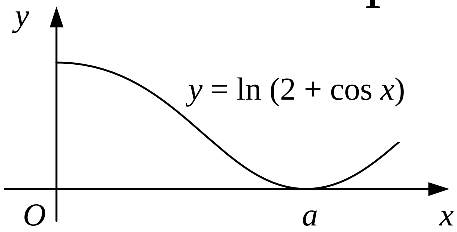
Core Mathematics 3 Paper G

1.



The diagram shows the curve with equation $y = \ln(2 + \cos x)$, $x \geq 0$.
The smallest value of x for which the curve meets the x-axis is a as shown.

(i) Find the value of a. [2]

(ii) Use Simpson's rule with four strips of equal width to estimate the area of the region bounded by the curve in the interval $0 \leq x \leq a$ and the coordinate axes. [3]

2. The functions f and g are defined by

$$\mathrm{f} : x \to 2 - x^2, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \to \frac{3x}{2x-1}, \quad x \in \mathbb{R}, \quad x \neq \tfrac{1}{2}.$$

(i) Evaluate fg(2). [2]

(ii) Solve the equation $\mathrm{gf}(x) = \frac{1}{2}$. [4]

3. Find the coordinates of the stationary points of the curve with equation

$$y = \frac{x-1}{x^2 - 2x + 5}.$$ [6]

4. *(i)* Sketch the graph of $y = 2 + \sec\left(x - \frac{\pi}{6}\right)$ for x in the interval $0 \leq x \leq 2\pi$.

Show on your sketch the coordinates of any turning points and the equations of any asymptotes. [4]

(ii) Find, in terms of π, the x-coordinates of the points where the graph crosses the x-axis. [4]

5. A curve has the equation $y = \sqrt{3x + 11}$.

The point P on the curve has x-coordinate 3.

(i) Show that the tangent to the curve at P has the equation

$$3x - 4\sqrt{5}y + 31 = 0.$$ [5]

The normal to the curve at P crosses the y-axis at Q.

(ii) Find the y-coordinate of Q in the form $k\sqrt{5}$. [3]

6. *(i)* Express $3\cos x^\circ + \sin x^\circ$ in the form $R\cos(x - \alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$. [3]

(ii) Using your answer to part *(a)*, or otherwise, solve the equation

$$6\cos^2 x^\circ + \sin 2x^\circ = 0,$$

for x in the interval $0 \le x \le 360$, giving your answers to 1 decimal place where appropriate. [5]

7. The finite region R is bounded by the curve with equation $y = x + \frac{2}{x}$, the x-axis and the lines $x = 1$ and $x = 4$.

(i) Find the exact area of R. [4]

The region R is rotated completely about the x-axis.

(ii) Find the volume of the solid formed, giving your answer in terms of π. [5]

8. The population in thousands, P, of a town at time t years after 1st January 1980 is modelled by the formula

$$P = 30 + 50e^{0.002t}.$$

Use this model to estimate

(i) the population of the town on 1st January 2010, [2]

(ii) the year in which the population first exceeds 84 000. [3]

The population in thousands, Q, of another town is modelled by the formula

$$Q = 26 + 50e^{0.003t}.$$

(iii) Show that the value of t when $P = Q$ is a solution of the equation

$$t = 1000\ln(1 + 0.08e^{-0.002t}).$$ [3]

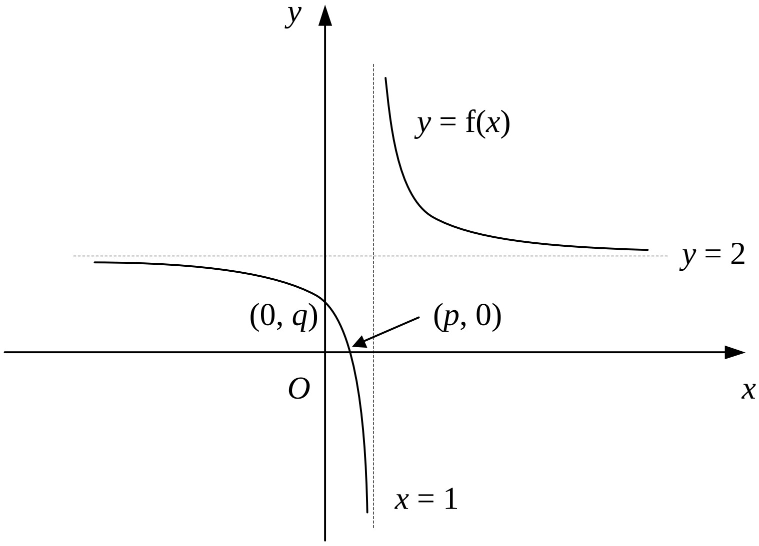
(iv) Use the iterative formula

$$t_{n+1} = 1000\ln(1 + 0.08e^{-0.002t_n})$$

with $t_0 = 50$ to find t_1, t_2 and t_3 and hence, the year in which the populations of these two towns will be equal according to these models. [3]

Turn over

9.



The diagram shows the curve with equation $y = \mathrm{f}(x)$. The curve crosses the axes at $(p, 0)$ and $(0, q)$ and the lines $x = 1$ and $y = 2$ are asymptotes of the curve.

(a) Showing the coordinates of any points of intersection with the axes and the equations of any asymptotes, sketch on separate diagrams the graphs of

(i) $y = |\mathrm{f}(x)|$, [2]

(ii) $y = 2\mathrm{f}(x + 1)$. [3]

Given also that

$$\mathrm{f}(x) \equiv \frac{2x-1}{x-1}, \quad x \in \mathbb{R}, \quad x \neq 1,$$

(b) find the values of p and q, [3]

(c) find an expression for $\mathrm{f}^{-1}(x)$. [3]