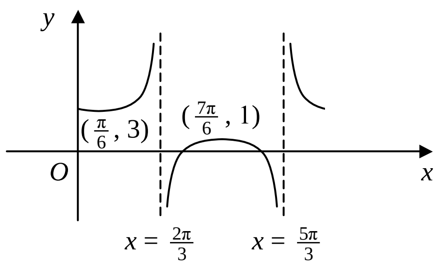
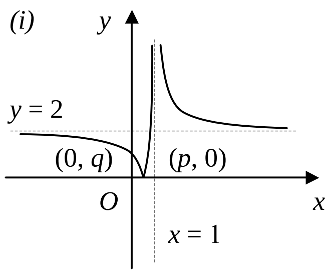
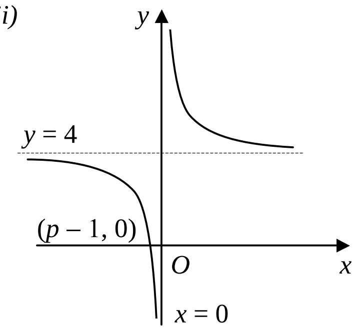
C3 Paper G – Marking Guide

1.	*(i)*	$\ln(2+\cos a)=0,\quad 2+\cos a=1,\quad \cos a=-1,\quad a=\pi$	M1 A1	
	(ii)	x: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, π		
		y: 1.0986, 0.9959, 0.6931, 0.2569, 0	M1	
		area $\approx \frac{1}{3}\times\frac{\pi}{4}\times[1.0986+0+4(0.9959+0.2569)+2(0.6931)]$	M1	
		$=1.96$ (3sf)	A1	**(5)**
2.	*(i)*	$=\mathrm{f}(2)=-2$	M1 A1	
	(ii)	$\mathrm{gf}(x)=\mathrm{g}(2-x^2)=\dfrac{3(2-x^2)}{2(2-x^2)-1}=\dfrac{6-3x^2}{3-2x^2}$	M1 A1	
		$\therefore\ \dfrac{6-3x^2}{3-2x^2}=\frac{1}{2},\quad 2(6-3x^2)=3-2x^2$		
		$x^2=\frac{9}{4}$	M1	
		$x=\pm\frac{3}{2}$	A1	**(6)**
3.		$\dfrac{dy}{dx}=\dfrac{1\times(x^2-2x+5)-(x-1)(2x-2)}{(x^2-2x+5)^2}=\dfrac{-x^2+2x+3}{(x^2-2x+5)^2}$	M1 A2	
	SP:	$\dfrac{-x^2+2x+3}{(x^2-2x+5)^2}=0,\quad -x^2+2x+3=0$		
		$-(x+1)(x-3)=0$	M1	
		$x=-1,\ 3$	A1	
		$\therefore\ (-1,-\frac{1}{4}),\ (3,\frac{1}{4})$	A1	**(6)**
4.	*(i)*		M2 A2	
	(ii)	$2+\sec(x-\frac{\pi}{6})=0$		
		$\sec(x-\frac{\pi}{6})=-2,\quad \cos(x-\frac{\pi}{6})=-\frac{1}{2}$	M1	
		$x-\frac{\pi}{6}=\pi-\frac{\pi}{3},\ \pi+\frac{\pi}{3}=\frac{2\pi}{3},\ \frac{4\pi}{3}$	M1	
		$x=\frac{5\pi}{6},\ \frac{3\pi}{2}$	A2	**(8)**
5.	*(i)*	$x=3,\ y=\sqrt{20}=2\sqrt{5}$	B1	
		$\dfrac{dy}{dx}=\frac{1}{2}(3x+11)^{-\frac{1}{2}}\times 3=\frac{3}{2}(3x+11)^{-\frac{1}{2}}$	M1	
		grad $=\dfrac{3}{4\sqrt{5}}$	A1	
		$\therefore\ y-2\sqrt{5}=\dfrac{3}{4\sqrt{5}}(x-3)$	M1	
		$4\sqrt{5}\,y-40=3x-9$		
		$3x-4\sqrt{5}\,y+31=0$	A1	
	(ii)	normal: $y-2\sqrt{5}=-\dfrac{4\sqrt{5}}{3}(x-3)$	M1	
		at Q, $x=0\ \therefore\ y-2\sqrt{5}=4\sqrt{5}$	M1	
		$y=6\sqrt{5}$	A1	**(8)**

6.	*(i)*	$3\cos x + \sin x = R\cos x\cos\alpha + R\sin x\sin\alpha$		
		$R\cos\alpha = 3,\ \ R\sin\alpha = 1$	M1	
		$\therefore\ R = \sqrt{3^2+1^2} = \sqrt{10}$	A1	
		$\tan\alpha = \frac{1}{3},\ \ \alpha = 18.4$ (3sf)	A1	
		$\therefore\ 3\cos x° + \sin x° = \sqrt{10}\cos(x - 18.4)°$		
	(ii)	$6\cos^2 x + 2\sin x\cos x = 0$	M1	
		$2\cos x(3\cos x + \sin x) = 0$	M1	
		$\cos x = 0$ or $3\cos x + \sin x = \sqrt{10}\cos(x - 18.4) = 0$	A1	
		$x = 90, 270$ or $x - 18.4 = 90, 270$		
		$x = 90, 108.4$ (1dp), $270, 288.4$ (1dp)	A2	**(8)**
7.	*(i)*	$= \int_1^4 (x + \frac{2}{x})\ dx = [\frac{1}{2}x^2 + 2\ln\lvert x\rvert]_1^4$	M1 A1	
		$= (8 + 2\ln 4) - (\frac{1}{2} + 0) = 7\frac{1}{2} + 2\ln 4$	M1 A1	
	(ii)	$= \pi\int_1^4 (x + \frac{2}{x})^2\ dx = \pi\int_1^4 (x^2 + 4 + 4x^{-2})\ dx$	M1	
		$= \pi[\frac{1}{3}x^3 + 4x - 4x^{-1}]_1^4$	M1 A1	
		$= \pi[(\frac{64}{3} + 16 - 1) - (\frac{1}{3} + 4 - 4)] = 36\pi$	M1 A1	**(9)**
8.	*(i)*	$P = 30 + 50e^{0.002 \times 30} = 83.1$	M1	
		$\therefore$ population $= 83\,100$ (3sf)	A1	
	(ii)	$30 + 50e^{0.002t} > 84, \qquad e^{0.002t} > \frac{54}{50}$	M1	
		$t > \frac{1}{0.002}\ln\frac{54}{50},\ \ t > 38.5\ \ \therefore\ 2018$	M1 A1	
	(iii)	$30 + 50e^{0.002t} = 26 + 50e^{0.003t}, \qquad 50e^{0.003t} - 50e^{0.002t} = 4$		
		$e^{0.003t} - e^{0.002t} = 0.08, \qquad e^{0.002t}(e^{0.001t} - 1) = 0.08$	M1	
		$e^{0.001t} - 1 = 0.08e^{-0.002t}$	M1	
		$0.001t = \ln(1 + 0.08e^{-0.002t})$		
		$t = 1000\ln(1 + 0.08e^{-0.002t})$	A1	
	(iv)	$t_1 = 69.887,\ \ t_2 = 67.251,\ \ t_3 = 67.595$	M1 A1	
		$\therefore\ 2047$	A1	**(11)**
9.	*(a)*	*(i)*  *(ii)* 	M1 A1 M2 A1	
	(b)	$y = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2}\ \ \therefore p = \frac{1}{2}$	M1 A1	
		$x = 0 \Rightarrow y = 1\ \ \therefore q = 1$	B1	
	(c)	$y = \frac{2x-1}{x-1}, \qquad y(x - 1) = 2x - 1, \qquad x(y - 2) = y - 1$	M1	
		$x = \frac{y-1}{y-2}$		
		$\therefore\ f^{-1}(x) = \frac{x-1}{x-2}$	M1 A1	**(11)**
			Total	**(72)**