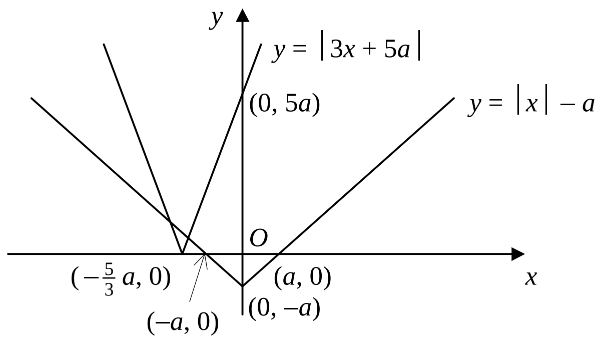
C3 Paper F – Marking Guide

1.		$= [\,\frac{2}{9}(3x-2)^{\frac{3}{2}}\,]_2^6$	M1 A1	
		$= \frac{2}{9}(64-8) = 12\frac{4}{9}$	M1 A1	**(4)**
2.	*(i)*	$= -3(2x-7)^{-\frac{3}{2}} \times 2 = -\dfrac{6}{(2x-7)^{\frac{3}{2}}}$	M1 A1	
	(ii)	$= 2x \times e^{-x} + x^2 \times (-e^{-x}) = xe^{-x}(2-x)$	M1 A2	**(5)**
3.	*(i)*	LHS $\equiv \sqrt{2}(\cos x \cos 45 - \sin x \sin 45) + 2(\cos x \cos 30 + \sin x \sin 30)$	M1 A1	
		$\equiv \sqrt{2}(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x) + 2(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x)$	M1	
		$\equiv \cos x - \sin x + \sqrt{3}\cos x + \sin x \equiv (1+\sqrt{3})\cos x \equiv$ RHS	A1	
	(ii)	let $x = 75$, $\sqrt{2}\cos 120° + 2\cos 45° = (1+\sqrt{3})\cos 75°$	M1	
		$\sqrt{2}(-\frac{1}{2}) + 2(\frac{1}{\sqrt{2}}) = (1+\sqrt{3})\cos 75°$		
		$\frac{1}{2}\sqrt{2} = (1+\sqrt{3})\cos 75°$	M1	
		$\cos 75° = \dfrac{\sqrt{2}}{2(1+\sqrt{3})} = \dfrac{1}{\sqrt{2}+\sqrt{6}}$	A1	**(7)**
4.	*(i)*	$f(1) = 2.30$, $f(1.5) = -18.5$	M1	
		sign change, $f(x)$ continuous $\therefore$ root	A1	
	(ii)	$x^2 + 5x - 2\sec x = 0 \Rightarrow x^2 + 5x = \dfrac{2}{\cos x}$	M1	
		$\cos x = \dfrac{2}{x^2+5x}$	M1	
		$x = \cos^{-1}(\dfrac{2}{x^2+5x}) \therefore x_{n+1} = \cos^{-1}(\dfrac{2}{x_n^2+5x_n})$	A1	
	(iii)	$x_0 = 1.25$, $x_1 = 1.31191$, $x_2 = 1.32686$, $x_3 = 1.33024$,	M1 A1	
		$x_4 = 1.33100$, $x_5 = 1.33116$, $x_6 = 1.33120$ $\therefore \alpha = 1.331$ (3dp)	A1	**(8)**
5.	*(i)*	$= f(2) = 2 + \ln 4$	M1 A1	
	(ii)	$f'(x) = \dfrac{1}{3x-2} \times 3 = \dfrac{3}{3x-2}$	M1	
		$x = 1$, $y = 2$, grad $= 3$	A1	
		$y - 2 = 3(x-1)$ $[\,y = 3x - 1\,]$	M1 A1	
	(iii)	$y = 2 + \ln(3x-2)$, $3x - 2 = e^{y-2}$	M1	
		$x = \frac{1}{3}(2 + e^{y-2})$, $f^{-1}(x) = \frac{1}{3}(2 + e^{x-2})$	A1	**(8)**
6.	*(i)*		B3 B2	
	(ii)	$-x - a = 3x + 5a \Rightarrow x = -\frac{3}{2}a$	M1 A1	
		$-x - a = -(3x + 5a) \Rightarrow x = -2a$, $x = -2a, -\frac{3}{2}a$	M1 A1	**(9)**

7. *(i)* $= \int_2^4 (2x - e^{\frac{1}{2}x})\, dx$

$= [x^2 - 2e^{\frac{1}{2}x}]_2^4$ — M1 A1

$= (16 - 2e^2) - (4 - 2e) = 12 + 2e - 2e^2$ — M1 A1

(ii) $V = \pi \int_2^4 (2x - e^{\frac{1}{2}x})^2\, dx$ — M1

x	2	3	4
$(2x - e^{\frac{1}{2}x})^2$	1.6428	2.3053	0.3733

M1

$I \approx \frac{1}{3} \times 1 \times [1.6428 + 0.3733 + 2(2.3053)] = 3.7458$ — M1 A1

$\therefore\ V \approx 3.7458\pi = 11.8$ (3sf) — A1 **(9)**

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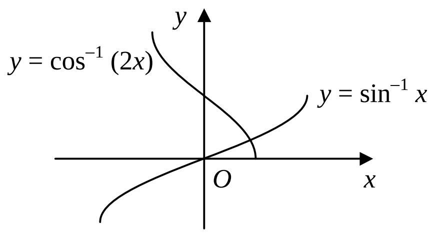
8. *(i)*



B3

(ii) $b = \sin^{-1} a \ \Rightarrow\ a = \sin b$

$b = \cos^{-1} 2a \ \Rightarrow\ 2a = \cos b$ — M1

$\therefore\ 2 \sin b = \cos b$ — M1

$\frac{\sin b}{\cos b} = \frac{1}{2}$

$\tan b = \frac{1}{2}$ — A1

(iii) $\tan^2 b = \frac{1}{4}$

$\sec^2 b = 1 + \frac{1}{4} = \frac{5}{4}$ — M1

$\cos^2 b = \frac{4}{5}$

$\cos b = \pm\frac{2}{\sqrt{5}}$ — A1

$a = \frac{1}{2}\cos b = \pm\frac{1}{\sqrt{5}}$ — M1

from diagram, $a > 0\ \therefore\ a = \frac{1}{\sqrt{5}} = \frac{1}{5}\sqrt{5}$ — A1 **(10)**

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9. *(i)* $f(x) > -2$ — B1

(ii) $x = 0,\ y = e - 2\ \therefore\ P\,(0, e - 2)$ — B1

$y = 0,\ 0 = e^{3x+1} - 2$

$3x + 1 = \ln 2$ — M1

$x = \frac{1}{3}(\ln 2 - 1)\ \therefore\ Q\,(\frac{1}{3}(\ln 2 - 1), 0)$ — A1

(iii) $f'(x) = 3e^{3x+1}$ — M1

at P, grad $= 3e$ — A1

$\therefore\ y - (e - 2) = 3e(x - 0)$ — M1

$y = 3ex + e - 2$ — A1

(iv) at Q, grad $= 6$ — B1

tangent at Q: $y - 0 = 6(x - \frac{1}{3}(\ln 2 - 1))$ — M1

$y = 6x - 2\ln 2 + 2$

intersect: $3ex + e - 2 = 6x - 2\ln 2 + 2$

$x(3e - 6) = 4 - e - 2\ln 2$ — M1

$x = \frac{4 - e - 2\ln 2}{3e - 6} = -0.0485$ (3sf) — A1 **(12)**

Total **(72)**