Core Mathematics 3 Paper F

1.   Evaluate

$$\int_{2}^{6} \sqrt{3x-2} \ dx.$$ [4]

2.   Differentiate each of the following with respect to x and simplify your answers.

(i) $\dfrac{6}{\sqrt{2x-7}}$ [2]

(ii) $x^2 e^{-x}$ [3]

3.   *(i)*   Prove the identity

$$\sqrt{2} \cos (x + 45)° + 2 \cos (x - 30)° \equiv (1 + \sqrt{3}) \cos x°.$$ [4]

(ii)   Hence, find the exact value of $\cos 75°$ in terms of surds.   [3]

4. $$f(x) = x^2 + 5x - 2 \sec x, \quad x \in \mathbb{R}, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

(i)   Show that the equation $f(x) = 0$ has a root, α, such that $1 < \alpha < 1.5$ [2]

(ii)   Show that a suitable rearrangement of the equation $f(x) = 0$ leads to the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{2}{x_n^2 + 5x_n}\right).$$ [3]

(iii)   Use the iterative formula in part *(ii)* with a starting value of 1.25 to find α correct to 3 decimal places. You should show the result of each iteration.   [3]

5.   The function f is defined by

$$f(x) \equiv 2 + \ln (3x - 2), \quad x \in \mathbb{R}, \quad x > \tfrac{2}{3}.$$

(i)   Find the exact value of $ff(1)$.   [2]

(ii)   Find an equation for the tangent to the curve $y = f(x)$ at the point where $x = 1$.   [4]

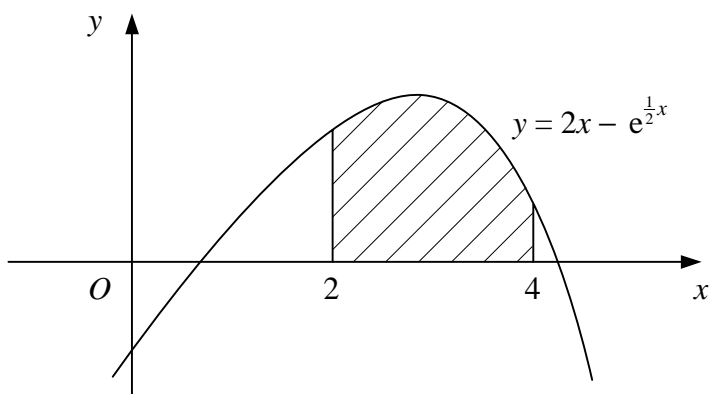*(iii)*   Find an expression for $f^{-1}(x)$.   [2]

6. *(i)* Sketch on the same diagram the graphs of $y = |x| - a$ and $y = |3x + 5a|$, where a is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes. [5]

(ii) Solve the equation

$$|x| - a = |3x + 5a|.$$ [4]

7.

$$y = 2x - e^{\frac{1}{2}x}$$

The diagram shows the curve with equation $y = 2x - e^{\frac{1}{2}x}$.

The shaded region is bounded by the curve, the *x*-axis and the lines $x = 2$ and $x = 4$.

(i) Find the area of the shaded region, giving your answer in terms of e. [4]

The shaded region is rotated through four right angles about the *x*-axis.

(ii) Using Simpson's rule with two strips, estimate the volume of the solid formed. [5]

8. *(i)* Sketch on the same diagram the graphs of

$$y = \sin^{-1} x, \ -1 \le x \le 1$$

and $$y = \cos^{-1}(2x), \ -\tfrac{1}{2} \le x \le \tfrac{1}{2}.$$ [3]

Given that the graphs intersect at the point with coordinates (a, b),

(ii) show that $\tan b = \tfrac{1}{2}$, [3]

(iii) find the value of a in the form $k\sqrt{5}$. [4]

Turn over

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9. $$f(x) = e^{3x+1} - 2, \quad x \in \mathbb{R}.$$

    (i)   State the range of f.   [1]

The curve $y = f(x)$ meets the *y*-axis at the point *P* and the *x*-axis at the point *Q*.

    (ii)   Find the exact coordinates of *P* and *Q*.   [3]

    (iii)   Show that the tangent to the curve at *P* has the equation

$$y = 3ex + e - 2.$$ [4]

    (iv)   Find to 3 significant figures the *x*-coordinate of the point where the tangent to the curve at *P* meets the tangent to the curve at *Q*.   [4]