C3 Paper D – Marking Guide

1. *(i)* LHS = $\sin x \cos 30 + \cos x \sin 30 + \sin x \cos 30 - \cos x \sin 30$ — M1 A1
$= 2 \sin x \cos 30$
$= \sqrt{3} \sin x \qquad [\, a = \sqrt{3} \,]$ — A1

(ii) let $x = 45$, $\quad \sin 75 + \sin 15 = \sqrt{3} \sin 45$ — M1
$= \sqrt{3} \times \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{6}$ — M1 A1 **(6)**

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2. *(i)* $2x - 3 = e$ — M1
$x = \frac{1}{2}(e + 3)$ — A1

(ii) $3e^{2y} - 16e^{y} + 5 = 0$ — M1
$(3e^{y} - 1)(e^{y} - 5) = 0$ — M1
$e^{y} = \frac{1}{3}, 5$ — A1
$y = \ln \frac{1}{3}, \ln 5$ — M1 A1 **(7)**

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3. *(i)* $\frac{dy}{dx} = 2e^{x} - \frac{6}{x}$ — M1
$x = 1,\ y = 2e,$ grad $= 2e - 6$ — A1
$\therefore\ y - 2e = (2e - 6)(x - 1)$ — M1 A1
$[\, y = (2e - 6)x + 6 \,]$

(ii) $x = 0 \Rightarrow y = 6$
$y = 0 \Rightarrow (2e - 6)x + 6 = 0$
$x = \frac{-6}{2e-6} = \frac{3}{3-e}$ — M1 A1
area $= \frac{1}{2} \times 6 \times \frac{3}{3-e} = \frac{9}{3-e}$ — M1 A1 **(8)**

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4. *(i)* $= \int_1^2 \frac{1}{2x-1}\, dx$
$= [\frac{1}{2} \ln |2x - 1|]_1^2$ — M1 A1
$= \frac{1}{2}(\ln 3 - 0) = \frac{1}{2} \ln 3$ — M1 A1

(ii) $= \pi \int_1^2 \frac{1}{(2x-1)^2}\, dx$
$= \pi[-\frac{1}{2}(2x - 1)^{-1}]_1^2$ — M1 A1
$= \pi[-\frac{1}{6} - (-\frac{1}{2})] = \frac{1}{3}\pi$ — M1 A1 **(8)**

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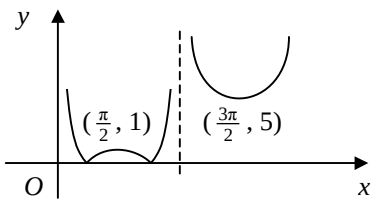
5. *(i)*



B2

(ii) $(\frac{\pi}{2}, -1) \Rightarrow -1 = a + b$
$(\frac{3\pi}{2}, -5) \Rightarrow -5 = a - b$ — B1
adding, $\quad -6 = 2a \ \therefore\ a = -3,\ b = 2$ — M1 A1

(iii) $-3 + 2 \operatorname{cosec} x = 0$
$\operatorname{cosec} x = \frac{3}{2}, \qquad \sin x = \frac{2}{3}$ — M1
$x = 0.73, \pi - 0.7297$
$x = 0.73, 2.41$ (2dp) — A2 **(8)**

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6. *(i)* LHS $\equiv \frac{2\cos 2x}{\sin 2x} + \frac{\sin x}{\cos x}$ M1

$\equiv \frac{\cos 2x}{\sin x \cos x} + \frac{\sin x}{\cos x}$ M1

$\equiv \frac{\cos 2x + \sin^2 x}{\sin x \cos x}$ A1

$\equiv \frac{(\cos^2 x - \sin^2 x) + \sin^2 x}{\sin x \cos x}$ M1

$\equiv \frac{\cos^2 x}{\sin x \cos x} \equiv \frac{\cos x}{\sin x} \equiv \cot x \equiv$ RHS A1

(ii) $\cot x = \operatorname{cosec}^2 x - 7$, $\cot x = 1 + \cot^2 x - 7$ M1

$\cot^2 x - \cot x - 6 = 0$, $(\cot x + 2)(\cot x - 3) = 0$ M1

$\cot x = -2$ or 3 A1

$\tan x = -\frac{1}{2}$ or $\frac{1}{3}$ M1

$x = \pi - 0.4636$ or 0.32

$x = 0.32, 2.68$ (2dp) A2 **(11)**

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7. *(i)* $f(x) > 0$ B1

(ii) $y = 3e^{x-1}$

$x - 1 = \ln \frac{y}{3}$ M1

$x = 1 + \ln \frac{y}{3}$

$f^{-1}(x) = 1 + \ln \frac{x}{3}$, $x \in \mathbb{R}$, $x > 0$ A2

(iii) $f(\ln 2) = 3e^{\ln 2 - 1} = 3e^{-1}e^{\ln 2} = 6e^{-1}$ M1 A1

$gf(\ln 2) = g(6e^{-1}) = 30e^{-1} - 2$ A1

(iv) $f^{-1}g(x) = f^{-1}(5x - 2) = 1 + \ln \frac{5x-2}{3}$ M1 A1

$\therefore\ 1 + \ln \frac{5x-2}{3} = 4, \quad \frac{5x-2}{3} = e^3$ M1

$x = \frac{1}{5}(3e^3 + 2)$ A1 **(11)**

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8. *(i)* $\frac{dy}{dx} = 2x - \frac{1}{2}(4 + \ln x)^{-\frac{1}{2}} \times \frac{1}{x} = 2x - \frac{1}{2x\sqrt{4 + \ln x}}$ M1 A1

$x = 1$, $y = -1$, grad $= \frac{7}{4}$ A1

$\therefore\ y + 1 = \frac{7}{4}(x - 1)$ M1

$4y + 4 = 7x - 7$

$7x - 4y = 11$ A1

(ii) SP: $2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0$ M1

let $f(x) = 2x - \frac{1}{2x\sqrt{4 + \ln x}}$

$f(0.3) = -0.40$, $f(0.4) = 0.088$ M1

sign change, $f(x)$ continuous $\therefore$ root A1

(iii) $2x - \frac{1}{2x\sqrt{4 + \ln x}} = 0 \quad \Rightarrow \quad 2x = \frac{1}{2x\sqrt{4 + \ln x}}$

$x^2 = \frac{1}{4\sqrt{4 + \ln x}} = \frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}$ M1

$x = \sqrt{\frac{1}{4}(4 + \ln x)^{-\frac{1}{2}}} = \frac{1}{2}(4 + \ln x)^{-\frac{1}{4}}$ A1

(iv) $x_1 = 0.381512$, $x_2 = 0.378775$, $x_3 = 0.378999$, M1 A1

$x_4 = 0.378981$, $x_5 = 0.378982$, $\therefore\ \alpha = 0.37898$ (5dp) A1 **(13)**

Total **(72)**