}(x)$ onto the graph of $y = \sqrt{x}$, $x \geq 0$. [3]

(v) Find an equation for the normal to the curve $y = f^{-1}(x)$ at the point where $x = 8$. [4]