Core Mathematics 3 Paper C

1. The region bounded by the curve $y = x^2 - 2x$ and the x-axis is rotated through 360° about the x-axis.

Find the volume of the solid formed, giving your answer in terms of π. [5]

2. *(i)* Solve the equation

$$\ln (3x + 1) = 2$$

giving your answer in terms of e. [2]

(ii) Prove, by counter-example, that the statement

"$\ln (3x^2 + 5x + 3) \geq 0$ for all real values of x"

is false. [5]

3. Differentiate each of the following with respect to x and simplify your answers.

(i) $\ln (3x - 2)$ [2]

(ii) $\dfrac{2x+1}{1-x}$ [3]

(iii) $x^{\frac{3}{2}} e^{2x}$ [3]

4. *(i)* Given that $\cos x = \sqrt{3} - 1$, find the value of $\cos 2x$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

(ii) Given that

$$2 \cos (y + 30)° = \sqrt{3} \sin (y - 30)°,$$

find the value of $\tan y$ in the form $k\sqrt{3}$ where k is a rational constant. [5]

5. The functions f and g are defined by

$$f(x) \equiv x^2 - 3x + 7, \quad x \in \mathbb{R},$$

$$g(x) \equiv 2x - 1, \quad x \in \mathbb{R}.$$

(i) Find the range of f. [3]

(ii) Evaluate $gf(-1)$. [2]

(iii) Solve the equation

$$fg(x) = 17.$$ [4]

6. *(i)* Express $4\sin x + 3\cos x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(ii) State the minimum value of $4\sin x + 3\cos x$ and the smallest positive value of x for which this minimum value occurs. [3]

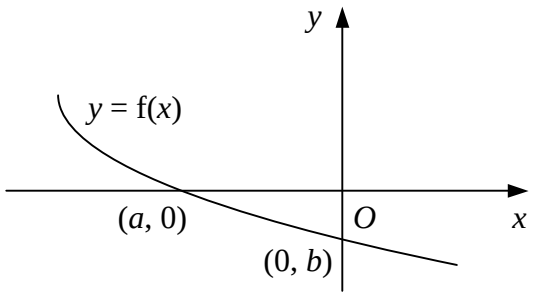
(iii) Solve the equation

$$4\sin 2\theta + 3\cos 2\theta = 2,$$

for θ in the interval $0 \leq \theta \leq \pi$, giving your answers to 2 decimal places. [4]

Turn over

7.



The diagram shows the graph of $y = \mathrm{f}(x)$ which meets the coordinate axes at the points $(a, 0)$ and $(0, b)$, where a and b are constants.

(a) Showing, in terms of a and b, the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

(i) $y = \mathrm{f}^{-1}(x)$, [2]

(ii) $y = 2\mathrm{f}(3x)$. [3]

Given that

$$\mathrm{f}(x) = 2 - \sqrt{x+9}, \quad x \in \mathbb{R}, \quad x \geq -9,$$

(b) find the values of a and b, [3]

(c) find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. [4]

8. The curve C has the equation $y = \sqrt{x} + \mathrm{e}^{1-4x}$, $x \geq 0$.

(i) Find an equation for the normal to the curve at the point $(\frac{1}{4}, \frac{3}{2})$. [4]

The curve C has a stationary point with x-coordinate α where $0.5 < \alpha < 1$.

(ii) Show that α is a solution of the equation

$$x = \tfrac{1}{4}[1 + \ln(8\sqrt{x})].$$ [3]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{4}[1 + \ln(8\sqrt{x_n})],$$

with $x_0 = 1$ to find x_1, x_2, x_3 and x_4, giving the value of x_4 to 3 decimal places. [2]

(iv) Show that your value for x_4 is the value of α correct to 3 decimal places. [2]

(v) Another attempt to find α is made using the iterative formula

$$x_{n+1} = \tfrac{1}{64}\mathrm{e}^{8x_n - 2},$$

with $x_0 = 1$. Describe the outcome of this attempt. [2]