C3 Paper H – Marking Guide

1. $f'(x) = \frac{4\times(2x+1)-(4x-1)\times 2}{(2x+1)^2} = \frac{6}{(2x+1)^2}$ M1 A1

$x = -2 \Rightarrow y = 3$, grad $= \frac{2}{3}$ A1

$\therefore\ y - 3 = \frac{2}{3}(x + 2)$ M1

$3y - 9 = 2x + 4$

$2x - 3y + 13 = 0$ A1 **(5)**

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2. *(i)* $x = \frac{1}{3}e^{2y}$ M1 A1

(ii) $= \pi\int_0^1 \frac{1}{9}e^{4y}\ dy$ M1

$= \pi[\frac{1}{36}e^{4y}]_0^1$ M1 A1

$= \frac{1}{36}\pi(e^4 - 1)$ M1 A1 **(7)**

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3. *(i)* LHS $\equiv \sin(2x + x)$

$\equiv \sin 2x \cos x + \cos 2x \sin x$ M1

$\equiv 2\sin x \cos^2 x + \sin x(1 - 2\sin^2 x)$ M1

$\equiv 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$ M1

$\equiv 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$\equiv 3\sin x - 4\sin^3 x \equiv$ RHS A1

(ii) $3\sin x - 4\sin^3 x - \sin x = 0$

$2\sin x(1 - 2\sin^2 x) = 0$ M1

$\sin x = 0, \pm\frac{1}{\sqrt{2}}$ A1

$x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$ A2 **(8)**

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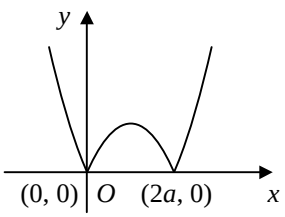
4. *(i)*



B3

(ii) $= f(3a^2) = 9a^4 - 6a^3$ M1 A1

(iii) $gf(x) = 3a(x^2 - 2ax)$ M1

$\therefore\ 3a(x^2 - 2ax) = 9a^3$

$x^2 - 2ax - 3a^2 = 0$ A1

$(x + a)(x - 3a) = 0$ M1

$x = -a, 3a$ A1 **(9)**

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5. *(i)* $(e^x - 3)(e^x - 5) = 0$ M1

$e^x = 3, 5$

$x = \ln 3, \ln 5$ M1 A1

(ii) assume $\log_2 3$ is rational B1

$\therefore\ \log_2 3 = \frac{p}{q}$ where p and q are integers and $q \neq 0$ M1

$\Rightarrow\quad 2^{\frac{p}{q}} = 3$ M1

$\Rightarrow\quad 2^p = 3^q$ A1

2 and 3 are co-prime $\therefore$ only solution is $p = q = 0$ M1

but $q \neq 0$ $\therefore$ contradiction $\therefore\ \log_2 3$ is irrational A1 **(9)**

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6. *(i)* $2x^2 + 3\ln(2-x) = 0 \Rightarrow 3\ln(2-x) = -2x^2$

$\ln(2-x) = -\frac{2}{3}x^2$ M1

$2 - x = e^{-\frac{2}{3}x^2}$ M1

$x = 2 - e^{-\frac{2}{3}x^2}$ $[k = -\frac{2}{3}]$ A1

(ii) $x_1 = 1.90988,\ x_2 = 1.91212,\ x_3 = 1.91262,\ x_4 = 1.91273,\ x_5 = 1.91275$ M1 A1

$\therefore\ \alpha = 1.913$ (3dp) A1

(iii) $f'(x) = 4x + \frac{3}{2-x} \times (-1) = 4x - \frac{3}{2-x}$ M1 A1

$\therefore\ 4x - \frac{3}{2-x} = 0,\quad 4x = \frac{3}{2-x},\quad 4x(2-x) = 3$ M1

$4x^2 - 8x + 3 = 0,\quad (2x-3)(2x-1) = 0$ M1

$x = \frac{1}{2}, \frac{3}{2}$ A1 **(11)**

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7. *(i)* $\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$

let $A = B = \frac{x}{2}$ $\cos x \equiv \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$ M1

$\cos x \equiv (1 - \sin^2\frac{x}{2}) - \sin^2\frac{x}{2}$

$\cos x \equiv 1 - 2\sin^2\frac{x}{2}$ A1

(ii) LHS $\equiv \frac{1-(1-2\sin^2\frac{x}{2})}{2\sin\frac{x}{2}\cos\frac{x}{2}}$ M1

$\equiv \frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} \equiv \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$ M1

$\equiv \tan\frac{x}{2} \equiv$ RHS A1

(iii) $\tan\frac{x}{2} = 2\sec^2\frac{x}{2} - 5,\quad \tan\frac{x}{2} = 2(1 + \tan^2\frac{x}{2}) - 5$ M1

$2\tan^2\frac{x}{2} - \tan\frac{x}{2} - 3 = 0,\quad (2\tan\frac{x}{2} - 3)(\tan\frac{x}{2} + 1) = 0$ M1

$\tan\frac{x}{2} = -1$ or $\frac{3}{2}$ A1

$\frac{x}{2} = 135$ or 56.310 M1

$x = 112.6°$ (1dp), $270°$ A2 **(11)**

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8. *(i)* $f(x) = (x-1)^2 - 1 + 5 = (x-1)^2 + 4$ M1 A1

(ii) $f(x) \geq 4$ B1

(iii) $y = (x-1)^2 + 4$

$(x-1)^2 = y - 4$

$x - 1 = \pm\sqrt{y-4}$ M1

$x = 1 \pm \sqrt{y-4}$

$f^{-1}(x) = 1 + \sqrt{x-4}$ A1

(iv) translation by 4 units in negative x direction

translation by 1 unit in negative y direction (either first) B3

(v) $\frac{dy}{dx} = \frac{1}{2}(x-4)^{-\frac{1}{2}}$ M1

$x = 8,\ y = 3,$ grad $= \frac{1}{4}$ A1

$\therefore$ grad of normal $= -4$

$\therefore\ y - 3 = -4(x-8)$ $[y = 35 - 4x]$ M1 A1 **(12)**

Total **(72)**