Core Mathematics 3 Paper B

1. Find the set of values of x such that

$$|2x - 3| > |x + 2|. \quad [5]$$

2. Find, to 2 decimal places, the solutions of the equation

$$3\cot^2 x - 4\operatorname{cosec} x + \operatorname{cosec}^2 x = 0$$

in the interval $0 \le x \le 2\pi$. [6]

3. A curve has the equation $x = y^2 - 3\ln 2y$.

(i) Show that

$$\frac{dy}{dx} = \frac{y}{2y^2 - 3}. \quad [3]$$

(ii) Find an equation for the tangent to the curve at the point where $y = \frac{1}{2}$.
Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [3]

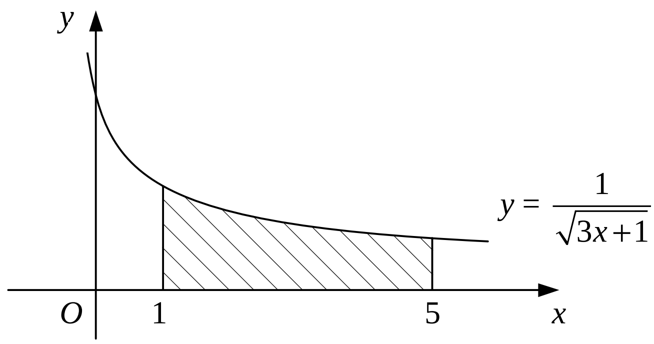
4. *(i)* Use Simpson's rule with four intervals, each of width 0.25, to estimate the value of the integral

$$\int_0^1 x\mathrm{e}^{2x}\ dx. \quad [3]$$

(ii) Find the exact value of the integral

$$\int_{\frac{1}{2}}^1 \mathrm{e}^{1-2x}\ dx. \quad [4]$$

5.



The diagram shows the curve with equation $y = \dfrac{1}{\sqrt{3x+1}}$.

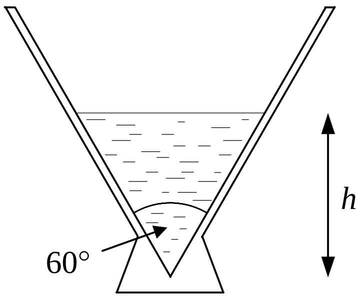
The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 5$.

(i) Find the area of the shaded region. [4]

The shaded region is rotated through four right angles about the x-axis.

(ii) Find the volume of the solid formed, giving your answer in the form $k\pi \ln 2$. [4]

6.



The diagram shows a vertical cross-section through a vase.

The inside of the vase is in the shape of a right-circular cone with the angle between the sides in the cross-section being 60°. When the depth of water in the vase is h cm, the volume of water in the vase is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$. [2]

The vase is initially empty and water is poured in at a constant rate of 120 cm^3 s^{-1}.

(b) Find, to 2 decimal places, the rate at which h is increasing

(i) when $h = 6$, [5]

(ii) after water has been poured in for 8 seconds. [2]

Turn over

7. *(i)* Prove that, for $\cos x \neq 0$,

$$\sin 2x - \tan x \equiv \tan x \cos 2x.$$ [5]

(ii) Hence, or otherwise, solve the equation

$$\sin 2x - \tan x = 2 \cos 2x,$$

for x in the interval $0 \leq x \leq 180°$. [4]

8. A rock contains a radioactive substance which is decaying. The mass of the rock, m grams, at time t years after initial observation is given by

$$m = 400 + 80\mathrm{e}^{-kt},$$

where k is a positive constant.

Given that the mass of the rock decreases by 0.2% in the first 10 years, find

(i) the value of k, [5]

(ii) the value of t when $m = 475$, [2]

(iii) the rate at which the mass of the rock is decreasing when $t = 100$. [4]

9. $$\mathrm{f}(x) = 3 - \mathrm{e}^{2x}, \quad x \in \mathbb{R}.$$

(i) State the range of f. [1]

(ii) Find the exact value of ff(0). [2]

(iii) Define the inverse function $\mathrm{f}^{-1}(x)$ and state its domain. [3]

Given that α is the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$,

(iv) explain why α satisfies the equation

$$x = \mathrm{f}^{-1}(x),$$ [2]

(v) use the iterative formula

$$x_{n+1} = \mathrm{f}^{-1}(x_n)$$

with $x_0 = 0.5$ to find α correct to 3 significant figures. [3]